OUT-OF-EQUILIBRIUM PHASE TRANSITIONS FOR THE LARGE SCALES OF 2D TURBULENCE

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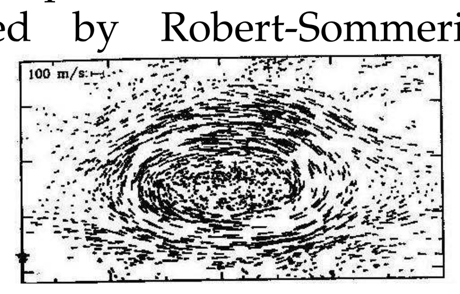
Statistical Mechanics of Large Scale Geophysical Flows

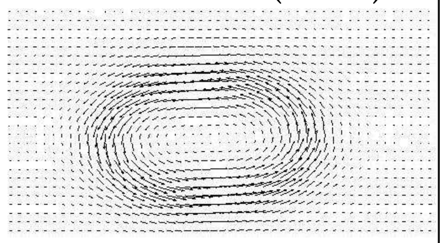
Large scale statistics of turbulent flows

In many applications of fluid dynamics, one of the most important problem is the prediction of the very high Reynolds' large-scale flows. The highly turbulent nature of such flows, for instance ocean circulation or atmosphere dynamics, renders a probabilistic description desirable, if not necessary. A statistical mechanics explanation of the self-organization of geophysical flows has been proposed by Robert-Sommeria and Miller (RSM).

The RSM theory has been successfully applied to the Jupiter's troposphere : cyclones, anticyclones and jets have been quantitatively described by this theory (F. Bouchet and J. Sommeria)



Observation (Voyager)


Statistical Equilibrium

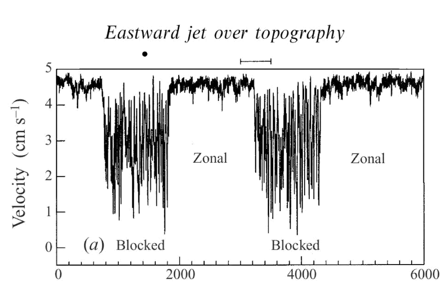
Velocity field of Jupiter's Great Red Spot

For applications : statistical mechanics beyond equilibrium ?

The RSM theory starts from the conservative dynamics and parametrizes equilibria by the energy and other dynamical invariants. However, the theory does not predict the long-term effects of the forcing, which is a relevant issue for any application. It is a practical and fundamental problem to understand how the invariants are selected by the presence of a weak forcing and dissipation, what are the associated fluctuations, are all forcings compatible with RSM equilibria ? The relaxation towards equilibrium of 2-D flows has been considered in the past, however the out-of-equilibrium statistical mechanics has never been considered yet. From a statistical mechanics point of view, this problem is a logical continuation of the RSM theory.

Out-of-Equilibrium Phase Transitions in Geophysics

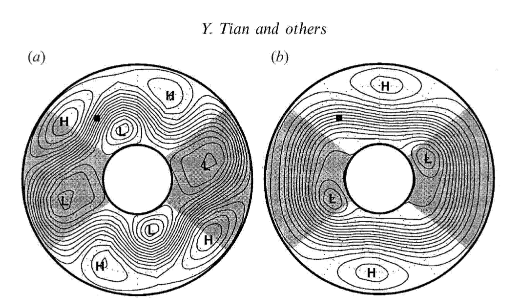
In many turbulent geophysical flows, one can see transitions, at random time, between two states with different large scale flow. The most famous example are probably the time reversal of the earth magnetic field, or the Milankovitch cylcle for the earth ice age cycles. Such general phenomena correspond to systems with a large number of degrees of freedom. The case of simple turbulent flows may be studied in much details theoreticaly and numericaly.



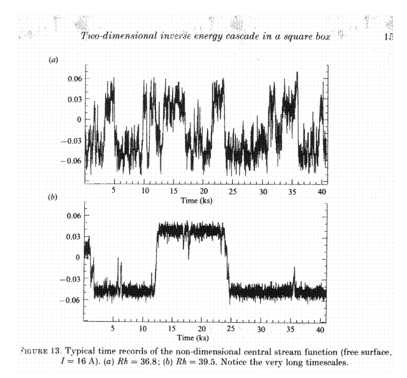
Velocity signal

Rotating tank experiment, where a transition from a blocking state to a zonal state is observed.

