La (dé)Skolémisation en logique du 1^{er} ordre

Optimisation et certification en preuve automatique

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Stage au LIRMM

Introduction

Contexte

- Assistant de preuve : automatisation.
- Facile pour les théories décidables.
- Difficile pour la logique du 1$^{\text{er}}$ ordre.

Introduction

Contexte

- Assistant de preuve : automatisation.
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Objectifs du stage

- Certifier les preuves en *méthode des tableaux*.
- Implémentation dans un prouveur automatique : Goéland.
- Méthode générique pour différentes variantes.

La méthode des tableaux à variables libres

Principe

- Preuve par réfutation.
- Forme d'arbre.
- Clôture d'une branche si contradiction.
- Preuve si toutes les branches sont closes.

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Traitement des quantificateurs

- Variable libre : « trou » qui attend l'instanciation ($\forall$).
- Skolémisation : fonction paramétrée par des variables libres ($\exists$).

Exemple de preuve tableau

$$\forall y. ((P(y) \land (\exists x. \neg P(x))) \lor (\exists x. Q(x) \land \neg Q(x))))$$

Exemple de preuve tableau

$$\frac{\forall y.\,((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))))}{(P(Y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x)))}\ \gamma\forall$$

Exemple de preuve tableau

$$\dfrac{\dfrac{\forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))))}{(P(Y) \wedge (\exists x.\, \neg P(x)))\vee(\exists x.\, Q(x) \wedge \neg Q(x)))}\,\gamma_\forall}{P(Y) \wedge \exists x.\, \neg P(x) \qquad \exists x.\, Q(x) \wedge \neg Q(x)}\,\beta_\vee$$

Exemple de preuve tableau

$$
\dfrac{\dfrac{\forall y.\, ((P(y) \land (\exists x.\, \neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x))))}{(P(Y) \land (\exists x.\, \neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x)))}\,\gamma_\forall}{P(Y) \land \exists x.\, \neg P(x) \qquad \dfrac{\exists x.\, Q(x) \land \neg Q(x)}{Q(c') \land \neg Q(c')}\,\delta_\exists}\,\beta_\lor
$$

Exemple de preuve tableau

$$
\cfrac{\forall y.\, ((P(y) \land (\exists x.\, \neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x)))}{\cfrac{(P(Y) \land (\exists x.\, \neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x))}{P(Y) \land \exists x.\, \neg P(x) \qquad \cfrac{\exists x.\, Q(x) \land \neg Q(x)}{\cfrac{Q(c') \land \neg Q(c')}{Q(c'), \neg Q(c')}\ \alpha_\land}\ \delta_\exists}\ \beta_\lor}\ \gamma_\forall
$$

Exemple de preuve tableau

$$
\cfrac{\forall y.\,((P(y)\wedge(\exists x.\,\neg P(x)))\vee(\exists x.\,Q(x)\wedge\neg Q(x)))}{\cfrac{(P(Y)\wedge(\exists x.\,\neg P(x)))\vee(\exists x.\,Q(x)\wedge\neg Q(x))}{P(Y)\wedge\exists x.\,\neg P(x) \qquad \cfrac{\exists x.\,Q(x)\wedge\neg Q(x)}{\cfrac{Q(c')\wedge\neg Q(c')}{\cfrac{Q(c'),\,\neg Q(c')}{\odot}\;\odot}\;\alpha_\wedge}\;\delta_\exists}\;\beta_\vee}\;\gamma_\forall
$$

Exemple de preuve tableau

$$
\dfrac{\forall y.\, ((P(y) \land (\exists x.\, \neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x))))}{\dfrac{(P(Y) \land (\exists x.\, \neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x)))}{\dfrac{P(Y) \land \exists x.\, \neg P(x)}{P(Y), \exists x.\, \neg P(x)}\,\alpha_\land \qquad \dfrac{\exists x.\, Q(x) \land \neg Q(x)}{\dfrac{Q(c') \land \neg Q(c')}{\dfrac{Q(c'), \neg Q(c')}{\odot}\,\odot}\,\alpha_\land}\,\delta_\exists}\,\beta_\lor}\,\gamma_\forall
$$

