Aspects structurels et algorithmiques des ordres partiels sur les graphes

Soutenance de thèse

Jean-Florent Raymond

cotutelle entre l'**Université de Varsovie** (Pologne) et l'**Université de Montpellier** (France)

Directeurs de thèse: **Marcin Kamiński** (Univ. Varsovie) et **Dimitrios M. Thilikos** (LIRMM)

18 novembre 2016

Strukturalne i algorytmiczne aspekty relacji zawierania się w grafach

Obrona doktorska

Jean-Florent Raymond

cotutelle pomiędzy **Uniwersytet Warszawski** i **Uniwersytet w Montpellier**

Promotorzy: **Marcin Kamiński** (Uniw. Warszawski) i **Dimitrios M. Thilikos** (LIRMM)

18 listopada 2016

Structural and algorithmic aspects of partial orderings of graphs

PhD defense

Jean-Florent Raymond

cotutelle between **University of Warsaw** (Poland) and **University of Montpellier** (France)

Advisors: **Marcin Kamiński** (Univ. Warsaw) and **Dimitrios M. Thilikos** (LIRMM)

18th of November 2016

Outline

1. Ordering objects
2. Exclusion theorems
3. Well-quasi-ordering
4. The Erdős–Pósa property

Outline

Ordering objects

raspberry grape cherry

Ordering objects

raspberry grape cherry

raspberry $\geqslant$ grape

cherry $\geqslant$ grape

Ordering objects

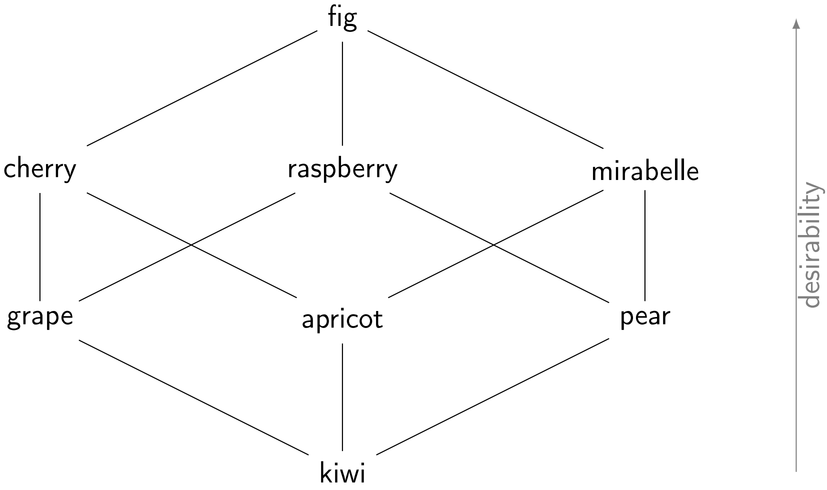
raspberry grape cherry

raspberry $\geqslant$ grape

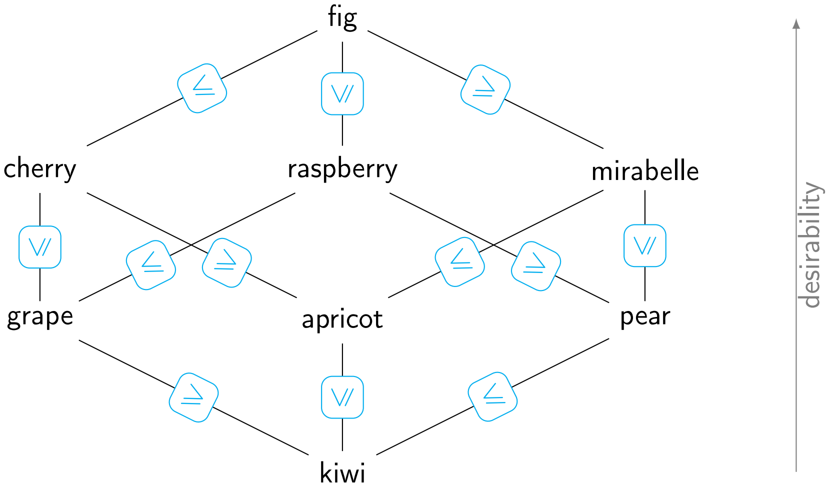
cherry $\geqslant$ grape

raspberry $\not\geqslant$ $\not\leqslant$ cherry

Ordering fruits



Ordering fruits



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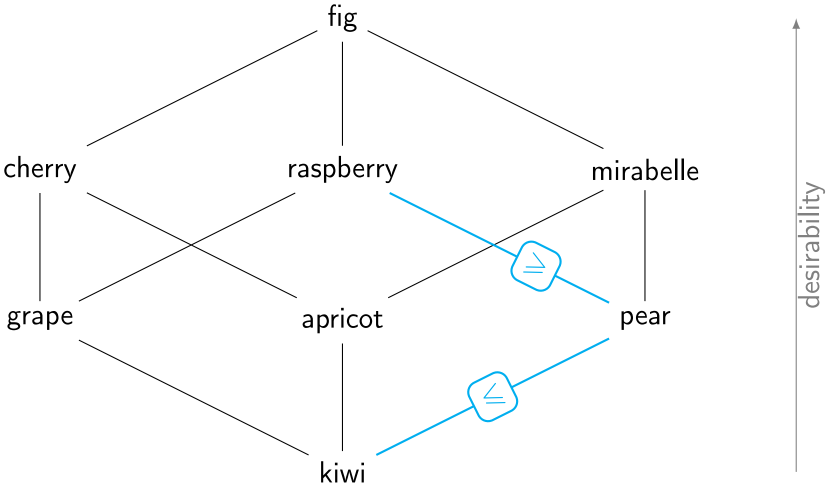

