
LUBIN’S CONJECTURE FOR FULL p -ADIC DYNAMICAL SYSTEMS

by

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Abstract. — We give a short proof of a conjecture of Lubin concerning certain families of p -adic power series that commute under composition. We prove that if the family is *full* (large enough), there exists a Lubin-Tate formal group such that all the power series in the family are endomorphisms of this group. The proof uses ramification theory and some p -adic Hodge theory.

Résumé (La conjecture de Lubin pour les systèmes dynamiques p -adiques pleins)

Nous donnons une démonstration courte d’une conjecture de Lubin concernant certaines familles de séries formelles p -adiques qui commutent pour la composition. Nous montrons que si la famille est *pleine* (assez grosse), il existe un groupe formel de Lubin-Tate tel que toutes les séries de la famille sont des endomorphismes de ce groupe. La démonstration utilise la théorie de la ramification et un peu de théorie de Hodge p -adique.

Introduction

Let K be a finite extension of \mathbf{Q}_p , and let \mathcal{O}_K be its ring of integers. In [Lub94], Lubin studied *p -adic dynamical systems*, namely families of elements of $T \cdot \mathcal{O}_K[[T]]$ that commute under composition, and remarked that “experimental evidence seems to suggest that for an invertible series to commute with a noninvertible series, there must be a formal group somehow in the background”. This observation has motivated the work of a number of people (Hsia, Laubie, Li, Movaheddi, Salinier, Sarkis, Specter, ...) who proved various results in that direction. The purpose of this note is to give a proof of a special case of the above observation, which is referred to as “Lubin’s conjecture” in §3.1 of [Sar10]. Let us consider a family \mathcal{F} of commuting power series $F(T) \in T \cdot \mathcal{O}_K[[T]]$. We say that such a family is *full* if for all $\alpha \in \mathcal{O}_K$ there exists $F_\alpha(T) \in \mathcal{F}$ such that $F'_\alpha(0) = \alpha$ and if $\text{wided}(F_\pi(T)) = q$, where $\text{wided}(F(T))$ denotes the Weierstrass degree of $F(T)$, π is any uniformizer of \mathcal{O}_K and q is the cardinality of the residue field of \mathcal{O}_K .

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Theorem. — *If \mathcal{F} is a full family of commuting power series, there exists a Lubin-Tate formal group G such that $F_\alpha(T) \in \text{End}(G)$ for all $\alpha \in \mathcal{O}_K$.*

This result already appears as theorem 2 of [HL16]. Our proof is however considerably shorter than that of *ibid.*, and does not use the theory of the field of norms. It is very similar to that of the main result of [Spe16], which treats the case $K = \mathbf{Q}_p$. The main ingredients are ramification theory and some p -adic Hodge theory. In order to simplify the use of p -adic Hodge theory, we assume that K is a Galois extension of \mathbf{Q}_p .

1. p -adic dynamical systems

In this note, we consider a set $\mathcal{F} = \{F_\alpha(T)\}_{\alpha \in \mathcal{O}_K}$ of power series $F_\alpha(T) \in T \cdot \mathcal{O}_K[[T]]$ such that $F'_\alpha(0) = \alpha$ and $F_\alpha \circ F_\beta(T) = F_\beta \circ F_\alpha(T)$ whenever $\alpha, \beta \in \mathcal{O}_K$. Recall that π is a uniformizer of \mathcal{O}_K , and that q is the cardinality of the residue field k of \mathcal{O}_K . If $F(T)$ is a power series and $n \geq 0$, we denote by $F^{\circ n}(T)$ the n -th fold iteration $F \circ \cdots \circ F(T)$. If $F(T)$ has an inverse for the composition, this definition extends to $n \in \mathbf{Z}$. Recall that the *Weierstrass degree* $\text{wdeg}(F(T))$ of $F(T) = \sum_{i=1}^{+\infty} f_i T^i \in T \cdot \mathcal{O}_K[[T]]$ is the smallest integer i such that $f_i \in \mathcal{O}_K^\times$. By the Weierstrass preparation theorem, if $\text{wdeg}(F) \neq +\infty$, then F has $\text{wdeg}(F)$ zeroes in $\mathfrak{m}_{\mathbf{C}_p}$.

