RIGIDITY AND UNLIKELY INTERSECTIONS FOR FORMAL GROUPS

by

Laurent Berger

Abstract. — Let K be a p-adic field and let F and G be two formal groups over the integers of K. We prove that if F and G have infinitely many torsion points in common, then $F = G$. This follows from a rigidity result: any bounded power series that sends infinitely many torsion points of F to torsion points of F is an endomorphism of F.

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Introduction

Let K be a finite extension of $\mathbf{Q}_p$ (or, more generally, a finite extension of $W(k)[1/p]$ where k is a perfect field of characteristic p). Let $\overline{K}$ be an algebraic closure of K and let $\mathbf{C}_p$ be the p-adic completion of $\overline{K}$. Let $\mathcal{O}_K$ denote the ring of integers of K, and let $F(X,Y) = X \oplus Y \in \mathcal{O}_K[\![X,Y]\!]$ be a formal group law over $\mathcal{O}_K$. Let $\mathrm{Tors}(F)$ be the set of torsion points of F in $\mathfrak{m}_{\mathbf{C}_p} = \{z \in \mathbf{C}_p,\ |z|_p < 1\}$. The question that motivates this paper is: to what extent is a formal group F determined by $\mathrm{Tors}(F)$? Our main result is an "unlikely intersections" result.

Theorem A. — *If F and G are two formal groups over $\mathcal{O}_K$ and if* $\mathrm{Tors}(F) \cap \mathrm{Tors}(G)$ *is infinite, then $F = G$.*

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If $n \geqslant 2$ and $[n](X)$ denotes the multiplication by n map on F, then $\mathrm{Tors}(F)$ is also the set of preperiodic points of $[n](X)$ in $\mathfrak{m}_{\mathbf{C}_p}$. We can therefore think of $\mathrm{Tors}(F)$ as the set $\mathrm{Preper}(F)$ of preperiodic points of a p-adic dynamical system attached to F. Theorem A then becomes a statement about preperiodic points of certain dynamical systems.

Some analogues of theorem A are known in other contexts. For example, if two elliptic curves over $\overline{\mathbf{Q}}$ have infinitely many torsion points in common (in a suitable sense), then they are isomorphic (Bogomolov and Tschinkel, see §4 of [**BT07**]). In another context, if f and g are two rational fractions of degree at least 2 with coefficients in the complex numbers, and if $\mathrm{Preper}(f) \cap \mathrm{Preper}(g)$ is infinite, then $\mathrm{Preper}(f) = \mathrm{Preper}(g)$ (Baker and DeMarco, theorem 1.2 of [**BD11**]). In this case, f and g have the same Julia set (corollary 1.3 of ibid.). One can then show that, if f and g are polynomials of the same degree, then in most cases they are equal up to a linear symmetry that preserves their common Julia set (see for instance [**BE87**] and [**SS95**]).

Our proof of theorem A relies on a rigidity result for formal groups. We say that a subset $Z \subset \mathfrak{m}_{\mathbf{C}_p}^d$ is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}^d$ if every power series $h(X_1, \dots, X_d) \in \mathcal{O}_K[\![X_1, \dots, X_d]\!]$ that vanishes on Z is necessarily equal to zero. For example, if $d = 1$ then $Z \subset \mathfrak{m}_{\mathbf{C}_p}$ is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}$ if and only if it is infinite.

Theorem B. — *If F is a formal group over $\mathcal{O}_K$ and if $h(X) \in X \cdot \mathcal{O}_K[\![X]\!]$ is such that $h(z) \in \mathrm{Tors}(F)$ for infinitely many $z \in \mathrm{Tors}(F)$, then $h \in \mathrm{End}(F)$.*

More generally, if $h(X_1, \dots, X_d) \in \mathcal{O}_K[\![X_1, \dots, X_d]\!]$ is such that $h(0) = 0$ and $h(z) \in \mathrm{Tors}(F)$ for all z in a subset of $\mathrm{Tors}(F)^d$ that is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}^d$, then there exists $h_1, \dots, h_d \in \mathrm{End}(F)$ such that $h = h_1(X_1) \oplus \cdots \oplus h_d(X_d)$.