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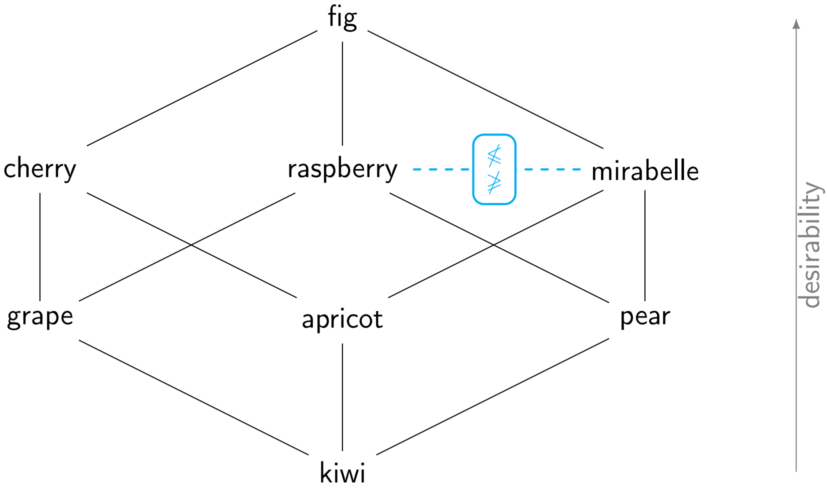
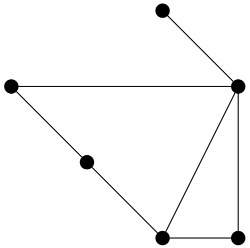


Graphs



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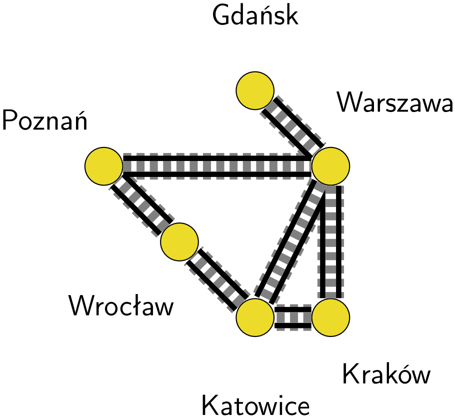
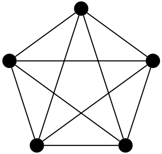
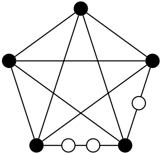


Ordering graphs

Subdivision partial order:



$\leqslant$



Ordering graphs

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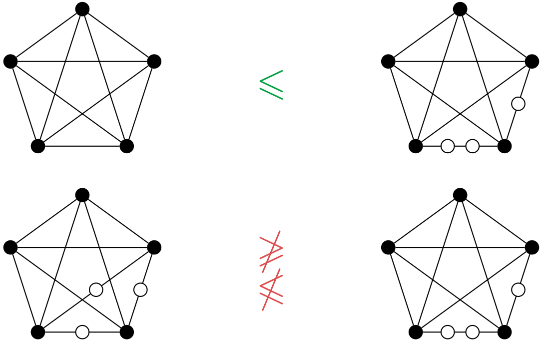
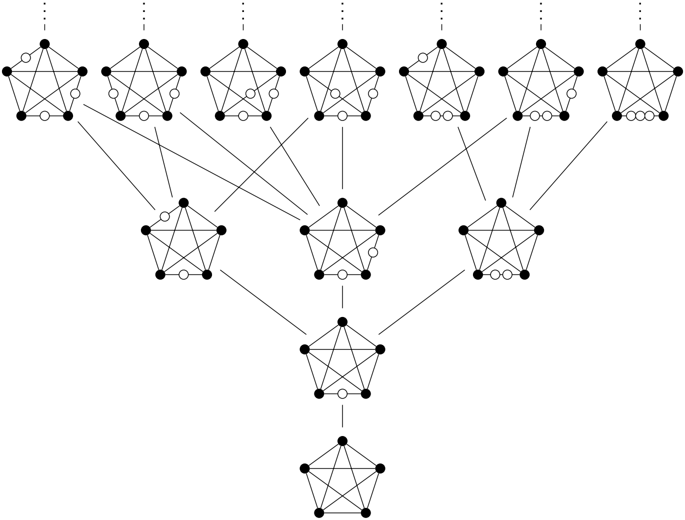


Ordering graphs



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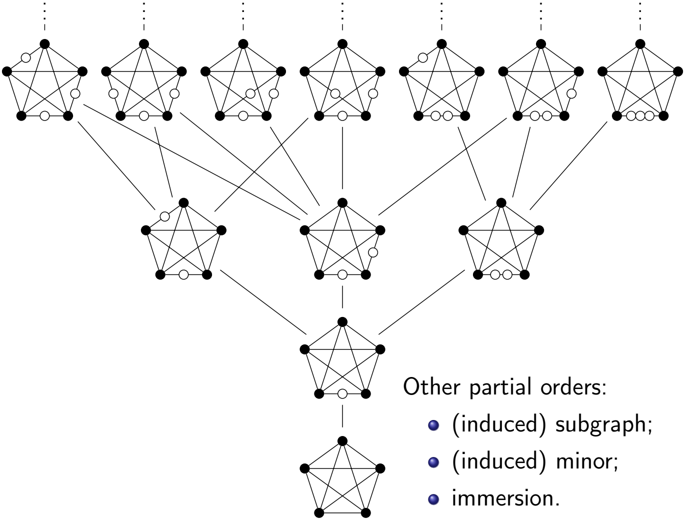

Other partial orders:

- (induced) subgraph;
- (induced) minor;
- immersion.

Excluding an object

Objects *excluding* x: objects that are not more desirable than x.