Y. Tian and col, J. Fluid. Mech. (2001)

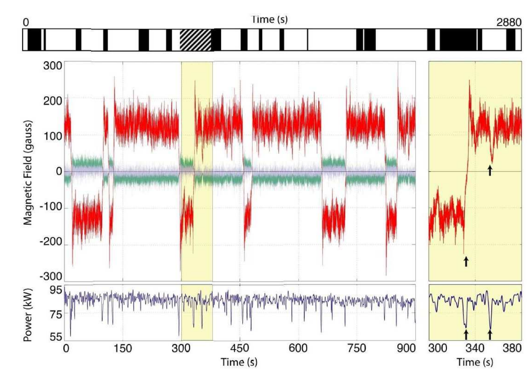


Stream function for both states


J. Sommeria, JFM (1986)

Left : Amplitude of the first mode vs time for an experimental 2D flow, in a square box.

Right : Earth (top) and experimental (bottom) magnetic field reversal (VKS experiment).


M. Berhanu and col. arxiv:physics/0701076

The Stochastic Navier-Stokes Equation

We consider the 2-D Navier-Stokes equation with weak stochastic forcing and dissipation (Euler limit).

$$\frac{\partial w}{\partial t} + \mathbf{u}.\nabla w = \nu \Delta w - \alpha w + f_d + f_s$$

where f_d is deterministic, f_s is a random force. This is the usual framework of turbulence studies. However, we are interested on large scales :

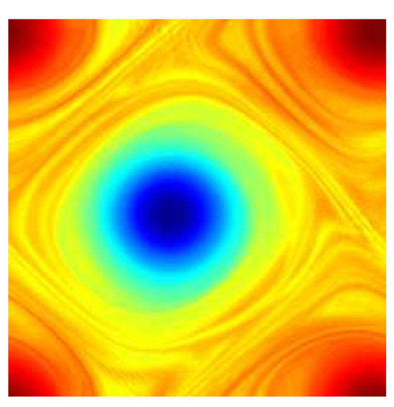
- We don't care with self-similar behavior
- Because it is relevant for applications, our forcing is not localized in Fourier space
- We use very small Rayleigh friction, to observe the large scale energy condensation

We study the out-of-equilibrium invariant measure resulting from the statistical balance between forcing and dissipation. Some recent mathematical results : S. Kuksin (Sinai, Shirikyan, Bricmont, Kupianen, ...) :

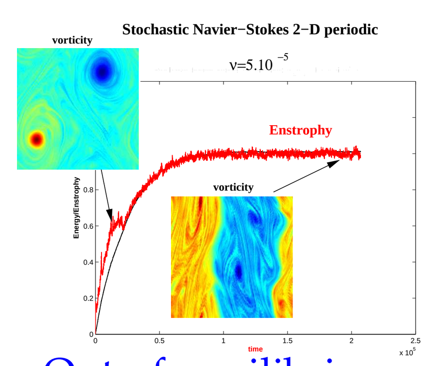
- Existence of a stationary measure μ_ν. Existence of $lim_{\nu \to 0}\ \mu_\nu$
- In this limit, almost all trajectories are solutions of the Euler equation

We would like to obtain more physical results : what determines the large scale flow, is the measure concentrated close to RSM equilibria, what is the fluctuations level, ... ?

The forcing : $f_S(\mathbf{x},t) = \sum_{\mathbf{k}} f_{\mathbf{k}} \eta_{\mathbf{k}}(t) e_{\mathbf{k}}(\mathbf{x})$ where $e_{\mathbf{k}}$'s are the Fourier modes and $< \eta_{\mathbf{k}}(t)\eta_{\mathbf{k}'}(t') >= \delta_{\mathbf{k},\mathbf{k}'}\delta(t-t')$ (white in time). For instance $f_{\mathbf{k}} = A \exp -\frac{(|\mathbf{k}|-m)^2}{2\sigma^2}$ with $\frac{1}{2}\sum \frac{|f_{\mathbf{k}}|^2}{|\mathbf{k}|^2} = 1$ (smooth in space).

