$$\Omega = \binom{N}{N_+}$$ on a montré $S \cong -NK_B\left(c\ln c + (1-c)\ln(1-c)\right)$ $\quad c = \frac{N_+}{N}$

donc $\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{\partial S}{\partial c}\cdot\frac{\partial c}{\partial U}$ $\quad$ or $U = -\mu_B N(2c-1)B$

donc $\frac{\partial U}{\partial c} = -2\mu_B NB$

soit $$\frac{1}{T} = +NK_B\left(\ln c + 1 - \ln(1-c) - 1\right)\frac{1}{2\mu_B NB}$$

soit $$\frac{1}{T} = \frac{K_B}{2\mu_B B}\ln\left(\frac{c}{1-c}\right)$$

$$M = \mu_B(2N_+ - N)$$
$$= \frac{\mu_B}{N^{-1}}(2c-1)$$

on cherche c : $e^{\left(\frac{2\mu_B B}{K_B T}\right)} = \frac{c}{1-c}$

$$\Rightarrow \frac{e^{\frac{2\mu_B B}{K_B T}}}{1 + e^{\frac{2\mu_B B}{K_B T}}} = c$$

donc finalement : $$M = \frac{\mu_B}{N^{-1}}\left(\frac{e^{-} - 1}{e^{-} + 1}\right) = \mu_B N \, \mathrm{th}\left(\frac{\mu_B B}{K_B T}\right)$$

$$\left[\text{ et donc } \chi = \frac{\partial M}{\partial B} = \frac{\mu_B^2 N}{K_B T}\left(1 - \mathrm{th}^2\left(\frac{\mu_B B}{K_B T}\right)\right)\right]$$