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Classification des feuilletages totalement géodésiques de codimension un. (French)
[Classification of totally geodesic foliations of codimension one]

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Given a transversely orientable, C^∞, codimension 1 foliation $\mathcal{F}$ on a closed orientable manifold M^n, when does there exist a Riemannian metric on M such that all leaves of $\mathcal{F}$ are totally geodesic submanifolds? The answer was known in dimension 3 [Y. Carriere and the author, An. Acad. Brasil. Cienc. 53 (1981), no. 3, 427–432; MR0663239 (83m:57019)]. Here it is proved in any dimension, namely, that such a metric exists if and only if either $\mathcal{F}$ is transverse to the orbits of a locally free circle action (generalized Seifert fibration), or $\mathcal{F}$ is differentiably conjugate to a "model foliation"; these model foliations, explicitly constructed by the author, are a generalization of the well-known Anosov foliations on T^2-bundles over the circle: in particular, they are transverse to the fibers of a torus bundle, and on each fiber they induce a linear foliation with dense leaves. There is a generalization of the theorem to noncompact manifolds, provided either $\pi_1 M$ is finitely generated or one considers only analytic foliations.

The proof relies heavily on the fact that, if $\mathcal{F}$ is totally geodesic, then the orthogonal foliation $\mathcal{F}^\perp$ is Riemannian. It is known that to $(\mathcal{F},\mathcal{F}^\perp)$ is naturally associated a principal $\mathrm{SO}(n-1)$-bundle $p\colon \hat{M}\to M$ such that the closures of the orbits of $[p^{-1}(\mathcal{F})]^\perp$ define a fibration of $\hat{M}$ by tori. One of the main steps in the proof consists in showing that, if $\mathcal{F}$ is not transverse to a circle action, then the structure group of this fibration can be reduced to a discrete group (possibly after changing the metric on M).

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