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Gambaudo, Jean-Marc (F-DJONST-IM); **Ghys, Étienne** (F-ENSLY-PM)
Braids and signatures. (English, French summaries)
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For a complex number $\omega = e^{i\theta} \neq 1$ and a braid $\beta \in B_n$, the ω-signature of the link closure $\widehat{\beta}$ defines a map $\text{sign}_\omega \colon B_n \to \mathbf{Z}$. This map is not a homomorphism, and this is related to nonadditivity of the signature under gluing two 4-manifolds with corners. For 4-manifolds $M = M_1 \cup M_2$ glued along submanifolds of their boundary, the Novikov-Wall formula expresses the defect $\sigma(M) - \sigma(M_1) - \sigma(M_2)$ as a Maslov triple index, and the authors apply this to understand the defect $\text{sign}_\omega(\widehat{\alpha \cdot \beta}) - \text{sign}_\omega(\widehat{\alpha}) - \text{sign}_\omega(\widehat{\beta})$ for $\alpha, \beta \in B_n$. Their main result equates this defect with the Meyer cocycle of the Burau-Squier representation $\mathcal{B}_\omega$ specialized to ω in case ω is a root of unity.

For $n = 3$, they show that the sum $\text{sign} + \frac{2}{3}\text{lk}_3$ descends to a function on $B_3/\langle \Delta_3^4 \rangle \cong \text{SL}(2, \mathbf{Z})$ which coincides with $-\frac{1}{3}$ times the Rademacher function.

They use their result to shed light on the global geometry on the Gordian metric space of knots. Any two knots $f_0, f_1 \colon \mathbf{S}^1 \to \mathbf{R}^3$ can be connected by a homotopy of immersions $(f_t)_{t\in[0,1]} \colon \mathbf{S}^1 \looparrowright \mathbf{R}^3$ such that each f_t has at most one generic double point. The Gordian distance between two knots is the minimum, over all such homotopies, of the total number of double points. The authors prove that the Gordian metric space contains quasi-Euclidean subspaces of arbitrary dimension.

Reviewed by *Hans U. Boden*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.