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Rigidité différentiable des groupes fuchsiens. (French) [Differentiable rigidity of Fuchsian groups]

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Let Γ_g be the fundamental group of a closed oriented surface of genus $g \geq 2$. In the paper under review, the author considers representations $\Phi\colon \Gamma_g \to \mathrm{Diff}^r_+(S^1)$ into the orientation-preserving C^r-diffeomorphisms of the circle. For the usual presentation of Γ_g with generators $a_1, b_1, \cdots, b_g$, one chooses lifts $\widetilde{A_i}$ and $\widetilde{B_i}$ of $\Phi(a_i)$ and $\Phi(b_i)$ to diffeomorphisms of $\mathbf{R}$, and defines the Euler number $\mathrm{eu}(\Phi)$ to be the integer $\widetilde{A_1}\widetilde{B_1}\widetilde{A_1}^{-1}\cdots\widetilde{B_g}^{-1}$. J. Milnor and J. Wood proved that $\mathrm{eu}(\Phi)$ is at most $2g-2$, and $\mathrm{eu}(\Phi) = 2g-2$ if Φ is an imbedding with image a discrete subgroup of $\mathrm{PSL}(2,\mathbf{R})$. Up to conjugacy, such imbeddings parameterize the Teichmüller space of the surface.

The author's main theorem says that if $r \geq 3$ and $\mathrm{eu}(\Phi) = 2g-2$, then there is a diffeomorphism of the circle, of class C^r, which conjugates f to an imbedding onto a discrete cocompact subgroup of $\mathrm{PSL}(2,\mathbf{R})$. The existence of a homeomorphism conjugating Φ to have values in $\mathrm{PSL}(2,\mathbf{R})$ is a result of S. Matsumoto [Invent. Math. **90** (1987), no. 2, 343–358; MR0910205 (88k:58016)]. The author also notes that any two imbeddings as discrete cocompact subgroups of $\mathrm{PSL}(2,\mathbf{R})$ are conjugate by a homeomorphism of S^1, but are conjugate by a C^1-diffeomorphism only when they are conjugate in $\mathrm{PSL}(2,\mathbf{R})$, and hence represent the same point of Teichmüller space.

The proof of the main theorem involves examination of Anosov diffeomorphisms of the torus and Anosov flows on 3-manifolds. In particular, the author proves that an Anosov flow of class C^r ($r \geq 2$) on a closed 3-manifold whose stable and weakly unstable leaves are of class $C^{1,1}$ must be C^r-equivalent to either a quasi-Fuchsian flow or to the suspension of a diffeomorphism of the torus. In the final section the author gives an application of the main theorem to holomorphic deformations of Fuchsian groups.

Reviewed by *Darryl McCullough*