MR1703323 (2000j:22014) 22E40 (57M60 57S25 57S30)
Ghys, Étienne (F-ENSLY-PM)
Actions de réseaux sur le cercle. (French) [Lattice actions on the circle]
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A lattice Γ in a connected semisimple Lie group G is called irreducible if there does not exist a nontrivial locally direct decomposition $G = G_1 G_2$ such that $(G_1 \cap \Gamma)(G_2 \cap \Gamma)$ is of finite index in Γ. Suppose that the real rank of G is ≥ 2 and that G has no simple factors isomorphic to $\mathrm{PSL}(2, \mathbf{R})$. Let an action of an irreducible lattice $\Gamma \subset G$ by orientation-preserving diffeomorphisms of $\mathbf{S}^1$ be given. It is proved that the action is reduced to an action of a finite cyclic quotient group of Γ. This main result is actually a consequence of a general theorem concerning actions of an irreducible lattice $\Gamma \subset G$ by orientation-preserving homeomorphisms of $\mathbf{S}^1$, supposing that the real rank of G is ≥ 2. Such an action either preserves a probability measure on $\mathbf{S}^1$, or is conjugate (in a certain weak sense) to an action induced by a surjection $G \to \mathrm{PSL}(2, \mathbf{R})$ and by the projective action of $\mathrm{PSL}(2, \mathbf{R})$ on $\mathbf{S}^1$. The main part of the proof is the verification of this property for each non-compact simple group of real rank ≥ 2 and for $\mathrm{PSL}(2, \mathbf{R}) \times \mathrm{PSL}(2, \mathbf{R})$.

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