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À propos d'un théorème de J.-P. Jouanolou concernant les feuilles fermées des feuilletages holomorphes. (French. English summary) [On a theorem of J.-P. Jouanolou concerning the closed leaves of holomorphic foliations]

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This paper presents an amazing proof of a result generalizing a theorem of J.-P. Jouanolou about closed leaves of holomorphic foliations. More precisely, in [Math. Ann. **232** (1978), no. 3, 239–245; MR0481129 (58 #1274)] Jouanolou considered a holomorphic codimension one foliation $\mathcal{F}$ on a compact, connected, complex manifold X and addressed the problem of finiteness of closed leaves of $\mathcal{F}$. By assuming that all holomorphic 1-forms on X are closed and that a certain morphism associated to the Hodge spectral sequence vanishes, Jouanolou showed that $\mathcal{F}$ has a finite number of closed leaves except when it admits a meromorphic first integral, in which case all leaves are closed (H. Cartan showed that the hypothesis concerning the 1-forms is unnecessary). In this paper Ghys drops all hypotheses in Jouanolou's result and proves the following: If $\mathcal{F}$ is a codimension one (possibly singular) holomorphic foliation on a compact, connected complex manifold, then $\mathcal{F}$ has only a finite number of closed leaves except when $\mathcal{F}$ admits a meromorphic first integral, in which case all leaves are closed. The proof is a very nice simplification of Jouanolou's original proof. The paper ends with some interesting and useful remarks, examples and questions on this subject.

Reviewed by *M. G. Soares*