Proposition 1.1. — *There exists a power series $G(T) \in T \cdot k[[T]]$ and an integer $d \geq 1$ such that $G'(0) \in k^\times$ and $F_\pi(T) \equiv G(T^{p^d})$.*

Proof. — See (the proof of) theorem 6.3 and corollary 6.2.1 of [Lub94]. □

Proposition 1.2. — *There exists a power series $L_{\mathcal{F}}(T) \in K[[T]]$ such that*

1. $L_{\mathcal{F}}(T) = T + \mathcal{O}(T^2)$;
2. $L_{\mathcal{F}}(T)$ converges on the open unit disk;
3. $L_{\mathcal{F}} \circ F_\alpha(T) = \alpha \cdot L_{\mathcal{F}}(T)$ for all $\alpha \in \mathcal{O}_K$.

Proof. — See propositions 1.2 and 2.2 of [Lub94] for the construction of a unique power series $L_{\mathcal{F}}(T)$ that satisfies (1), (2) and (3) for α a uniformizer of \mathcal{O}_K . If $\beta \in \mathcal{O}_K \setminus \{0\}$, then $\beta^{-1} \cdot L_{\mathcal{F}} \circ F_\beta$ also satisfies (1), (2) and (3) for α as above, so that $L_{\mathcal{F}} \circ F_\beta(T) = \beta \cdot L_{\mathcal{F}}(T)$ for all $\beta \in \mathcal{O}_K$. □

The hypothesis that \mathcal{F} is full implies that $p^d = q$, so that $\text{wdeg}(F_\pi(T)) = q$. For $n \geq 1$, let Λ_n denote the set of $u \in \mathfrak{m}_{\mathbf{C}_p}$ such that $F_\pi^{\circ n}(u) = 0$ and $F_\pi^{\circ n-1}(u) \neq 0$ and let $\Lambda_\infty = \cup_{n \geq 1} \Lambda_n$. Proposition 1.1 implies that $F'_\pi(T)/\pi$ is a unit of $\mathcal{O}_K[[T]]$, so that the roots of $F_\pi^{\circ n}(T)$ are simple for all $n \geq 1$. The set Λ_n therefore has $q^{n-1}(q-1)$ elements.

The series $F_\alpha(T)$ is invertible if $\alpha \in \mathcal{O}_K^\times$ so that in this case, $F_\alpha(z) = 0$ if and only if $z = 0$. If $u \in \Lambda_n$ and $\alpha \in \mathcal{O}_K^\times$, then $F_\pi^{\circ n} \circ F_\alpha(u) = F_\alpha \circ F_\pi^{\circ n}(u) = 0$ and $F_\pi^{\circ n-1} \circ F_\alpha(u) = F_\alpha \circ F_\pi^{\circ n-1}(u) \neq 0$ so that the action of $F_\alpha(T)$ permutes the elements of Λ_n .

Let $K_n = K(\Lambda_n)$, so that $\Lambda_i \subset K_n$ if $i \leq n$, and let $K_\infty = \cup_{n \geq 1} K_n$. If $\alpha \in \mathcal{O}_K^\times$, let $n(\alpha)$ be the largest integer $n \geq 0$ such that $\alpha \in 1 + \pi^n \mathcal{O}_K$.

Proposition 1.3. — *If $n \geq 1$ and $u \in \Lambda_n$, then*

1. $F_\alpha(u) = u$ if and only if $n(\alpha) \geq n$;
2. If $n(\alpha) = n$, then $\text{wided}(F_\alpha(T) - T) = q^n$;
3. $\Lambda_n = \{F_\alpha(u)\}_{\alpha \in \mathcal{O}_K^\times}$.