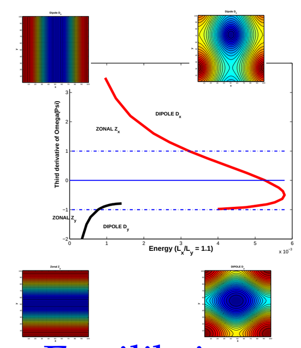

Dipole stationary state (vorticity)

Out-of-Equilibrium Flows and the RSM Theory



Out-of-equilibrium (DNS of Navier Stokes)

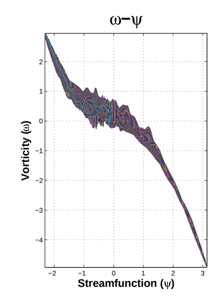
These two figures help to compare the predictions of equilibrium statistical mechanics (conservative) to the Quasi-Stationary large scale flows obtained in the Navier-Stokes equation (dissipative) with random forcing. It illustrates that the **qualitative** properties of the later can be understood using the former.

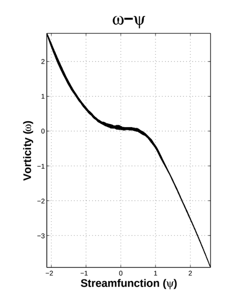

Equilibrium Phase Diagram

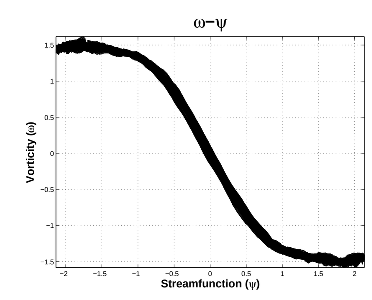
The convergence towards equilibrium occurs on a time scale scaling like ν.

Vorticity-Streamfunction Relation

Are out-of-equilibrium flow close to steady states of the Euler equation ? In order to addrees this, we plot the vorticity-streamfunction relation, which is a curve for Euler equilibria.

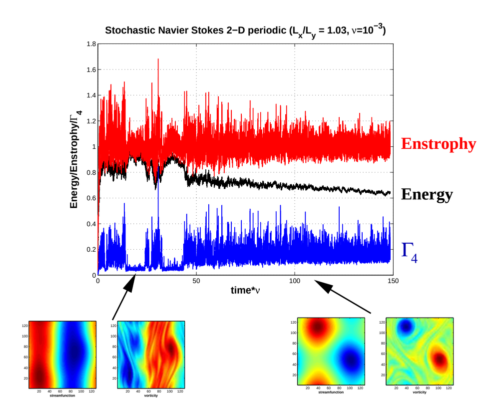

Dipole (snapshot)


Dipole (Temporal average)

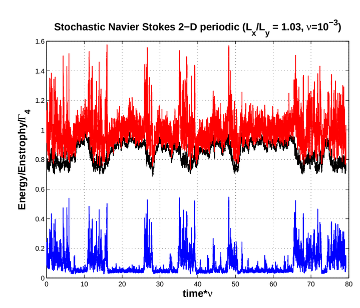

Zonal (unidirectional) flow

We are close to some conservative steady states. The zonal case seems to show a non-monotonic vorticity-streamfunction relationship. Such states would be uncompatible with RSM equilibria (To be confirmed by computation at lower ν),