This theorem generalizes corollary 4.2 of Hida's [**Hid14**], which concerns the case $F = \mathbf{G}_m$. Our proof uses ideas coming from the theory of p-adic dynamical systems (developed in large part by Lubin, see [**Lub94**]) rather than the "special subvarieties" argument of Hida (which is in the spirit of Chai's [**Cha08**]). Other kinds of "unlikely intersections" results for certain formal groups can be found in [**Ser18**].

1. Formal groups

For the basic definitions and results about formal groups that we need, we refer for instance to Lubin's [**Lub64, Lub67**]. Let $F(X, Y) = X \oplus Y \in \mathcal{O}_K[\![X, Y]\!]$ be a formal group law over $\mathcal{O}_K$. If $n \in \mathbf{Z}$, let $[n](X)$ denote the multiplication by n map on F. More generally, if $a \in \mathcal{O}_K$, let $[a](X)$ be the unique endomorphism of F such that $[a]'(0) = a$ if it exists (it always does if $a \in \mathbf{Z}_p$). Let $\mathrm{Tors}(F)$ be the set of torsion points of F. If F

is of finite height, then $\mathrm{Tors}(F)$ is infinite, while if F is of infinite height, then $\mathrm{Tors}(F)$ is finite and our results are vacuous. We therefore assume from now on that F is of finite height h.

Let $T_pF = \varprojlim_n F[p^n]$ be the Tate module of F. If F is of height h, then T_pF is a free $\mathbf{Z}_p$-module of rank h, equipped with an action of $\mathrm{Gal}(\overline{K}/K)$. If we choose a basis of T_pF, this gives rise to a Galois representation $\rho_F : \mathrm{Gal}(\overline{K}/K) \to \mathrm{GL}_h(\mathbf{Q}_p)$. Let E be the fraction field of $\mathrm{End}(F)$. It is a finite extension of $\mathbf{Q}_p$ whose degree e divides h (theorem 2.3.2 of [**Lub64**]), so that we can view $\mathrm{GL}_{h/e}(E)$ as a subgroup of $\mathrm{GL}_h(\mathbf{Q}_p)$.

Theorem 1.1. — *The image of ρ_F has an open subgroup that is an open subgroup of a conjugate of* $\mathrm{GL}_{h/e}(E)$.

Proof. — This is an unpublished theorem of Serre (see however the remark after theorem 5 on page 130 of [**Ser67**]), which is proved in [**Sen73**] (see theorem 3 on page 168 and the remark that follows). □

Corollary 1.2. — *The image of ρ_F contains an open subgroup of $\mathbf{Z}_p^\times \cdot \mathrm{Id}$.*

Note that if $\sigma \in \mathrm{Gal}(\overline{K}/K)$ and $a \in \mathbf{Z}_p^\times$ are such that $\rho_F(\sigma) = a \cdot \mathrm{Id}$, then $\sigma(z) = [a](z)$ for all $z \in \mathrm{Tors}(F)$.

2. p-adic dynamical systems

In this §, we prove a number of results about power series that commute under composition (sometimes also called permutable power series). These results are all inspired by Lubin's theory of p-adic dynamical systems (see [**Lub94**]).

A power series $h(X) \in X \cdot K[\![X]\!]$ is said to be stable if $h'(0)$ is neither 0 nor a root of unity. If $h'(0) \neq 0$, then there exists a unique power series $h^{\circ -1}(X) \in X \cdot K[\![X]\!]$ such that $h \circ h^{\circ -1} = h^{\circ -1} \circ h = X$. If in addition $h(X) \in X \cdot \mathcal{O}_K[\![X]\!]$ and $h'(0) \in \mathcal{O}_K^\times$, then $h^{\circ -1}(X) \in X \cdot \mathcal{O}_K[\![X]\!]$.

Theorem 2.1. — *Let $u(X) \in X \cdot K[\![X]\!]$ be a stable power series.*

A power series $h(X_1, \ldots, X_d) \in K[\![X_1, \ldots, X_d]\!]$ such that $h(0) = 0$ and such that $h \circ u = u \circ h$ is determined by $\{dh/dX_i(0)\}_{1 \leqslant i \leqslant d}$.