Proof. — If $n = 1$ and $F_\alpha(u) = u$, then u is a root of $F_\alpha(T) - T = (\alpha - 1)T + O(T^2)$, so that $\alpha - 1 \in \pi \mathcal{O}_K$. This implies that $\{F_\alpha(u)\}_{\alpha \in \mathcal{O}_K^\times}$ has at least $q - 1$ distinct elements. These elements are all roots of $F_\pi(T)/T$, whose wided is $q - 1$, so $\{F_\alpha(u)\}_{\alpha \in \mathcal{O}_K^\times}$ has precisely $q - 1$ elements. These elements all have valuation $1/(q - 1)$, and if $n(\alpha) = 1$, the Newton polygon of $F_\alpha(T) - T$ starts at the point $(1, 1)$, so that it can have only one segment, and $\text{wided}(F_\alpha(T) - T) = q$. This implies the proposition for $n = 1$.

Assume now that the proposition holds up to some $n \geq 1$ and take $u \in \Lambda_{n+1}$. If $n(\alpha) \leq n$, then $F_\alpha(T) - T$ has at most q^n roots by (2), contained in $\Lambda_0 \cup \dots \cup \Lambda_n$ by (1). Therefore $F_\alpha(u) = u$ implies $n(\alpha) \geq n + 1$. The set $\{F_\alpha(u)\}_{\alpha \in \mathcal{O}_K^\times}$ therefore has at least $q^n(q - 1)$ distinct elements, all of them roots of $F_\pi^{\circ n+1}(T)/F_\pi^{\circ n}(T)$.

This implies that $\{F_\alpha(u)\}_{\alpha \in \mathcal{O}_K^\times}$ has exactly $q^n(q - 1)$ elements. If $n(\alpha) = n + 1$, the Newton polygon of $F_\alpha(T) - T$ starts at the point $(1, n + 1)$, with $n + 1$ segments of height one and slopes $-1/q^k(q - 1)$ with $0 \leq k \leq n$, so that it reaches the point $(q^{n+1}, 0)$ and hence $\text{wided}(F_\alpha(T) - T) = q^{n+1}$. This implies the proposition for $n + 1$. \square

Corollary 1.4. — *The field K_∞ is an abelian totally ramified extension of K , and if $g \in \text{Gal}(K_\infty/K)$, there is a unique $\eta(g) \in \mathcal{O}_K^\times$ such that $g(u) = F_{\eta(g)}(u)$ for all $u \in \Lambda_\infty$.*

The map $\eta : \text{Gal}(K_\infty/K) \rightarrow \mathcal{O}_K^\times$ is an isomorphism.

Proof. — Take $u \in \Lambda_n$ and $\alpha \in \mathcal{O}_K^\times$. As we have seen above, $F_\alpha(u) \in \Lambda_n$, so that the map $u \mapsto F_\alpha(u)$ induces a field automorphism of $K(u)$. By (3) of proposition 1.3, this implies that $K_n = K(u)$ and that every element of $\text{Gal}(K_n/K)$ comes from $u \mapsto F_\alpha(u)$ for some $\alpha \in \mathcal{O}_K^\times$. The extension K_n/K is therefore abelian, and so is K_∞/K . Since $K_n = K(u)$, the extension K_n/K is totally ramified, and so is K_∞/K .

The map η is surjective because every $F_\alpha(T)$ gives rise to an automorphism of K_∞ , and it is injective because if $\eta(g) = 1$, then $g(u) = u$ for all $u \in \Lambda_\infty$ so that $g = 1$. \square

In order to prove our main theorem, we study the p -adic periods of η . Corollary 1.4 and local class field theory imply that the extension K_∞/K is attached to a uniformizer ϖ of \mathcal{O}_K . Let $\chi_\varpi : G_K \rightarrow \mathcal{O}_K^\times$ denote the corresponding Lubin-Tate character.

2. p -adic Hodge theory

Let R be the p -adic completion of $\varinjlim_{n \geq 1} \mathcal{O}_K[[X_n]]$ where $\mathcal{O}_K[[X_n]]$ is seen as a subring of $\mathcal{O}_K[[X_{n+1}]]$ via the identification $X_n = F_\pi(X_{n+1})$ (this ring is defined in [Spe16], where it is denoted by A_∞). We define an action of G_K on R by $g(H(X_n)) = H(F_{\eta(g)}(X_n))$. This is well-defined since $F_\pi \circ F_{\eta(g)}(T) = F_{\eta(g)} \circ F_\pi(T)$. We have $R/\pi R = \varinjlim_{n \geq 1} k[[X_n]]$.

