AMERICAN MATHEMATICAL SOCIETY
MathSciNet Mathematical Reviews on the Web
Previous | Up | Next Article
Citations
From References: 3
From Reviews: 0

MR1316551 (95k:53039) 53C12 (58F18)
Brunella, Marco (I-BOLO); **Ghys, Étienne** (F-ENSLY)
Umbilical foliations and transversely holomorphic flows.
J. Differential Geom. **41** (1995), *no. 1,* 1–19.

A codimension-1 foliation $\mathcal{F}$ of a closed oriented Riemannian 3-manifold M is called umbilical if all its leaves are totally umbilical, i.e., if for each point $x \in M$ the second quadratic form on the leaf $\mathcal{F}_x$ at the point x is proportional to the induced metric on the tangent space to $\mathcal{F}_x$, or, equivalently, if the holonomy of the orthogonal 1-dimensional foliation $\mathcal{N}$ acts conformally on leaves of $\mathcal{F}$. It can easily be shown that the class of foliations which are umbilical for a certain metric on M coincides with the class of transversely holomorphic foliations. The purpose of this paper is to classify such foliations.

The authors obtain a complete solution of this problem by giving an exhaustive list of classes of such foliations. In particular, any umbilical foliation with dense leaves either corresponds to a Seifert fibration, or is the foliation determined by an algebraic hyperbolic automorphism of the 2-dimensional torus, or else is just a linear foliation of the 3-dimensional torus.

The proof is based on using the notion of a harmonic measure of a foliation due to L. Garnett [J. Funct. Anal. **51** (1983), no. 3, 285–311; MR0703080 (84j:58099)]. Namely, for any foliation of a compact Riemannian manifold M there exists a probability measure on M whose leafwise densities are harmonic functions of the leafwise Laplacians. For codimension-1 foliations one can also define conditional measures of the harmonic measure on the leaves of the orthogonal foliation $\mathcal{N}$. Taking the values of these conditional measures on arcs of $\mathcal{N}$ joining two leaves of $\mathcal{F}$ gives rise to functions on the leaves of $\mathcal{F}$ measuring "distance" between leaves of $\mathcal{F}$ in the orthogonal direction. Since for umbilical foliations the holonomy of $\mathcal{N}$ acts conformally on leaves of $\mathcal{F}$, in this case the "distance" functions are leafwise harmonic.

This fact is then used (this is the crucial point of the proof) to show that if $\mathcal{F}$ is an umbilical foliation of M with dense leaves, then the lifts of $\mathcal{F}$ and $\mathcal{N}$ to the universal cover of M are product foliations, which implies that $\mathcal{F}$ belongs to one of the three classes mentioned above. The case when $\mathcal{F}$ has an exceptional minimal set is treated by using the nucleus theorem, and the case when $\mathcal{F}$ has compact leaves by using a surgery technique.

Reviewed by *Vadim A. Kaĭmanovich*

© *Copyright American Mathematical Society 1995, 2006*