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Ghys, Étienne (F-ENSLY-PM)

★**A singular mathematical promenade.**

ENS Éditions, Lyon, 2017. *viii*+302 *pp. ISBN* 978-2-84788-939-0

It is hard to think of a better title than *A singular mathematical promenade* for Étienne Ghys's recent book. Indeed, it feels like a stroll—with some singularities—through a mathematical landscape with a friendly guide. He points out a bird here, a tree there, and tells you a story about the person who dug the pond in the middle. As a guide, Ghys is wise, curious, and generous. His generosity is in fact quite literal: the book is available as a PDF for free from his website, and he has released it and the figures he created for it into the public domain. Both the paper and the PDF versions have their advantages. The paper version allows for easier perusing and fuller appreciation of the lovely illustrations, but the PDF may be easier to transport, and it can be (and has been) updated.

Ghys begins the book with an engaging nugget of a theorem Maxim Kontsevich doodled for him on a Paris metro ticket: there do not exist four polynomials $P_1(x), P_2(x), P_3(x), P_4(x)$ such that in a small neighborhood of the origin, $P_1(x) > P_2(x) > P_3(x) > P_4(x)$ when $x < 0$ and $P_2(x) > P_4(x) > P_1(x) > P_3(x)$ when $x > 0$. In the book, Ghys spins out the story to a much more general one about singularities of algebraic curves in the plane, taking a look at how the ideas developed historically. The plentiful illustrations are both mathematically helpful and visually engaging. Several chapters are fairly independent from the rest of the book, side trails from the main promenade, and can be skipped or saved for later if desired.

A promenade may be a walk in the park, but it is good to remember that a walk in the park can be as strenuous as you'd like. If you want, you can read the book in a leisurely way, letting the theorems wash over you and not worrying too much about the details. Alternatively, you can dig into the nuts and bolts, pondering the stimulating questions Ghys includes and filling in the blanks in sketched proofs. Ghys says he wrote the book for one specific reader: himself as an undergraduate. It would probably be a good jumping-off point for independent studies for both undergraduate and graduate students.

In some ways, *A singular mathematical promenade* feels like a Mathematical Review of the topic of singularities of planar curves. Ghys summarizes the story, pointing the reader to technical details in the extensive (and—because they are in the margins instead of being hidden in footnotes or endnotes—easy to notice and fit into one's mental map of the topic) references. As I was reading it, my notes became more or less an evergrowing list of other references I wanted to consult. (One I haven't quite gotten to yet: why did d'Alembert include a proof of the fundamental theorem of algebra in a paper about the cause of winds?) At the end of one chapter, Ghys concludes with this advice: "stop reading this book and read (some of) Euler's papers. Now!" Perhaps I should now follow suit. Stop reading this review and read Ghys's book. Now! *Evelyn James Lamb*