Proof. — Suppose that $h^{(1)}$ and $h^{(2)}$ are two such power series, and that they coincide in degrees $\leqslant m$. Let h_m be the sum of the terms of $h^{(i)}$ of total degree $\leqslant m$. We have

$h^{(i)} = h_m + r^{(i)}$ with $r^{(i)}$ of degree $\geqslant m+1$, and

$$(h_m + r^{(i)}) \circ u = h_m \circ u + r^{(i)} \circ u \equiv h_m \circ u + u'(0)^{m+1} r^{(i)} \bmod \deg(m+2),$$
$$u \circ (h_m + r^{(i)}) \equiv u \circ h_m + r^{(i)} u'(h_m) \equiv u \circ h_m + r^{(i)} u'(0) \bmod \deg(m+2).$$

Since $u'(0)^m \neq 1$, the fact that $h^{(i)} \circ u = u \circ h^{(i)}$ implies that

$$r^{(i)} \equiv \frac{h_m \circ u - u \circ h_m}{u'(0) - u'(0)^{m+1}} \bmod \deg(m+2).$$

If $h^{(1)}$ and $h^{(2)}$ coincide in degrees $\leqslant m$, they therefore have to coincide in degrees $\leqslant m+1$. This implies the theorem by induction on m. □

Let us say that an endomorphism of a formal group is stable if the corresponding power series is stable.

Corollary 2.2. — *Let F be a formal group and let u be a stable endomorphism of F. If $h(X) \in X \cdot \mathcal{O}_K[\![X]\!]$ is such that $h \circ u = u \circ h$, then h is an endomorphism of F.*

Proof. — The power series $F \circ h$ and $h \circ F$ both commute with u, and have the same derivatives at 0, so that $F \circ h = h \circ F$ by theorem 2.1. □

Corollary 2.3. — *If u is a stable endomorphism of a formal group and if $h(X_1, \dots, X_d) \in \mathcal{O}_K[\![X_1, \dots, X_d]\!]$ is such that $h(0) = 0$ and $h \circ u = u \circ h$, then there exists $a_1 \dots, a_d \in \mathcal{O}_K$ such that $h(X_1, \dots, X_d) = [a_1](X_1) \oplus \cdots \oplus [a_d](X_d)$.*

Proof. — Let $h_i(X)$ be the power series h evaluated at $X_i = X$ and $X_k = 0$ for $k \neq i$. We have $h_i \circ u = u \circ h_i$ and hence by corollary 2.2, $h_i(X) = [a_i](X)$ where $a_i = h_i'(0) \in \mathcal{O}_K$. The two power series $h(X_1, \dots, X_d)$ and $[a_1](X_1) \oplus \cdots \oplus [a_d](X_d)$ commute with u and have the same derivatives at 0, so that they are equal by theorem 2.1. □

3. Rigidity and unlikely intersections

We first recall and prove theorem B.

Theorem 3.1. — *If F is a formal group over $\mathcal{O}_K$ and if $h(X_1, \dots, X_d) \in \mathcal{O}_K[\![X_1, \dots, X_d]\!]$ is such that $h(0) = 0$ and $h(z) \in \mathrm{Tors}(F)$ for all z in a subset Z of $\mathrm{Tors}(F)^d$ that is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}^d$, then there exists $h_1, \dots, h_d \in \mathrm{End}(F)$ such that $h = h_1(X_1) \oplus \cdots \oplus h_d(X_d)$.*

Proof. — Since $\mathrm{Tors}(F)$ is infinite, F is of finite height. By corollary 1.2, there exists $\sigma \in \mathrm{Gal}(\overline{K}/K)$ and a stable endomorphism u of F such that $\sigma(z) = u(z)$ for all $z \in \mathrm{Tors}(F)$. If $z \in Z$, then we have $\sigma(h(z)) = u(h(z))$ as well as $\sigma(h(z)) = h(\sigma(z)) = h(u(z))$. The power series $u \circ h - h \circ u$ therefore vanishes on Z. Since Z is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}^d$, we have $u \circ h = h \circ u$. The theorem now follows from corollary 2.3. □

Remark 3.2. — If $Y_1, \ldots, Y_d$ are infinite subsets of $\mathrm{Tors}(F)$, then $Y_1 \times \cdots \times Y_d$ is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}^d$.

We now recall and prove theorem A.