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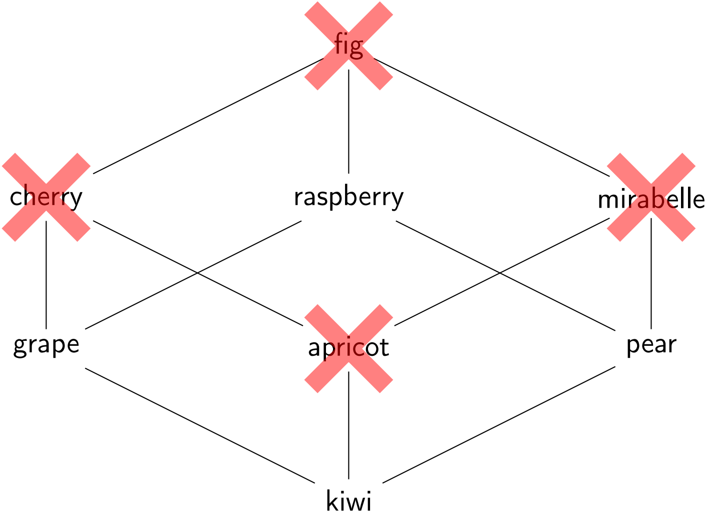
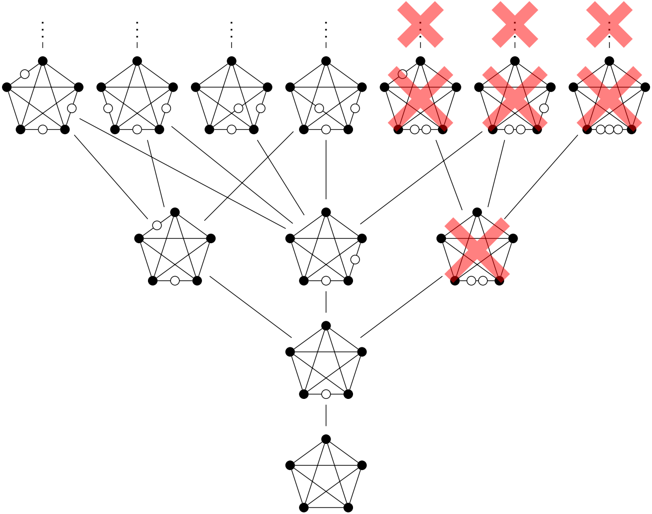


Excluding a graph



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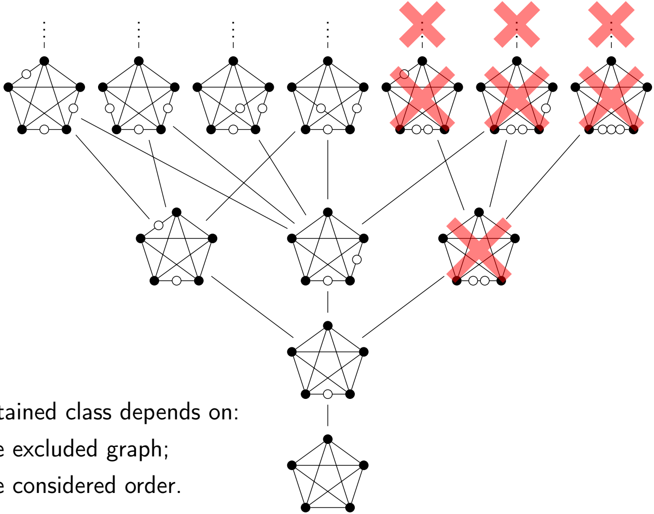
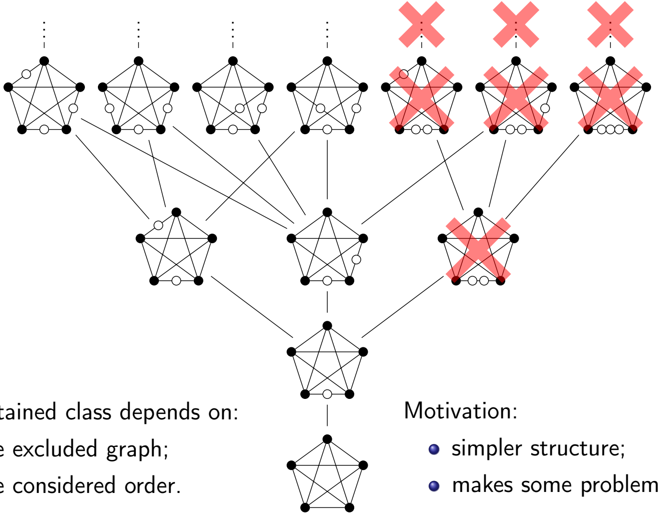


The obtained class depends on:

1. the excluded graph;
2. the considered order.

Excluding a graph



The obtained class depends on:

1. the excluded graph;
2. the considered order.

Motivation:

- simpler structure;
- makes some problems easier.

Outline

1. Ordering objects
2. Exclusion theorems
3. Well-quasi-ordering
4. The Erdős–Pósa property

Exclusion theorems

If G excludes H for $\preceq$, then ...

Exclusion theorems

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structural description of G

bound on a parameter of G

G looks like ...

$\mathbf{f}(G) \leqslant c$

Exclusion theorems

If G excludes H for $\preceq$, then ...

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If G excludes  as subdivision, then

G is a *forest*

$\delta(G) \leqslant 1$

Exclusion theorems

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If G excludes  as subdivision, then

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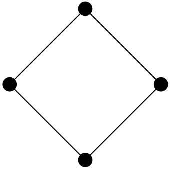
If G excludes  as subdivision, then

blocks of G are *series-parallel*

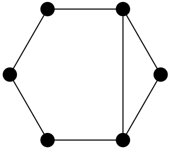
$\mathbf{tw}(G) \leqslant 2$

The minor ordering

H is a *minor* of G if it can be obtained by deleting vertices or edges and contracting edges.



$\leqslant_{\mathrm{m}}.$



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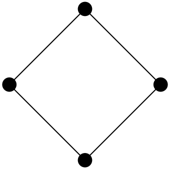
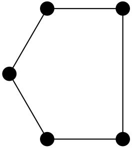
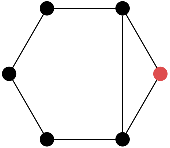


 $\leqslant_{\mathrm{m.}}$ 

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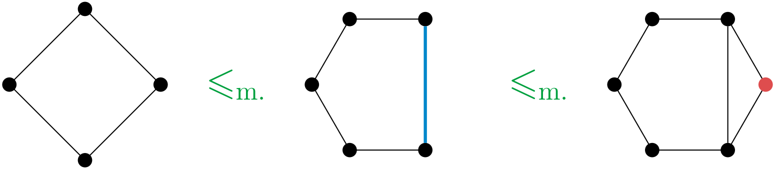

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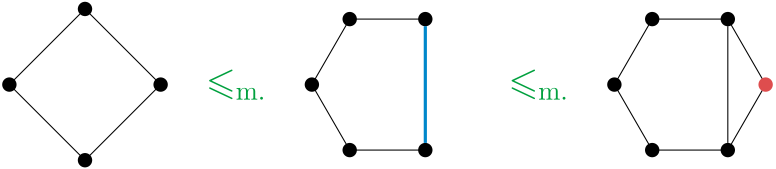

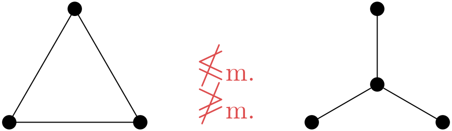

Bounds on parameters

Grid Exclusion Theorem [Robertson and Seymour, JCTB 1986]

There is a function f such that, for every planar graph H,
if G excludes H as minor, then $\mathbf{tw}(G) \leqslant f(\|H\|)$.