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\dfrac{\forall y.\,((P(y) \land (\exists x.\,\neg P(x))) \lor (\exists x.\,Q(x) \land \neg Q(x))))}{\dfrac{(P(Y) \land (\exists x.\,\neg P(x))) \lor (\exists x.\,Q(x) \land \neg Q(x)))}{\dfrac{P(Y) \land \exists x.\,\neg P(x)}{\dfrac{P(Y), \exists x.\,\neg P(x)}{\neg P(c)}\,\delta_{\exists}}\,\alpha_{\land} \qquad \dfrac{\exists x.\,Q(x) \land \neg Q(x)}{\dfrac{Q(c') \land \neg Q(c')}{\dfrac{Q(c'), \neg Q(c')}{\odot}\,\odot}\,\alpha_{\land}}\,\delta_{\exists}}\,\beta_{\lor}}\,\gamma_{\forall}
$$

Exemple de preuve tableau

$$\dfrac{\forall y.\,((P(y)\land(\exists x.\,\neg P(x)))\lor(\exists x.\,Q(x)\land\neg Q(x))))}{\dfrac{(P(Y)\land(\exists x.\,\neg P(x)))\lor(\exists x.\,Q(x)\land\neg Q(x)))}{\dfrac{P(Y)\land\exists x.\,\neg P(x)}{\dfrac{P(Y),\exists x.\,\neg P(x)}{\dfrac{\neg P(c)}{\{Y\mapsto c\}}\,\odot}\,\delta_\exists}\,\alpha_\land \qquad \dfrac{\exists x.\,Q(x)\land\neg Q(x)}{\dfrac{Q(c')\land\neg Q(c')}{\dfrac{Q(c'),\neg Q(c')}{\odot}\,\odot}\,\alpha_\land}\,\delta_\exists}\,\beta_\lor}\,\gamma_\forall$$

Exemple de preuve tableau

$$\dfrac{\forall y.\,((P(y)\land(\exists x.\,\neg P(x)))\lor(\exists x.\,Q(x)\land\neg Q(x)))}{\dfrac{(P(Y)\land(\exists x.\,\neg P(x)))\lor(\exists x.\,Q(x)\land\neg Q(x))}{\dfrac{P(Y)\land\exists x.\,\neg P(x)}{\dfrac{P(Y),\exists x.\,\neg P(x)}{\dfrac{\neg P(c)}{\{Y\mapsto c\}}\odot}\delta_{\exists}}\alpha_{\land}\qquad\dfrac{\exists x.\,Q(x)\land\neg Q(x)}{\dfrac{Q(c')\land\neg Q(c')}{\dfrac{Q(c'),\neg Q(c')}{\odot}\odot}\alpha_{\land}}\delta_{\exists}}\beta_{\lor}}\gamma_{\forall}$$

Exemple de preuve tableau

$$\cfrac{\forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x)))}{\cfrac{(P(c) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))}{\cfrac{P(c) \wedge \exists x.\, \neg P(x)}{\cfrac{P(c), \exists x.\, \neg P(x)}{\cfrac{\neg P(c)}{\odot}\ \odot}\ \delta_\exists}\ \alpha_\wedge \qquad \cfrac{\exists x.\, Q(x) \wedge \neg Q(x)}{\cfrac{Q(c') \wedge \neg Q(c')}{\cfrac{Q(c'), \neg Q(c')}{\odot}\ \odot}\ \alpha_\wedge}\ \delta_\exists}\ \beta_\vee}\ \gamma_\forall$$

Certification automatique

Transformation de language

- Assistant de preuve.
- Système adapté $\longrightarrow$ le calcul de Gentzen-Schütte (GS3).
- Séquent avec les règles des tableaux sans variable libres.

Certification automatique

Transformation de language

- Assistant de preuve.
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- Séquent avec les règles des tableaux sans variable libres.

Différences

- Règles sur les quantificateurs.
- Universel : instantiation avec un terme connu.
- Existentiel : instantiation avec une constante *fraîche* (pas de variables libres).