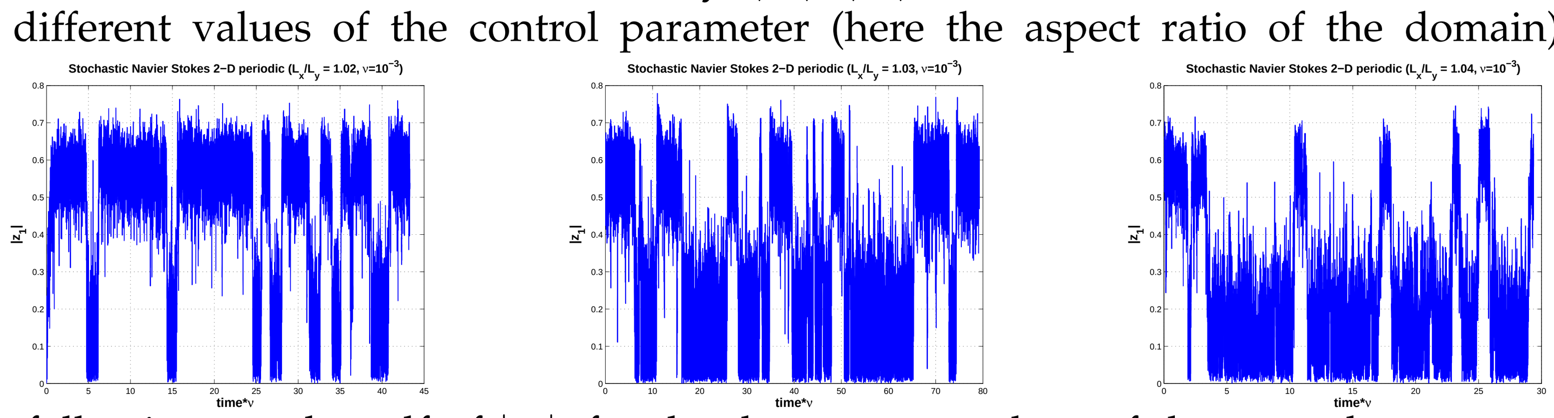
Out-of-Equilibrium Phase Transition



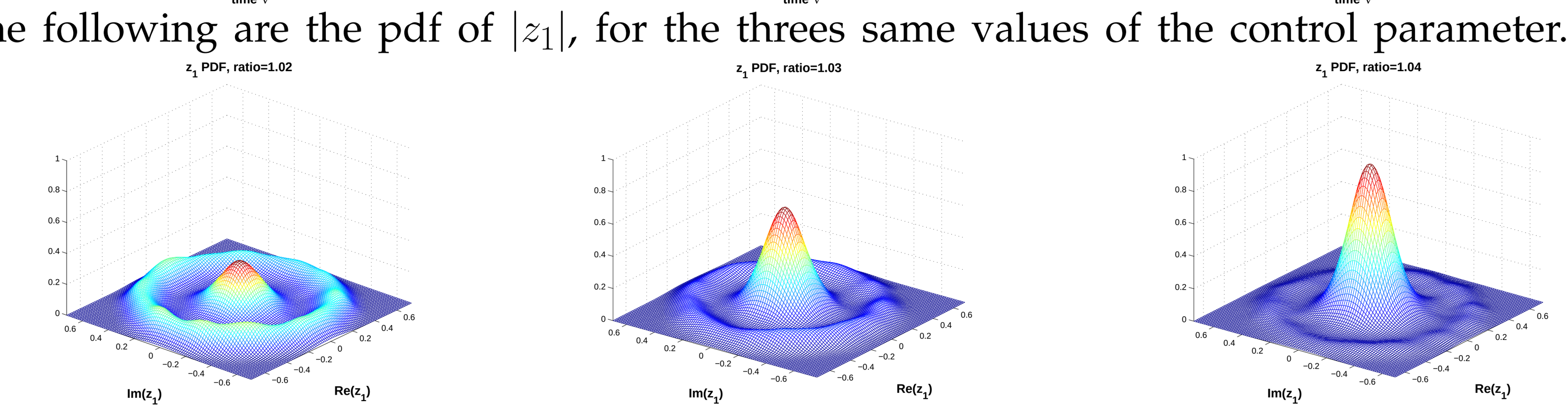
Left : Energy (black), enstrophy (red) and fourth order moment of the vorticity (blue), versus time. This clearly illustrates an out-of-equilibrium dipole-zonal phase transition.


Idem, in a stationary situation

In order to study this transition, we study the time evolution of the modulus of the first Fourrier coefficient for the vorticity, $|z_1|$ ($|z_1|$ is close to zero for zonal states), for different values of the control parameter (here the aspect ratio of the domain).



The following are the pdf of $|z_1|$, for the threes same values of the control parameter.



Conclusion

- The large scales of out-of-equilibrium flows are close to Euler steady states.
- What is the link between the control parameters (forcing) and the observed flows ?
- This requires a theory (the RSM theory only gives a qualitative understanding)
- Letting the energy pile up to larger scales may lead to very interesting phenomena
- Poorly studied by experimentalists, and poorly studied by numericians
- Probably very relevant for geophysical applications (with other models)

ANR Statflow

This project is funded by the french agency ANR. It is led by a physicist of the **statistical mechanics of geophysical flows** (F. Bouchet, sec. 02), by a specialist of the **numerical stability of 2-D and 3-D flows** (F. Gallaire, sec. 10), by a **mathematician of geophysical flows and kinetic theory** (F. Rousset, sec. 01) and a specialist of the low frequency variability of **ocean dynamics** (E. Simonnet, sec. 19)

F. Bouchet and E. Simonnet are based at INLN (Nice/Sophia Antipolis) and F. Gallaire and F. Rousset at the Nice university.