Bounds on parameters

Grid Exclusion Theorem [Robertson and Seymour, JCTB 1986]

There is a function f such that, for every planar graph H,
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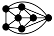
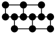
Theorem [Chekuri and Chuzhoy, FOCS 2013]

There is a polynomial[a] f such that, for every planar graph H,
if G excludes H as minor, then $\mathbf{tw}(G) \leqslant f(\|H\|)$.

[a] Currently: $f(k) = O(k^{19} \operatorname{polylog} k)$ [Chuzhoy, STOC 2015+].

Our bounds

excluded pattern	ex.	relation	par.	value of the parameter	
wheel of order k		minor	**tw**	$\Theta(k)$	
double wheel of order k		minor	**tw**	$O(k^2 \log^2 k)$	
H, $\mathbf{pw}(H) \leqslant 2$		minor	**tw**	$O\left((\lvert H\rvert + \lVert H\rVert)^2\right)$	
yurt graph of order k		minor	**tw**	$O(k^4)$	
$k \cdot \theta_r$		minor	**tw**	$\Theta(k \log k)$	
			δ	$\Theta(k)$	
edge-disj. union of k θ_r's		minor	Δ	$\Theta(k)$	$\star$
K_k		minor	θ_r-girth	$O(\log k)$	$\star$
H planar subcubic		immersion	**tcw**	$O\left(\lVert H\rVert^{29} \operatorname{polylog} \lVert H\rVert\right)$	

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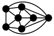
double wheel of order k		minor	**tw**	$O(k^2 \log^2 k)$
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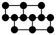
H planar subcubic		immersion	**tcw**	$O\left(\lVert H\rVert^{29} \operatorname{polylog} \lVert H\rVert\right)$

Improves the general Grid Exclusion Theorem for specific patterns.

Our bounds

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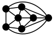
double wheel of order k		minor	**tw**	$O(k^2 \log^2 k)$	
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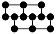
H planar subcubic		immersion	**tcw**	$O\left(\lVert H\rVert^{29} \operatorname{polylog} \lVert H\rVert\right)$	

Used in the proof of the Erdős–Pósa property of θ_r-minors.

Our bounds

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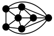
double wheel of order k		minor	**tw**	$O(k^2 \log^2 k)$	
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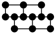
H planar subcubic		immersion	**tcw**	$O\left(\lVert H\rVert^{29} \operatorname{polylog} \lVert H\rVert\right)$	

General bound extending a result of [Kühn and Osthus, Random Structures & Algorithms 2003].

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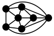
double wheel of order k		minor	**tw**	$O(k^2 \log^2 k)$	
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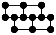
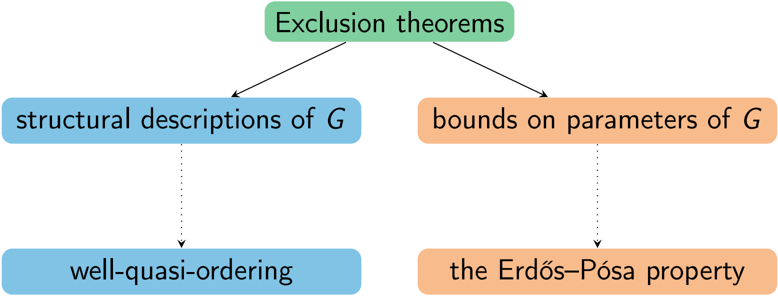
H planar subcubic		immersion	**tcw**	$O\left(\lVert H\rVert^{29} \operatorname{polylog} \lVert H\rVert\right)$	

Most general pattern for immersions and **tcw**
(relies on the results of [Wollan, JCTB 2015]).

Applications of exclusion theorems

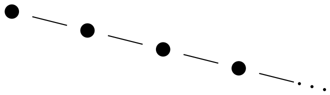


Outline

Well-quasi-ordering

Well order: total order where

- infinite decreasing sequences are not allowed



Well-quasi-ordering

Well-quasi-order: partial order where

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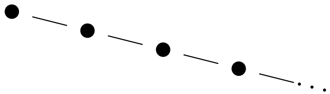

- infinite collections of incomparable elements are not allowed

(antichain)

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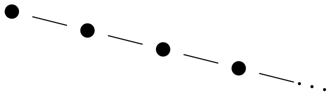

$\Rightarrow$ every set has minimal elements

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Well-quasi-ordering

Well-quasi-order: partial order where

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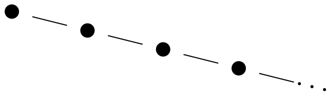

$\Rightarrow$ every set has minimal elements

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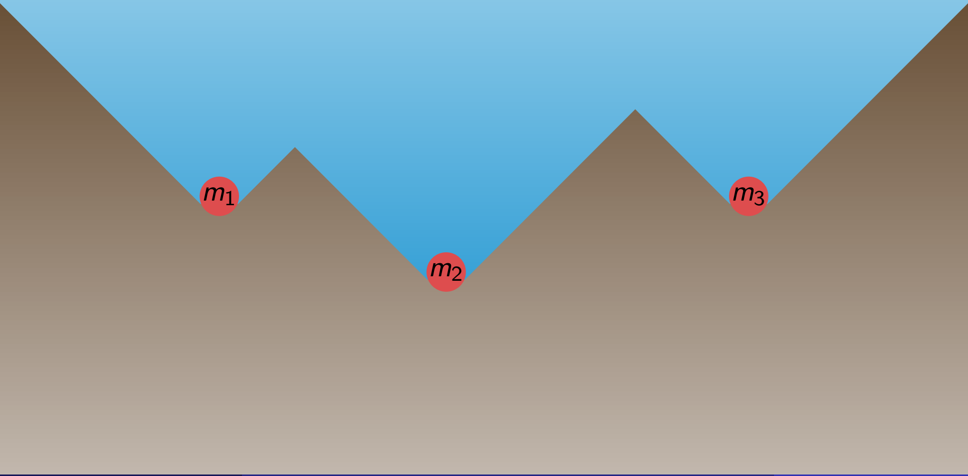
(antichain)

$\Rightarrow$ every set has finitely many minimal elements

Why do we like well-quasi-orders?