Traduction naive

$$
\dfrac{\forall y.\,((P(y) \land (\exists x.\,\neg P(x))) \lor (\exists x.\,Q(x) \land \neg Q(x)))}{\dfrac{(P(c) \land (\exists x.\,\neg P(x))) \lor (\exists x.\,Q(x) \land \neg Q(x))}{\dfrac{P(c) \land (\exists x.\,\neg P(x))}{\dfrac{P(c), \exists x.\,\neg P(x)}{\dfrac{P(c), \neg P(c)}{\odot}\;\odot}\;\delta_\exists}\;\alpha_\land \qquad \dfrac{\exists x.\,Q(x) \land \neg Q(x)}{\dfrac{Q(c') \land \neg Q(c')}{\dfrac{Q(c'), \neg Q(c')}{\odot}\;\odot}\;\alpha_\land}\;\delta_\exists}\;\beta_\lor}\;\gamma_\forall
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdots, P(c), \cdots, \neg P(c) \vdash}\;\mathsf{ax}}{\cdots, P(c), \exists x.\,\neg P(x) \vdash}\;\exists}{\cdots, P(c) \land \exists x.\,\neg P(x) \vdash}\;\land \qquad \dfrac{\dfrac{\dfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\;\mathsf{ax}}{\cdots, Q(c') \land \neg Q(c') \vdash}\;\land}{\cdots, \exists x.\,Q(x) \land \neg Q(x) \vdash}\;\exists}{\cdots, (P(c) \land (\exists x.\,\neg P(x))) \lor (\exists x.\,Q(x) \land \neg Q(x))) \vdash}\;\lor}{\forall y.\,((P(y) \land (\exists x.\,\neg P(x))) \lor (\exists x.\,Q(x) \land \neg Q(x))) \vdash}\;\forall
$$

$$\dfrac{\forall y.\,((P(y)\wedge(\exists x.\,\neg P(x)))\vee(\exists x.\,Q(x)\wedge\neg Q(x)))}{\dfrac{(P(c)\wedge(\exists x.\,\neg P(x)))\vee(\exists x.\,Q(x)\wedge\neg Q(x))}{\dfrac{P(c)\wedge(\exists x.\,\neg P(x))}{\dfrac{P(c),\exists x.\,\neg P(x)}{\dfrac{P(c),\neg P(c)}{\odot}\,\odot}\,\delta_\exists}\,\alpha_\wedge \qquad \dfrac{\exists x.\,Q(x)\wedge\neg Q(x)}{\dfrac{Q(c')\wedge\neg Q(c')}{\dfrac{Q(c'),\neg Q(c')}{\odot}\,\odot}\,\alpha_\wedge}\,\delta_\exists}\,\beta_\vee}\,\gamma_\forall$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdots,P(c),\cdots,\neg P(c)\vdash}\,\mathrm{ax}}{\cdots,P(c),\exists x.\,\neg P(x)\vdash}\,\exists}{\cdots,P(c)\wedge\exists x.\,\neg P(x)\vdash}\,\wedge \qquad \dfrac{\dfrac{\dfrac{}{\cdots,Q(c'),\neg Q(c')\vdash}\,\mathrm{ax}}{\cdots,Q(c')\wedge\neg Q(c')\vdash}\,\wedge}{\cdots,\exists x.\,Q(x)\wedge\neg Q(x)\vdash}\,\exists}{\cdots,(P(c)\wedge(\exists x.\,\neg P(x)))\vee(\exists x.\,Q(x)\wedge\neg Q(x)))\vdash}\,\vee}{\forall y.\,((P(y)\wedge(\exists x.\,\neg P(x)))\vee(\exists x.\,Q(x)\wedge\neg Q(x)))\vdash}\,\forall$$