Theorem 3.3. — *If F and G are two formal groups over $\mathcal{O}_K$ and if* $\mathrm{Tors}(F) \cap \mathrm{Tors}(G)$ *is infinite, then $F = G$.*

Proof. — By corollary 1.2, there exists an element $\sigma \in \mathrm{Gal}(\overline{K}/K)$ and a stable endomorphism u of F such that $\sigma(z) = u(z)$ for all $z \in \mathrm{Tors}(F)$. The set $\Lambda = \mathrm{Tors}(F) \cap \mathrm{Tors}(G)$ is stable under the action of $\mathrm{Gal}(\overline{K}/K)$. If $z \in \Lambda$, we therefore have $\sigma(z) \in \Lambda$ and hence $u(z) \in \mathrm{Tors}(G)$ for all $z \in \Lambda$, since $u(z) = \sigma(z)$. By theorem B applied to G, we get that $u \in \mathrm{End}(G)$. The power series F and G commute with u and have the same linear terms, hence $F = G$ by theorem 2.1. □

4. Generalizations and perspectives

4.1. Universal bounds. — In §4 of [**BT07**], Bogomolov and Tschinkel prove that two nonisomorphic elliptic curves over $\overline{\mathbf{Q}}$ have only finitely many torsion points in common. In [**BFT18**], the authors raise the question of the existence of a universal bound for the maximum number of torsion points that two nonisomorphic elliptic curves over $\overline{\mathbf{Q}}$ (or even over the complex numbers) can share. The same kind of question is raised, for preperiodic points of rational fractions, in conjecture 1.4 of [**DKY19**].

The following proposition shows that in our situation, there is no straightforward refinement of theorem A.

Proposition 4.1. — *For all $n \geqslant 1$, there exists a formal group F over $\mathbf{Z}_p$, of height 1, such that F is not isomorphic to $\mathbf{G}_m$ but such that* $\mathrm{Tors}(F) \cap \mathrm{Tors}(\mathbf{G}_m)$ *contains at least n points.*

Proof. — Take $n \geqslant 1$ and let $u(X) = 1 + p \cdot ((1+X)^{p^n} - 1)/X$ and $q(X) = (1+X)^p - 1$ and $f(X) = u(X)q(X)$. We have $f(X) = p(1+p^{n+1})X + \mathrm{O}(X^2)$ and $f(X) \equiv X^p \bmod p$.

By Lubin-Tate theory (see §1 of [**LT65**]) there exists a formal group F such that $[p(1+p^{n+1})](X) = f(X)$. This group is attached to the uniformizer $p(1+p^{n+1})$ of $\mathbf{Q}_p$. Likewise, $\mathbf{G}_m$ is attached to p. The formal group F is not isomorphic to $\mathbf{G}_m$ over $\mathbf{Q}_p$ as $p \neq p(1+p^{n+1})$ and any Lubin-Tate group attached to a uniformizer π determines π.

However, we have $f(\zeta_p - 1) = 0$ and $f(\zeta_{p^k} - 1) = \zeta_{p^{k-1}} - 1$ for all $k \leqslant n$, so that $\zeta_{p^k} - 1 \in \mathrm{Tors}(F)$ for all $k \leqslant n$. This proves the proposition. $\square$

If $\mathrm{Tors}(F) \cap \mathrm{Tors}(G)$ is large, then are F and G close to each other in some sense?

4.2. The logarithm of a formal group. — Using the logarithms of formals groups, we can give a very short proof of a weaker form of theorem A, namely: if $\mathrm{Tors}(F) = \mathrm{Tors}(G)$ (and this common set is infinite), then $F = G$. Indeed, Log_F is holomorphic on $\mathfrak{m}_{\mathbf{C}_p}$ and its zeroes are precisely the elements of $\mathrm{Tors}(F)$, with multiplicity 1. In addition, Log_F' is a bounded power series since dLog_F is the normalized invariant differential on F. If $\mathrm{Tors}(F) = \mathrm{Tors}(G)$, then Log_F and Log_G have the same zeroes, so that they differ by a unit u. A unit is necessarily bounded. We have $\mathrm{Log}_G = u \cdot \mathrm{Log}_F$ and hence $\mathrm{Log}_G' = u \cdot \mathrm{Log}_F' + u' \cdot \mathrm{Log}_F$. Since Log_G' and Log_F' and u are bounded, but not Log_F, we must have $u' = 0$ (the sup norms $\|\cdot\|_r$ on circles are multiplicative). This implies that $u \in \mathcal{O}_K^\times$ and then that $u = 1$ since $\mathrm{Log}_F'(0) = \mathrm{Log}_G'(0) = 1$, so that $\mathrm{Log}_F = \mathrm{Log}_G$ and $F = G$. The same argument gives the following characterization of the logarithm of a formal group of finite height.