Recall: in a wqo, every set has finitely many minimal elements



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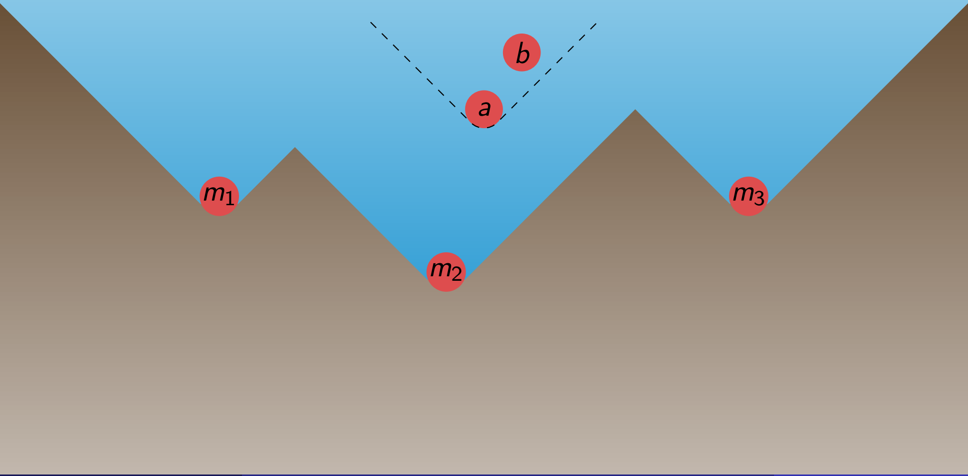
If U is *upward closed*:



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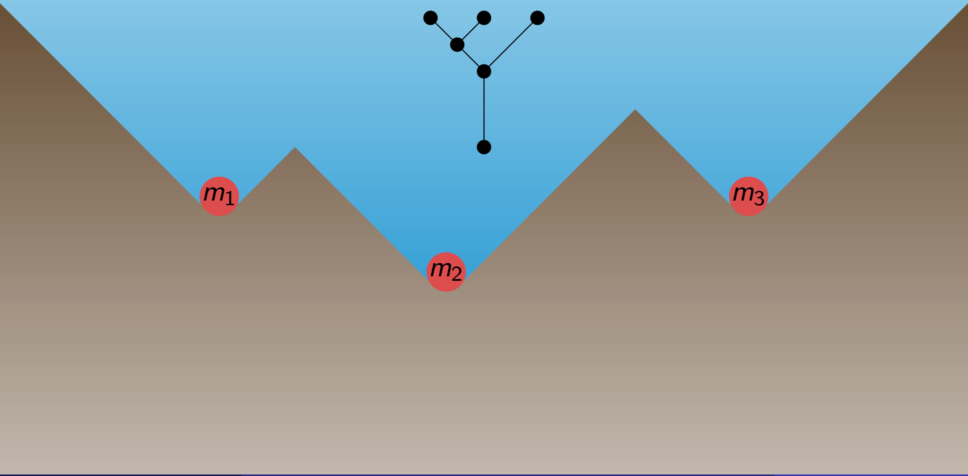

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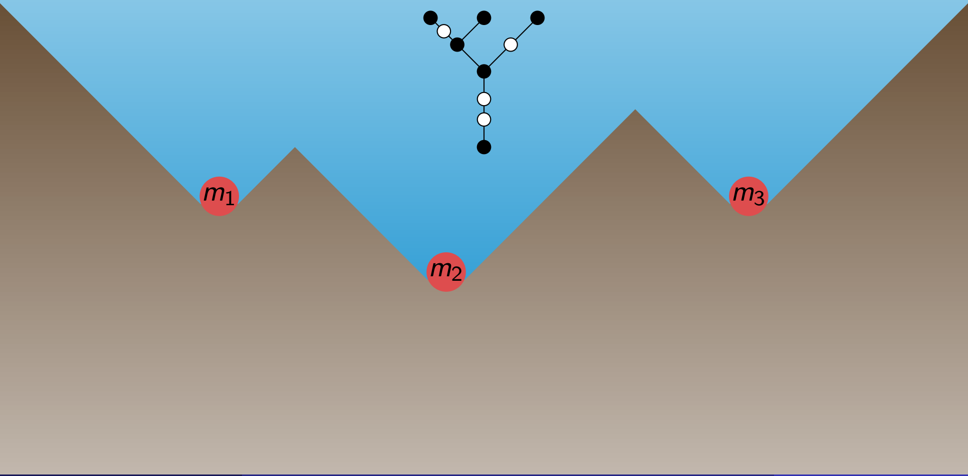

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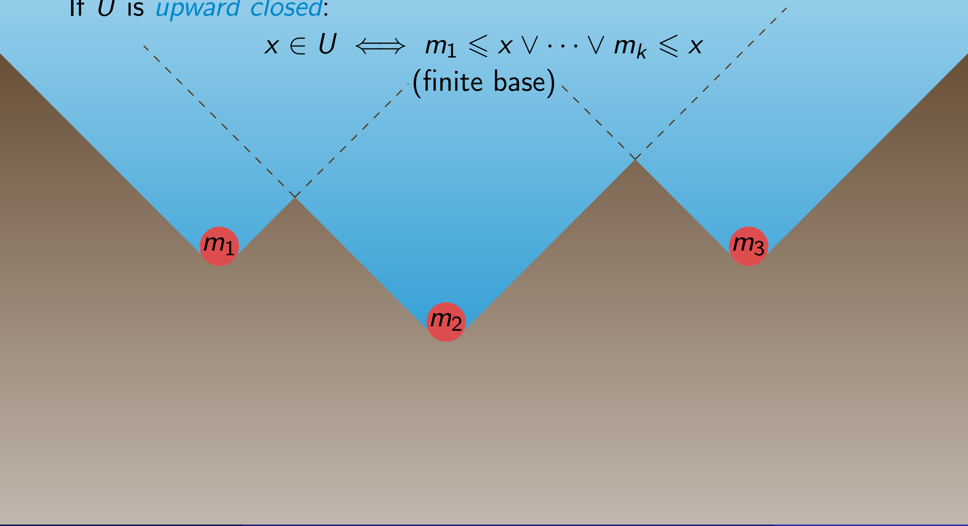
If U is *upward closed*:

$$x \in U \iff m_1 \leqslant x \vee \cdots \vee m_k \leqslant x$$

(finite base)