Traduction naive

$$\dfrac{\forall y. ((P(y) \wedge (\exists x. \neg P(x))) \vee (\exists x. Q(x) \wedge \neg Q(x)))}{\dfrac{(P(c) \wedge (\exists x. \neg P(x))) \vee (\exists x. Q(x) \wedge \neg Q(x))}{\dfrac{P(c) \wedge (\exists x. \neg P(x))}{\dfrac{P(c), \exists x. \neg P(x)}{\dfrac{P(c), \neg P(c)}{\odot}\,\odot}\,\delta_\exists}\,\alpha_\wedge \qquad \dfrac{\exists x. Q(x) \wedge \neg Q(x)}{\dfrac{Q(c') \wedge \neg Q(c')}{\dfrac{Q(c'), \neg Q(c')}{\odot}\,\odot}\,\alpha_\wedge}\,\delta_\exists}\,\beta_\vee}\,\gamma_\forall$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdots, P(c), \cdots, \neg P(c^*) \vdash}\,\cdots}{\cdots, P(c), \exists x. \neg P(x) \vdash}\,\exists}{\cdots, P(c) \wedge \exists x. \neg P(x) \vdash}\,\wedge \qquad \dfrac{\dfrac{\dfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\,\mathrm{ax}}{\cdots, Q(c') \wedge \neg Q(c') \vdash}\,\wedge}{\cdots, \exists x. Q(x) \wedge \neg Q(x) \vdash}\,\exists}{\cdots, (P(c) \wedge (\exists x. \neg P(x))) \vee (\exists x. Q(x) \wedge \neg Q(x))) \vdash}\,\vee}{\forall y. ((P(y) \wedge (\exists x. \neg P(x))) \vee (\exists x. Q(x) \wedge \neg Q(x))) \vdash}\,\forall$$

Traduction naive

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\cfrac{\forall y.\,((P(y) \land (\exists x.\,\neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x)))}{\cfrac{(P(c) \land (\exists x.\,\neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x))}{\cfrac{P(c) \land (\exists x.\,\neg P(x))}{\cfrac{P(c), \exists x.\,\neg P(x)}{\cfrac{P(c), \neg P(c)}{\odot}\;\odot}\;\delta_\exists}\;\alpha_\land \qquad \cfrac{\exists x.\, Q(x) \land \neg Q(x)}{\cfrac{Q(c') \land \neg Q(c')}{\cfrac{Q(c'), \neg Q(c')}{\odot}\;\odot}\;\alpha_\land}\;\delta_\exists}\;\beta_\lor}\;\gamma_\forall
$$

$$
\begin{prooftree}
\AxiomC{$\vdots$}
\RightLabel{$\cdots$}
\UnaryInfC{$\cdots, P(c), \cdots, \neg P(c^*) \vdash$}
\RightLabel{$\boxed{\exists}$}
\UnaryInfC{$\cdots, P(c), \exists x.\,\neg P(x) \vdash$}
\RightLabel{$\land$}
\UnaryInfC{$\cdots, P(c) \land \exists x.\,\neg P(x) \vdash$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\cdots, Q(c'), \neg Q(c') \vdash$}
\RightLabel{$\land$}
\UnaryInfC{$\cdots, Q(c') \land \neg Q(c') \vdash$}
\RightLabel{$\exists$}
\UnaryInfC{$\cdots, \exists x.\, Q(x) \land \neg Q(x) \vdash$}
\RightLabel{$\lor$}
\BinaryInfC{$\cdots, (P(c) \land (\exists x.\,\neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x))) \vdash$}
\RightLabel{$\forall$}
\UnaryInfC{$\forall y.\,((P(y) \land (\exists x.\,\neg P(x))) \lor (\exists x.\, Q(x) \land \neg Q(x))) \vdash$}
\end{prooftree}
$$

Solution envisagée

Déskolémisation

- Travail basé sur un algorithme connu.
- Effectuer toutes les Skolémisation au début.
- Atteint la borne max. (exponentielle).
- Objectifs : amélioration, généralisation et implémentation de l'algorithme dans Goéland.

Algorithme

Dépendance

- Ordre partiel sur les formules $\prec$.
- Si $F \prec G$, Skolémisation sur F doit être appliquée avant règle sur G.
- $F \prec G$ si et seulement si symbole de Skolem de F apparaît dans G.

Algorithme

Dépendance

- Ordre partiel sur les formules $\prec$.
- Si $F \prec G$, Skolémisation sur F doit être appliquée avant règle sur G.
- $F \prec G$ si et seulement si symbole de Skolem de F apparaît dans G.

Algorithme

- Appliquer les règles tant que non Skolémisation.
- Nettoyer toutes les formules dépendantes de la formule à Skolémiser.
- Appliquer la Skolémisation.
- Réappliquer les règles nécessaires pour récupérer les formules nettoyées.