Lemma 2.1. — *The ring $R/\pi R$ is perfect.*

Proof. — Let $G(T)$ be as in lemma 1.1. The fact that $X_n = F_\pi(X_{n+1})$ implies that $G^{\circ n}(X_n) = G^{\circ n+1}(X_{n+1})^q$. Since $G'(0) \in k^\times$, we have $k[[T]] = k[[G(T)]]$ and therefore

$$R/\pi R = \varinjlim_{G^{\circ n}(X_n) = G^{\circ n+1}(X_{n+1})^q} k[[G^{\circ n}(X_n)]]$$

is perfect. \square

Let $\tilde{\mathbf{E}}^+ = \varprojlim_{x \rightarrow x^q} \mathcal{O}_{\mathbf{C}_p}/\pi$. Choose a sequence $\{u_n\}_{n \geq 1}$ with $u_n \in \Lambda_n$ and $F_\pi(u_{n+1}) = u_n$. This sequence gives rise to a map $i : R/\pi R \rightarrow \tilde{\mathbf{E}}^+$, determined by the requirement that $i(X_n) = (G^{\circ 1}(\bar{u}_n), G^{\circ 2}(\bar{u}_{n+1}), \dots)$. The definition of the action of G_K on R and corollary 1.4 imply that i is G_K -equivariant.

Lemma 2.2. — *The map $i : R/\pi R \rightarrow \tilde{\mathbf{E}}^+$ is injective.*

Proof. — It is enough to show that $i : k[[X_n]] \rightarrow \tilde{\mathbf{E}}^+$ is injective. If it was not, there would be a nonzero polynomial $P(T) \in k[[T]]$ such that $P(i(X_n)) = 0$, and $i(X_n) = (G^{\circ 1}(\bar{u}_n), G^{\circ 2}(\bar{u}_{n+1}), \dots)$ would then belong to $\bar{\mathbf{F}}_p$, which is clearly not the case. \square

Let $K_0 = \mathbf{Q}_p^{\text{unr}} \cap K$ and let $\tilde{\mathbf{A}}^+ = \mathcal{O}_K \otimes_{\mathcal{O}_{K_0}} W(\tilde{\mathbf{E}}^+)$ (see [Fon94a]; note that $\tilde{\mathbf{A}}^+$ usually denotes $W(\tilde{\mathbf{E}}^+)$, and is denoted by A_{inf} in *ibid.*). We have $R = \mathcal{O}_K \otimes_{\mathcal{O}_{K_0}} W(R/\pi R)$ since R is a strict π -ring, and by the functoriality of Witt vectors, the map i extends to an injective and G_K -equivariant map $i : R \rightarrow \tilde{\mathbf{A}}^+$. We write x instead of $i(X_1) \in \tilde{\mathbf{A}}^+$. The G_K -equivariance of i implies that $g(x) = F_{\eta(g)}(x)$.

Let $\mathbf{B}_{\text{cris}}^+$ and \mathbf{B}_{dR} be some of Fontaine's rings of periods. Recall that \mathbf{B}_{dR} is a field, that there is a Frobenius map φ on $\mathbf{B}_{\text{cris}}^+$, a filtration $\{\text{Fil}^i \mathbf{B}_{\text{dR}}\}_{i \in \mathbf{Z}}$ on \mathbf{B}_{dR} , and an injective map $K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+ \rightarrow \mathbf{B}_{\text{dR}}$. There is also an action of G_K on $\mathbf{B}_{\text{cris}}^+$ and \mathbf{B}_{dR} compatible with the above structure, and $\mathbf{B}_{\text{dR}}^{G_K} = K$. Let $\varphi_q = \varphi^f$ on $\mathbf{B}_{\text{cris}}^+$, where $q = p^f$, extended

by K -linearity to $K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+$. We refer to [Fon94a] and [Fon94b] for the properties of these objects. Let $\Sigma = \text{Gal}(K/\mathbf{Q}_p)$. If $\tau \in \Sigma$, choose a $n(\tau) \in \mathbf{Z}_{\geq 0}$ such that $\tau|_{K_0} = \varphi^{n(\tau)}$. The map $\tau \otimes \varphi^{n(\tau)} : K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+ \rightarrow K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+$ is then well-defined and commutes with φ_q and the action of G_K .