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Recall: in a wqo, every set has finitely many minimal elements

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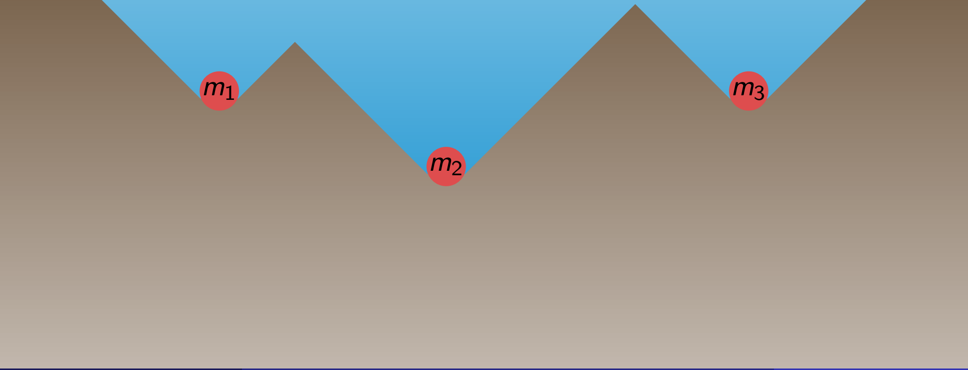
(finite base)

Membership testing can be done in a finite number of checks.



Graphs and well-quasi-ordering

Theorem [Robertson and Seymour, JCTB 2004 and JCTB 2010]

The minor and the immersion relations are well-quasi-orders of graphs.

Graphs and well-quasi-ordering

Theorem [Robertson and Seymour, JCTB 2004 and JCTB 2010]

The minor and the immersion relations are well-quasi-orders of graphs.

Folklore

The following relations are not well-quasi-orders of graphs:

- subgraph;
- induced subgraph;
- induced minor;
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Theorem [Robertson and Seymour, JCTB 2004 and JCTB 2010]

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Folklore

The following relations are not well-quasi-orders of graphs:

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What about classes excluding a graph?

Graphs and well-quasi-ordering

Theorem [Robertson and Seymour, JCTB 2004 and JCTB 2010]

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Folklore

The following relations are not well-quasi-orders of graphs:

- subgraph;
- induced subgraph;
- induced minor;
- topological minor.

What about classes excluding a graph? What is the dichotomy?

Graph exclusion and well-quasi-ordering

Theorem [Damaschke, JGT 1990]

Graphs excluding H as induced subgraph are wqo by induced subgraphs iff

$H \leqslant_{\text{i.sg.}}$ .

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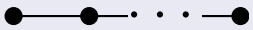
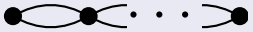
$H \leqslant_{\text{sg.}}$ .

Theorem (Liu and Thomas, 2013)

Graphs excluding H as topological minor are wqo by topological minors iff

$H \leqslant_{\text{t.m.}}$ .

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Induced minors and well-quasi-ordering

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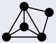
$H \leqslant_{\text{i.m.}}$  *or* $H \leqslant_{\text{i.m.}}$ .

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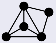
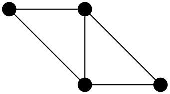
$H \leqslant_{\text{i.m.}}$  *or* $H \leqslant_{\text{i.m.}}$ .

We also obtained similar dichotomies for contractions of graphs and multigraphs.

From structure to well-quasi-ordering

Toy example

Subdivisions of H are well-quasi-ordered by the subdivision order.

$H =$ 

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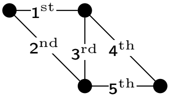
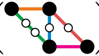
$H =$ 

1. choose an encoding of graphs as simple objects
   e.g. # of subdivisions for each edge, in some chosen order;

enc () $= (0, 2, 1, 1, 0)$

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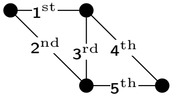
$H =$ 

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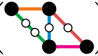
   enc () $= (0, 2, 1, 1, 0)$

2. choose an order on encodings s.t. $\mathtt{enc}(G) \leqslant \mathtt{enc}(G') \Rightarrow G \preceq G'$
   e.g. the product order, $(2, 1, 0, 3, 1) \leqslant (5, 1, 2, 4, 1)$

From structure to well-quasi-ordering

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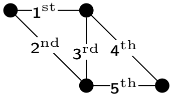
$H =$ 

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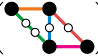
   enc () $= (0, 2, 1, 1, 0)$

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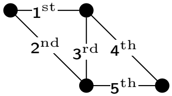
$H =$ 

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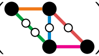
   enc () $= (0, 2, 1, 1, 0)$

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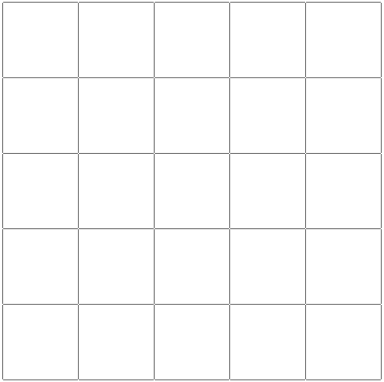
3. show that encodings are well-quasi-ordered by this order;
4. that's all!

   antichain $\{G_1, G_2, \ldots\} \Rightarrow$ antichain $\{\mathrm{enc}(G_1), \mathrm{enc}(G_2), \ldots\}$

