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Cocycles bornés et actions de groupes sur les arbres réels. (French) [Bounded cycles and actions of groups on real trees]

Group theory from a geometrical viewpoint (*Trieste*, 1990), 617–621, *World Sci. Publ.*, *River Edge*, *NJ*, 1991.

In this paper the authors consider the question of groups acting on $\mathbf{R}$-trees. It is well known that groups acting freely on combinatorial (simplicial) trees are free. Here the analogous question is which (finitely generated) groups can act freely on real trees. Such groups are the group $\mathbf{Z}^n$ and free products of free abelian groups and groups of orientable surfaces. The main result, in group-theoretic terminology, goes as follows: Let Γ be a group, Γ' its derived subgroup and $\gamma \in \Gamma'$. Let $|\gamma|$ be the minimum number of commutators with product γ. Let $\|\gamma\| = \lim_{n\to\infty}(1/n)|\gamma^n|$ be the "stable length of commutators" for γ. Let Γ be a nonabelian group with zero stable length of commutators on Γ'. Then the group does not act freely on a real tree. Recently E. Rips has announced that the only finitely generated groups which act freely on real trees are indeed free products of free abelian groups and surface groups.

Reviewed by *S. Andreadakis*