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Singularités des flots holomorphes. II. (French. English, French summaries) [Singularities of holomorphic flows. II]

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This work is a natural continuation of Part I [J. C. Rebelo, *Ann. Inst. Fourier (Grenoble)* **46** (1996), no. 2, 411–428; [MR1393520 \(97d:32059\)](#)], where the behavior of complete holomorphic vector fields on surfaces was considered. Let us recall that a holomorphic vector field on a complex manifold M is complete if it generates an action of the additive group \mathbf{C} on M . On the other hand, a semi-complete holomorphic vector field X , on an open subset U of a complex manifold M , is a vector field which generates a semi-global flow on U , that is, a holomorphic mapping $\Phi: \Omega \rightarrow U$, where Ω is an open subset of $\mathbf{C} \times U$, satisfying the usual conditions for a flow plus the following property: if $x \in U$ and T_i is a sequence of complex numbers such that $(T_i, x) \in \Omega$ and (T_i, x) approaches the boundary of Ω , then $\Phi(T_i, x)$ leaves any compact subset of U . An interesting property of a semi-complete vector field is that its restriction to an open subset is still semi-complete. The present paper displays a thorough study of semi-complete vector fields, with zero first jet at an isolated singularity, on complex surfaces. A list of normal forms, modulo holomorphic conjugacy, is given and one amazing consequence is the following: Let X be a holomorphic vector field on a compact complex surface M , and suppose X has an isolated singularity at which its first jet vanishes. Then M is Hirzebruch's surface F_n , $n \geq 0$.

Reviewed by *M. G. Soares*

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