Traduction valide

$$\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash$$

Traduction valide

$$\frac{\cdots, ((P(c) \wedge (\exists x. \neg P(x))) \vee (\exists x. Q(x) \wedge \neg Q(x))) \vdash}{\psi = \forall y. ((P(y) \wedge (\exists x. \neg P(x))) \vee (\exists x. Q(x) \wedge \neg Q(x))) \vdash} \forall$$

Traduction valide

$$\frac{\cdots, ((P(c^*) \land (\exists x. \neg P(x))) \lor (\exists x. Q(x) \land \neg Q(x))) \vdash}{\psi = \forall y. ((P(y) \land (\exists x. \neg P(x))) \lor (\exists x. Q(x) \land \neg Q(x))) \vdash} \forall$$

Traduction valide

$$\dfrac{\dfrac{\cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash \qquad \cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}{\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\ \vee}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\ \forall$$

Traduction valide

$$
\cfrac{
  \cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash
  \qquad
  \cfrac{
    \cfrac{
      \cfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\ \text{ax}
    }{\cdots, Q(c') \wedge \neg Q(c') \vdash}\ \wedge
  }{\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}\ \exists
}{
  \cfrac{\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\ \forall
}\ \vee
$$

Traduction valide

$$
\dfrac{\dfrac{\cdots, P(c^*), \exists x.\, \neg P(x) \vdash}{\cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash}\wedge \qquad \dfrac{\dfrac{\dfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\,\mathsf{ax}}{\cdots, Q(c') \wedge \neg Q(c') \vdash}\wedge}{\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}\exists}{\dfrac{\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\forall}\vee
$$

$$
\dfrac{
\dfrac{
\dfrac{\psi, \exists x.\, \neg P(x) \vdash}{\cdots, P(c), \exists x.\, \neg P(x) \vdash}\;\mathrm{w} \times 3
}{\cdots, P(c) \wedge (\exists x.\, \neg P(x)) \vdash}\;\wedge
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\dfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\;\mathrm{ax}
}{\cdots, Q(c') \wedge \neg Q(c') \vdash}\;\wedge
}{\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}\;\exists
}{
\dfrac{\cdots, ((P(c) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\;\forall
}\;\vee
$$

Traduction valide

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{\psi, \exists x.\, \neg P(x), \neg P(c) \vdash}{\psi, \exists x.\, \neg P(x) \vdash}\;\exists
    }{\cdots, P(c^*), \exists x.\, \neg P(x) \vdash}\;\mathsf{w} \times 3
  }{\cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash}\;\wedge
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\;\mathsf{ax}
    }{\cdots, Q(c') \wedge \neg Q(c') \vdash}\;\wedge
  }{\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}\;\exists
}{
  \dfrac{\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\;\forall
}\;\vee
$$

$$
\dfrac{
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\dfrac{
\dfrac{\psi, \exists x.\, \neg P(x), \neg P(c) \vdash}{\psi, \exists x.\, \neg P(x) \vdash}\;\exists
}{\cdots, P(c^*), \exists x.\, \neg P(x) \vdash}\;\mathsf{w} \times 3
}{\cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash}\;\wedge
\qquad
\dfrac{
\dfrac{
\dfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\;\mathsf{ax}
}{\cdots, Q(c') \wedge \neg Q(c') \vdash}\;\wedge
}{\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}\;\exists
}{
\dfrac{\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\;\forall
}\;\vee
$$

Traduction valide

$$
\begin{prooftree}
\AxiomC{$\psi, \exists x.\, \neg P(x), \neg P(c) \vdash$}
\RightLabel{$\exists$}
\UnaryInfC{$\psi, \exists x.\, \neg P(x) \vdash$}
\RightLabel{w $\times$ 3}
\UnaryInfC{$\cdots, P(c^*), \exists x.\, \neg P(x) \vdash$}
\RightLabel{$\wedge$}
\UnaryInfC{$\cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\cdots, Q(c'), \neg Q(c') \vdash$}
\RightLabel{$\wedge$}
\UnaryInfC{$\cdots, Q(c') \wedge \neg Q(c') \vdash$}
\RightLabel{$\exists$}
\UnaryInfC{$\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash$}
\RightLabel{$\vee$}
\BinaryInfC{$\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash$}
\RightLabel{$\forall$}
\UnaryInfC{$\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash$}
\end{prooftree}
$$