Outline

Hunting rats



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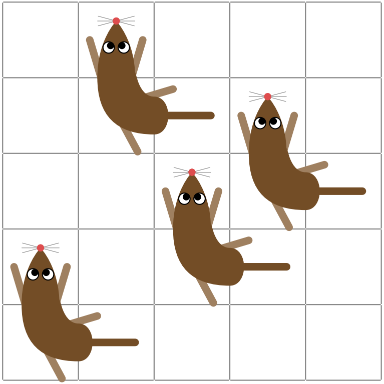

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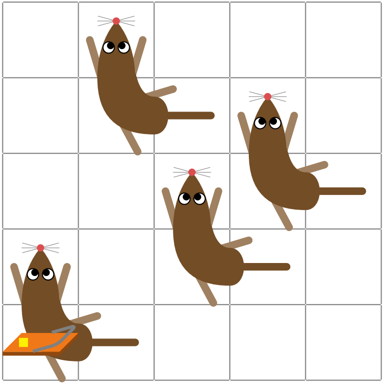

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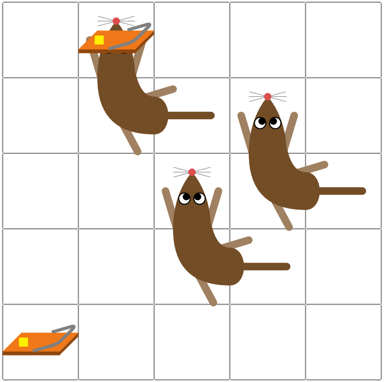

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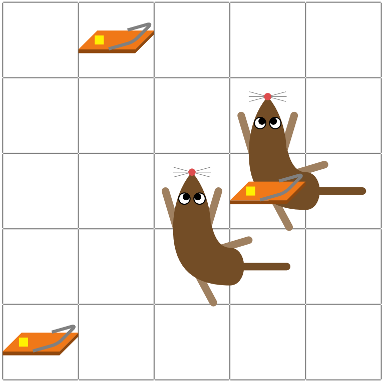

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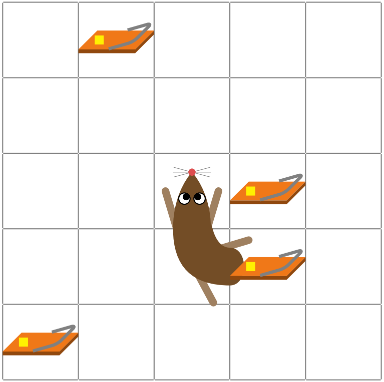

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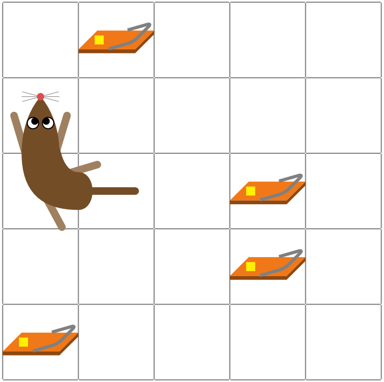

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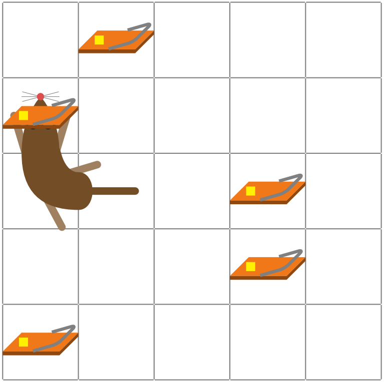

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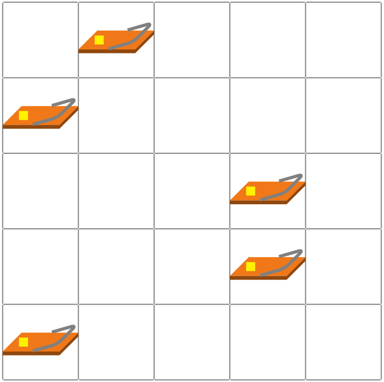

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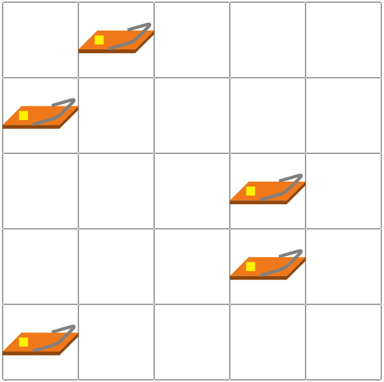
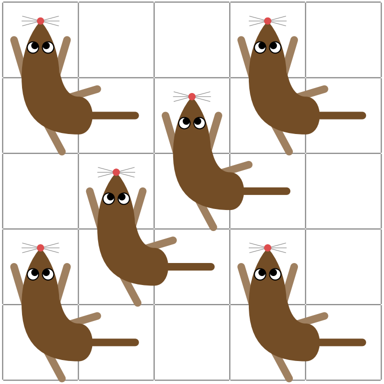
Hunting rats



How many traps are needed?

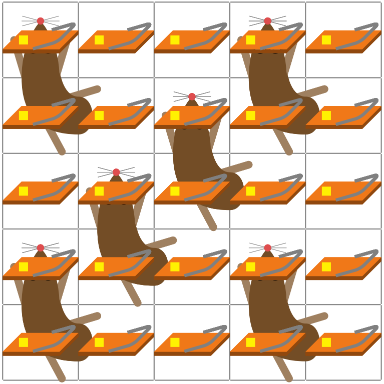
Hunting rats



How many traps are needed?

