MR1040572 (91h:57015) 57R30 (58F18)

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L'invariant de Godbillon-Vey. (French) [The Godbillon-Vey invariant]

Séminaire Bourbaki, Vol. 1988/89.
Astérisque No. 177-178 (1989), *Exp. No.* 706, 155–181.

This is an excellent survey of the theory of the classical Godbillon-Vey invariant of compact, foliated manifolds $(M,\mathcal{F})$. If $\operatorname{codim}(\mathcal{F}) = 1$, this invariant is a de Rham class $\operatorname{gv}(\mathcal{F}) \in H^3(M;\mathbf{R})$, the geometric meaning of which was rather mysterious when the invariant was first introduced. It was conjectured by D. Sullivan and Moussu-Pelletier that, in the absence of leaves with exponential growth, $\operatorname{gv}(\mathcal{F}) = 0$. Several authors gave partial verifications of this conjecture, culminating in a theorem of G. Duminy (unfortunately, never published) that greatly sharpened the conjecture. Duminy showed that, in the absence of a resilient leaf, $\operatorname{gv}(\mathcal{F}) = 0$. A leaf L is resilient if it has a holonomy contraction which "captures" L itself. It is easily proven that such a leaf L, and every leaf that approaches L, has exponential growth, but it is also possible for nonresilient leaves to grow exponentially. Resiliency is a dynamical property analogous to "Smale's horseshoe", so $\operatorname{gv}(\mathcal{F})$ reflects deep properties of the topological dynamics of $\mathcal{F}$. In this survey, the author gives a detailed outline of Duminy's work and a less detailed but helpful outline of Hurder's generalization to higher codimension. Subsequently, he describes work of Connes on the ergodic theory of foliations (of codimension one) with $\operatorname{gv}(\mathcal{F}) \neq 0$ and closes with an extensive discussion of the problem of topological invariance of $\operatorname{gv}(\mathcal{F})$, again in codimension one. The healthy bibliography (49 items) makes this valuable survey even more useful.

Reviewed by *Lawrence Conlon*