MR766280 (86c:57025) 57R30 (53C12 58F17)

Ghys, Étienne (F-LILL)

Feuilletages riemanniens sur les variétés simplement connexes. (French. English summary)
[Riemannian foliations on simply connected manifolds]

Ann. Inst. Fourier (Grenoble) **34** (1984), *no. 4,* 203–223.

In this brilliant paper, the author gives important properties of Riemannian foliations on simply connected closed manifolds.

His first result (Theorem A) says that, if $(M,\mathfrak{F})$ is such a foliation, there exists an arbitrarily near Riemannian foliation with all leaves compact ("generalized Seifert fibration"); moreover, if the Euler-Poincaré characteristic $\chi(M)$ is nonzero, then $\mathfrak{F}$ admits a compact leaf. The demonstration involves general structure theorems for Riemannian foliations, and the fact that $\pi_1(M)=0$ implies that the "structural Lie algebra" of $(M,\mathfrak{F})$ is abelian. A special proof is necessary in the case of codimension 2, because one uses the lifted foliation in the bundle of transverse orthonormal frames, and $\pi_1(\mathrm{SO}(q,\mathbf{R}))$ is infinite if $q=2$.

The second result (Theorem B) asserts that such a foliation is minimalizable. This property is obtained by using Rummler's characteristic [H. Rummler, Comment. Math. Helv. **54** (1979), no. 2, 224–239; MR0535057 (80m:57021)], and a generalization of Moser's lemma [J. Moser, Trans. Amer. Math. Soc. **120** (1965), 286–294; MR0182927 (32 #409)].

Finally (Theorem C), in the case where M is a sphere, either $\mathfrak{F}$ is a generalized Seifert fibration, or $\mathfrak{F}$ is an isometric flow, and admits a compact orbit. If M is a complex or quaternionic projective space, then $\mathfrak{F}$ is a generalized Seifert fibration. The argument used here involves the Wang sequence of a fibration over a sphere.

Reviewed by *P. Molino*