- $\tau \geqslant 6$

Hunting rats



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- $\tau \geqslant 6$
- $\tau \leqslant 25$ (size of the garden)

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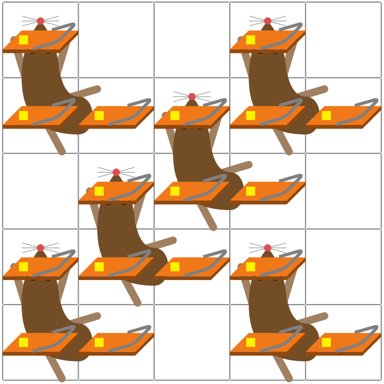
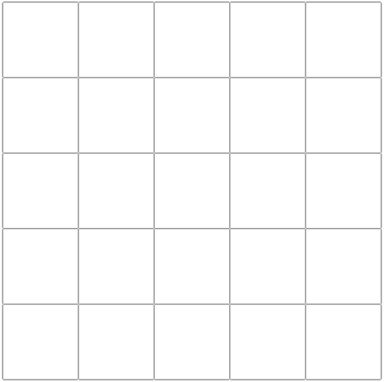


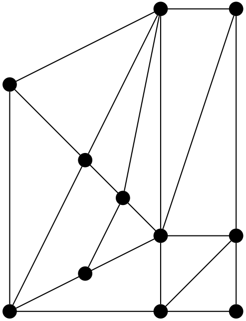
How many traps are needed?

- $\tau \geqslant 6$
- $\tau \leqslant 25$ (size of the garden)
- $\tau \leqslant 3 \times$ max. number of rats

Hunting graphs within graphs



garden ↔ graph



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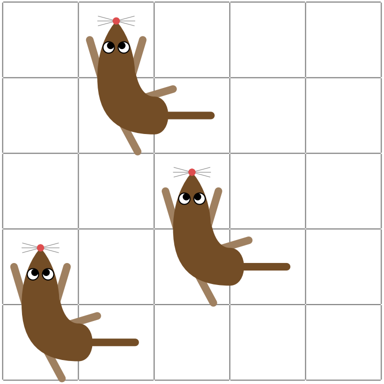

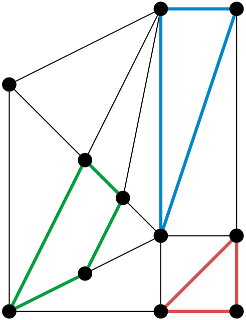

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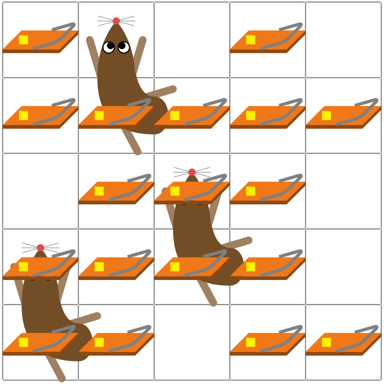

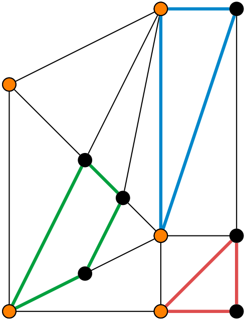

garden ↔ graph

rats ↔ disjoint subgraphs of a given type (here: cycles)

traps ↔ vertices covering all these subgraphs (cover)

The Erdős–Pósa property

Erdős-Pósa Theorem, 1965

For k the maximum number of disjoint cycles in a graph, the minimum number of vertices covering all cycles is at most $ck \log k$ (for some c).

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If such a theorem holds for a class $\mathcal{H}$ (instead of cycles), we say that $\mathcal{H}$ has the *Erdős–Pósa property*.

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Theorem [Robertson and Seymour, JCTB 1986]

There is a function f such that, for every planar graph H,
for k the maximum number of disjoint H-minors in a graph, the minimum number of vertices covering all H-minors is at most $f(k)$.

Edge version of the Erdős-Pósa Theorem, 1962

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Theorem (Giannopoulou, Kwon, R., Thilikos, 2016)

There is a polynomial f such that, for every planar subcubic graph H, for k the maximum number of edge-disjoint H-immersions in a graph, the minimum number of edges covering all H-immersions is $\leqslant f(k)$.

Three ways to the edge-Erdős–Pósa property

Typical statement

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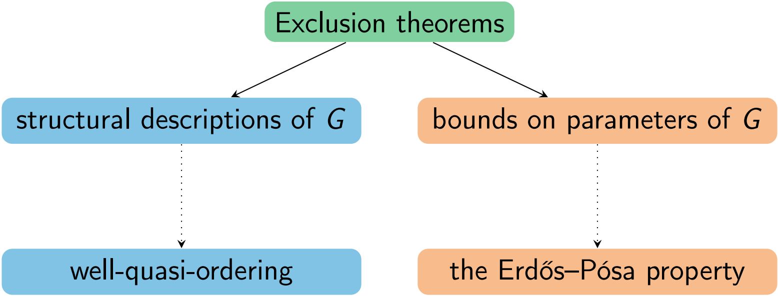
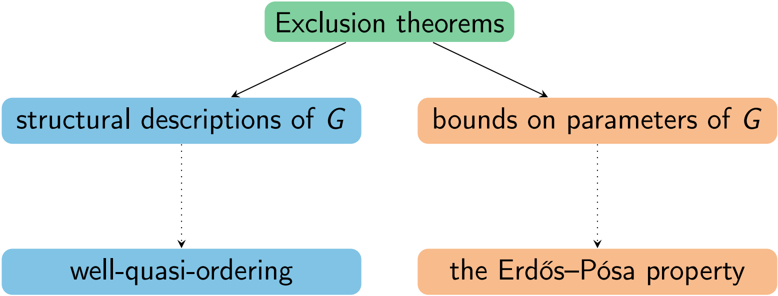
3. bound a girth-like parameter and construct step-by-step a small cover with edges.

Summary


Exclusion theorems
structural descriptions of G
bounds on parameters of G
well-quasi-ordering
the Erdős–Pósa property

Summary



Three directions for further research:

- graph modification problems;
- obstructions;
- directed graphs.

Not in this talk

- decomposition theorems when excluding some induced minor with Trunck and Kamiński;
- well-quasi-ordering and contraction, with Trunck and Kamiński;
- algorithms for packing and covering θ_r-minors, with Chatzidimitriou, Sau, Thilikos;
- more on the Erdős–Pósa property (θ_r-minors and girth, vertex version), with Chatzidimitriou, Giannopoulou, Kwon, Sau, Thilikos;

- kernels for cycle packing problems, with Atminas and Kamiński;
- on the Erdős–Pósa property for digraphs;
- bounding the size of obstructions for bounded cutwidth, with Giannopoulou, Mi. Pilipczuk, Thilikos, Wrochna;
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Dziękuję! *Thank you!* *Merci !*