We say that a character $\lambda : G_K \rightarrow \mathcal{O}_K^\times$ is *crystalline positive* if there exists a nonzero $z \in K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+$ such that $g(z) = \lambda(g) \cdot z$ for all $g \in G_K$. The following proposition summarizes the input that we need from the p -adic Hodge theory of characters.

Proposition 2.3. — *A character $\lambda : G_K \rightarrow \mathcal{O}_K^\times$ that factors through $\text{Gal}(K_\infty/K)$ is crystalline positive if and only if $\lambda = \prod_{\tau \in \Sigma} \tau \circ \chi_\varpi^{h_\tau}$ with $h_\tau \in \mathbf{Z}_{\geq 0}$.*

If $t_\varpi \in K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+$ is such that $g(t_\varpi) = \chi_\varpi(g) \cdot t_\varpi$ for all $g \in G_K$, then $t_\varpi \in \text{Fil}^1 \mathbf{B}_{\text{dR}}$ and $\varphi_q(t_\varpi) = \varpi \cdot t_\varpi$.

Sketch of proof. — If $\lambda : G_K \rightarrow \mathcal{O}_K^\times$ is a crystalline positive character and $h_\tau \in \mathbf{Z}_{\geq 0}$ denotes the Hodge-Tate weight of λ at $\tau \in \Sigma$, then $\lambda \cdot \prod_{\tau \in \Sigma} \tau \circ \chi_\varpi^{-h_\tau}$ is crystalline and has Hodge-Tate weight zero at all $\tau \in \Sigma$ so that it is unramified, and therefore trivial if λ factors through $\text{Gal}(K_\infty/K)$, since K_∞/K is totally ramified.

Let ω_E and t_E be the elements constructed in §9.2 and §9.3 of [Col02] (with $E = K$ and $\pi_E = \varpi$). We have $t_E \in K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+$ and $\varphi_q(t_E) = \varpi \cdot t_E$ and $t_E \in \text{Fil}^1 \mathbf{B}_{\text{dR}}$ (proposition 9.10 of ibid). If $g \in G_K$, then (in the notation of ibid and where $[\cdot]_{\text{LT}}$ denotes the endomorphisms of the Lubin-Tate group attached to ϖ) we have $g(\omega_E) = [\chi_\varpi(g)]_{\text{LT}}(\omega_E)$ and therefore $g(t_E) = g(F_E(\omega_E)) = F_E(g(\omega_E)) = F_E \circ [\chi_\varpi(g)]_{\text{LT}}(\omega_E) = \chi_\varpi(g) \cdot F_E(\omega_E) = \chi_\varpi(g) \cdot t_E$ since F_E is the logarithm of the Lubin-Tate group attached to ϖ . If $t_\varpi \in K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+$ is such that $g(t_\varpi) = \chi_\varpi(g) \cdot t_\varpi$ for all $g \in G_K$, then $t_\varpi/t_E \in \mathbf{B}_{\text{dR}}^{G_K} = K$, and this implies the rest of the proposition. \square

Recall that $L_{\mathcal{F}}(T) \in K[[T]]$ is the logarithm attached to \mathcal{F} . Since $L_{\mathcal{F}}(T)$ converges on the open unit disk, we can view $L_{\mathcal{F}}(x)$ as an element of $K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+$.

