MR880511 (88b:57032) 57R30
Ghys, Étienne (F-LILL)
Classe d'Euler et minimal exceptionnel. (French) [Euler classes and exceptional minimal sets]
Topology 26 (1987), no. 1, 93-105.
Let $\Sigma$ be a compact, oriented surface of nonpositive Euler characteristic. Each representation $\varphi: \pi_{1}(\Sigma) \rightarrow \operatorname{Diff}_{k}^{+}\left(S^{1}\right)$ defines a foliated circle bundle over $\Sigma$, many properties of which are related directly to the dynamics of $\varphi\left(\pi_{1}(\Sigma)\right)$ on $S^{1}$. In particular, the foliation has an exceptional minimal set if and only if there is a $\varphi\left(\pi_{1}(M)\right)$-invariant Cantor set $K \subset S^{1}$, each orbit of which is dense in $K$.
The author studies the relationship between the Euler number $\operatorname{eu}(\varphi)$ of the circle bundle and the existence of such a minimal Cantor set $K$. His main results are that, if $k \geq 2$, a necessary condition for the existence of $K$ is that $|\mathrm{eu}(\varphi)|<|\chi(\Sigma)|$ and that, if $k=\omega$, it is necessary that eu $(\varphi)=0$. It is a well-known result of Milnor and Wood that, in any case, $|\mathrm{eu}(\varphi)| \leq|\chi(\Sigma)|$. A $C^{\infty}$ construction due to the author and V . Sergiescu exhibits a representation $\varphi$ with an exceptional minimal set, having eu $(\varphi)=1$ and $\chi(\Sigma)=-22$ [Comment. Math. Helv. 62 (1987), no. 2, 185-239].

Reviewed by Lawrence Conlon
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