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Groupes d'holonomie des feuilletages de Lie. (French) [Holonomy groups of Lie foliations]
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Following A. Haefliger's point of view [see J. Differential Geom. **15** (1980), no. 2, 269–284; MR0614370 (82j:57027)], to a foliation $\mathfrak{F}$ on a manifold M is associated an equivalence class $[\mathfrak{H}]$ of pseudogroups (a representative of which is the pseudogroup $\mathfrak{H}$ induced by holonomy on a complete transversal submanifold). A natural problem is to study what sort of pseudogroup is this holonomy pseudogroup if M is a closed manifold.

In this paper, the author studies the case where $(M, \mathfrak{F})$ is a $\mathfrak{g}$-foliation on a closed manifold [E. Fedida, "Feuilletages du plan; feuilletages de Lie", Thèse, Univ. Louis Pasteur, Strasbourg, 1973; BullSig(110) 1974:2599; see *Differential topology, foliations and Gel′fand-Fuks cohomology* (Rio de Janeiro, 1976), 183–195, Lecture Notes in Math., 652, Springer, Berlin, 1978; see MR 80a: 57012]; if G is the simply connected Lie group with $\mathfrak{g}$ as Lie algebra, then $\mathfrak{H}$ is equivalent to a subgroup Γ of G. This "holonomy group" of $(M, \mathfrak{F})$ is a quotient of the fundamental group $\pi_1(M)$. Hence Γ is finitely generated. Moreover, as M fibers over $G/\overline{\Gamma}$, this homogeneous space is compact.

The author obtains a new property of such a subgroup Γ, involving the notion of "real cohomological dimension" of a CW-complex. If $\mathrm{rcd}(X)$ denotes this dimension, and if Γ is a group, then by definition, $\mathrm{rcd}(\Gamma) = \mathrm{rcd}(K(\Gamma, 1))$. The principal result of the author is the inequality $\mathrm{rcd}(\Gamma) \geq \dim G - \dim K$, where K is a maximal compact subgroup of G. Using this property, the author shows that, if G is the affine group of $\mathbf{R}$, and Γ is generated by elements $\left(\begin{smallmatrix} a_1 & b_1 \\ 0 & 1 \end{smallmatrix}\right), \cdots, \left(\begin{smallmatrix} a_l & b_l \\ 0 & 1 \end{smallmatrix}\right)$, a necessary condition in order that Γ may be the holonomy group of a $\mathfrak{g}$-Lie foliation is that $a_1, b_1, \cdots, a_l, b_l$ be algebraically dependent over $\mathbf{Q}$.

Reviewed by *P. Molino*