Proposition 2.4. — *The character $\eta : G_K \rightarrow \mathcal{O}_K^\times$ is crystalline positive.*

Proof. — If $g \in G_K$, then $g(L_{\mathcal{F}}(x)) = L_{\mathcal{F}}(g(x)) = L_{\mathcal{F}}(F_{\eta(g)}(x)) = \eta(g) \cdot L_{\mathcal{F}}(x)$. \square

Corollary 2.5. — *We have $L_{\mathcal{F}}(x) = \beta \cdot \prod_{\tau \in \Sigma} (\tau \otimes \varphi^{n(\tau)})(t_\varpi)^{h_\tau}$ where $h_\tau \in \mathbf{Z}_{\geq 0}$ and $\beta \in K^\times$.*

Proof. — This follows from the facts that $\eta = \prod_{\tau \in \Sigma} \tau \circ \chi_\varpi^{h_\tau}$, that $\chi_\varpi(g) = g(t_\varpi)/t_\varpi$ and that $\mathbf{B}_{\text{dR}}^{G_K} = K$. \square

Proposition 2.6. — *We have $\varphi_q(L_{\mathcal{F}}(x)) = \mu \cdot L_{\mathcal{F}}(x)$ for some $\mu \in \pi \mathcal{O}_K$.*

Proof. — Corollary 2.5 and proposition 2.3 imply the proposition with $\mu = \prod_{\tau} \tau(\varpi)^{h_{\tau}}$, and not all h_{τ} can be equal to 0 since $\eta \neq 1$. \square

Corollary 2.7. — *We have $\varphi_q(x) = F_{\mu}(x)$.*

Proof. — Proposition 2.6 implies that $L_{\mathcal{F}}(\varphi_q(x)) = L_{\mathcal{F}}(F_{\mu}(x))$. We would like to apply $L_{\mathcal{F}}^{\circ-1}(T)$ but we have to mind the convergence and need to proceed as follows. Since η is nontrivial, there is a $\tau \in \Sigma$ such that $h_{\tau^{-1}} \geq 1$. We have

$$(\tau \otimes \varphi^{n(\tau)})(L_{\mathcal{F}}(\varphi_q(x))) = (\tau \otimes \varphi^{n(\tau)})(L_{\mathcal{F}}(F_{\mu}(x)))$$

in $K \otimes_{K_0} \mathbf{B}_{\text{cris}}^+$ and $h_{\tau^{-1}} \geq 1$ now implies that $(\tau \otimes \varphi^{n(\tau)})(L_{\mathcal{F}}(\varphi_q(x)))$ is divisible by t_{ϖ} so that by proposition 2.3, it belongs to $\text{Fil}^1 \mathbf{B}_{\text{dR}}$. We can then apply $L_{\mathcal{F}}^{\tau \circ -1}(T)$ in \mathbf{B}_{dR} and get that $(\tau \otimes \varphi^{n(\tau)})(\varphi_q(x)) = (\tau \otimes \varphi^{n(\tau)})(F_{\mu}(x))$ in \mathbf{B}_{dR} . This equality also holds in $\tilde{\mathbf{A}}^+$, so that $\varphi_q(x) = F_{\mu}(x)$. \square

Theorem 2.8. — *There is a Lubin-Tate formal group G such that $F_{\alpha}(T) \in \text{End}(G)$ for all $\alpha \in \mathcal{O}_K$.*

Proof. — By corollary 2.7, we have $\varphi_q(x) = F_{\mu}(x)$. This implies that $F_{\mu}(T) \equiv T^q \pmod{\pi \mathcal{O}_K[[T]]}$. The Weierstrass degree of $F_{\mu}(T)$ is $q^{\text{val}(\mu)}$ so that $\text{val}(\mu) = 1$ and $F_{\mu}(T)$ is a Lubin-Tate power series. By [LT65], there is a Lubin-Tate formal group G such that $F_{\mu}(T) \in \text{End}(G)$. Since $F_{\alpha}(T)$ commutes with $F_{\mu}(T)$, we also have $F_{\alpha}(T) \in \text{End}(G)$ for all $\alpha \in \mathcal{O}_K$. \square

Remark 2.9. — We have $\mu = \varpi$ and $\eta = \chi_{\varpi}$. Indeed, the extension K_{∞}/K is generated by the torsion points of G , and is therefore attached to μ by local class field theory, so that $\mu = \varpi$. This in turn implies that $\eta = \chi_{\varpi}$.

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