Traduction valide

$$\pi = \dfrac{\dfrac{\dfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\;\text{ax}}{\cdots, Q(c') \wedge \neg Q(c') \vdash}\;\wedge}{\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}\;\exists$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\psi, \exists x.\, \neg P(x), \neg P(c) \vdash}{\psi, \exists x.\, \neg P(x) \vdash}\;\exists}{\cdots, P(c^*), \exists x.\, \neg P(x) \vdash}\;\text{w} \times 3}{\cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash}\;\wedge \qquad \pi}{\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\;\vee}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\;\forall$$

$$\pi = \dfrac{\dfrac{\dfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\ \text{ax}}{\cdots, Q(c') \wedge \neg Q(c') \vdash}\ \wedge}{\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}\ \exists$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\cdots, (P(c) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x)) \vdash}{\psi, \exists x.\, \neg P(x), \neg P(c) \vdash}\ \forall}{\psi, \exists x.\, \neg P(x) \vdash}\ \exists}{\cdots, P(c^*), \exists x.\, \neg P(x) \vdash}\ \text{w} \times 3}{\cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash}\ \wedge \qquad \pi}{\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\ \vee}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\ \forall$$

Traduction valide

$$\pi = \cfrac{\cfrac{\cfrac{}{\cdots, Q(c'), \neg Q(c') \vdash}\ \text{ax}}{\cdots, Q(c') \wedge \neg Q(c') \vdash}\ \wedge}{\cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}\ \exists$$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cdots, P(c) \wedge (\exists x.\, \neg P(x))) \vdash \qquad \cdots, \exists x.\, Q(x) \wedge \neg Q(x) \vdash}{\cdots, (P(c) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x)) \vdash}\ \boxed{\vee}}{\psi, \exists x.\, \neg P(x), \neg P(c) \vdash}\ \forall}{\psi, \exists x.\, \neg P(x) \vdash}\ \exists}{\cdots, P(c^*), \exists x.\, \neg P(x) \vdash}\ \mathrm{w} \times 3}{\cdots, P(c^*) \wedge (\exists x.\, \neg P(x)) \vdash}\ \wedge \qquad \pi}{\cdots, ((P(c^*) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\ \boxed{\vee}}{\psi = \forall y.\, ((P(y) \wedge (\exists x.\, \neg P(x))) \vee (\exists x.\, Q(x) \wedge \neg Q(x))) \vdash}\ \forall$$

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Grandes lignes de la correction

Idées

- Correspondance entre feuille séquent et tableau référence.
- Perdu après affaiblissement.
- Regagnée après réapplication des règles.
- Induction sur le nombre de règles effectuées, puis induction sur nombre de symboles de Skolem dépendants.

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Théorème

Soit T une preuve tableau. Alors l'application de l'algorithme sur T renvoie un séquent GS3 valide.

Démonstration.

- Soit μ la correspondance entre séquent et T.
- Pour toute feuille f du séquent, $\mu(f) \subseteq f$.
- $\mu(f) \vdash \perp$ donc $f \vdash \perp$.

$\square$

Implémentation et résultats

Implémentation

- Grande place dans le stage.
- Plus de 1000 lignes de code contribuées.
- Traduction automatique de preuve tableaux en Coq.
- 6 variantes de Goéland (3 Skolémisations et Déduction Modulo Théorie (DMT)).

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Résultats

	Problèmes Prouvés	Pourcentage Certifiés	Moy. Augm. Taille
Goéland	261	100%	0%
Goéland+δ^+	272	100%	8.1%
Goéland+δ^{+^+}	274	100%	10.3%
Goéland+DMT	363	100%	0%
Goéland+DMT+δ^+	375	100%	4.5%
Goéland+DMT+δ^{+^+}	377	100%	7.4%

Conclusion

Travail réalisé

- Amélioration d'un algorithme de déskolémisation.
- Prouvé correct pour les formes de Skolémisation optimisées.
- Implémentation de l'algorithme dans Goéland.
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Travail à venir

- Preuve de correction pour la version DMT.
- Extension à d'autres formes de Skolémisation.
- Extension à d'autres logiques.

The End

C’est la fin !

Merci pour votre attention, des questions ?