

## A singular mathematical promenade

This book is a small jewel for a mathematical library. As the author defines it, it is a promenade into the mathematical world. With the excuse of a simple mathematical question, the author enters into many realms of mathematics, some of them almost recreative, other bits are truly deep mathematics concepts, all of that flavoured with historical review of many developments in topology, geometry, algebra and analysis.

First the question proposed by the author: if one has a collection of real polynomials  $P_1(x), \dots, P_k(x)$ , that are ordered in some way for small  $x > 0$ , and in other way for small  $x < 0$ , how this ordering may change? This is rewritten as how the branches of real algebraic curves are cyclically ordered at a point. Surprisingly, there is a neat characterization (not any order is allowed), and one may count how many possibilities are, with nice combinatorics popping up. The book explores very different questions related to this problem, and follows on different ramifications. Questions in combinatorics, algebra, number theory, geometry and topology, algebraic topology, algebraic geometry, complex variables, analysis, come to the surface at every moment. The book is at no point shallow in the content: there are very deep mathematics told at a fairly easy level. A professional mathematician can (and will) learn a lot from this text. On the other hand, the book does not contain hard edge-cutting research with new theorems, although the last chapters have a more modern flavour directing to the proof of the main result on the order of the branches of a real analytic singularity. The divulgative nature of the book does not mean that one will encounter the typical stories for spreading maths to the general public. Certainly, the book is addressed to a person with knowledge at the level of undergraduate student in mathematics, increasing slowly and steadily where the last part contains results at the graduate level and beyond.

One very strong aspect of the book is the review of historical matters. The author has done good work in accounting on the history of some very classical notions (just to name a few, the fundamental theorem of algebra, or the theory of Puiseux series, the linking number of knots, but there are very many), and he has made a very loable effort (and very successful indeed!) to transmitting this in a friendly, amusing and rigorous manner. Notions of combinatorics, discrete mathematics (like the analysis of trees), algebra (operads), algebraic geometry (resolution of curve singularities, complex singularities), are explained in an enlightening way, changing mathematical rigour for clarifying drawings in a wise choice.

The layout of the book is also impressive: it contains many nice photographs, pictures and drawings, also scans of manuscripts, historical bibliographic references, references of papers and books on mathematical concepts, clarifying remarks, all distributed in a column at the right of the page (so that one has the information immediately available). The text contains many historical quotations in different languages, with translations, and interesting analysis of the mathematics of our "classics" (Newton, Gauss, Hipparchus, are among hundreds of other names mentioned along the text).

One last word. The book is available for free at the arxiv: <https://arxiv.org/ftp/arxiv/papers/1612/1612.06373.pdf>

It is published with a price of 27 euros. Just let me say that, after reading the copy of my library for writing this review, I will buy a personal copy for myself. I really enjoyed it!

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### Book details

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