Proposition 4.2. — *If F is a formal group of finite height, then the power series* Log_F *is the unique element of $X + X^2 \cdot K[\![X]\!]$ that is holomorphic on $\mathfrak{m}_{\mathbf{C}_p}$, whose zero set is precisely* $\mathrm{Tors}(F)$*, with multiplicity* 1*, and whose derivative is bounded.*

4.3. More rigidity. — A common generalization of theorems A and B would be the assertion that if a power series h maps infinitely many torsion points of F to torsion points of G, then $h \in \mathrm{Hom}(F, G)$. In order to prove this using the same method as in the proof of theorem B, we would need to show that there exists $\sigma \in \mathrm{Gal}(\overline{K}/K)$ that acts on $\mathrm{Tors}(F)$ and $\mathrm{Tors}(G)$ by two power series u_F and u_G, satisfying some stability condition. If G is a Lubin-Tate formal group (for some finite extension of $\mathbf{Q}_p$ contained in K), there is a character $\chi_G : \mathrm{Gal}(\overline{K}/K) \to \mathcal{O}_K^\times$ such that $\sigma(z) = [\chi_G(\sigma)](z)$ for all $z \in \mathrm{Tors}(G)$ (theorem 2 of [**LT65**]).

Theorem 4.3. — *If F is a formal group and G is a Lubin-Tate formal group, both defined over $\mathcal{O}_K$, and if $h(X) \in X \cdot \mathcal{O}_K[\![X]\!]$ is such that $h'(0) \neq 0$ and $h(z) \in \mathrm{Tors}(G)$ for infinitely many $z \in \mathrm{Tors}(F)$, then $h \in \mathrm{Hom}(F, G)$.*

Proof. — Since $\mathrm{Tors}(F)$ is infinite, F is of finite height. By corollary 1.2, there exists an element $\sigma \in \mathrm{Gal}(\overline{K}/K)$ and a stable endomorphism u_F of F such that $\sigma(z_F) = u_F(z_F)$ if $z_F \in \mathrm{Tors}(F)$. Let $u_G(X) = [\chi_G(\sigma)](X)$, so that $\sigma(z_G) = u_G(z_G)$ if $z_G \in \mathrm{Tors}(G)$.

If $z \in \mathrm{Tors}(F)$ is such that $h(z) \in \mathrm{Tors}(G)$, then $\sigma(h(z)) = u_G(h(z))$ and $\sigma(h(z)) = h(\sigma(z)) = h(u_F(z))$. The power series $u_G \circ h - h \circ u_F$ therefore vanishes at infinitely many points of $\mathfrak{m}_{\mathbf{C}_p}$, so that $u_G \circ h = h \circ u_F$. Since $h'(0) \neq 0$, we have $u_F'(0) = u_G'(0)$ and u_G is stable. The theorem now follows from lemma 4.4 below. □

Lemma 4.4. — *Let F and G be two formal groups and let f and g be endomorphisms of F and G, with g stable. If $h(X) \in X \cdot \mathcal{O}_K[\![X]\!]$ is such that $h'(0) \neq 0$ and $h \circ f = g \circ h$, then $h \in \mathrm{Hom}(F, G)$.*

Proof. — Consider the power series $K(X, Y) = h \circ F(h^{\circ -1}(X), h^{\circ -1}(Y))$. We have

$$K \circ g = h \circ F \circ h^{\circ -1} \circ g = h \circ F \circ f \circ h^{\circ -1} = h \circ f \circ F \circ h^{\circ -1} = g \circ h \circ F \circ h^{\circ -1} = g \circ K$$

Since K and G commute with g and have the same derivatives at 0, we have $K = G$ by theorem 2.1 and hence $h \circ F = G \circ h$, so that $h \in \mathrm{Hom}(F, G)$. □

Note that the hypotheses of the lemma imply that $f'(0) = g'(0)$ so that if one series is stable, then both are.

4.4. Homotheties and stable p-adic dynamical systems. — If F is a formal group of finite height, then $\mathrm{End}(F)$ is a set of power series that commute with each other under composition. One can forget about the formal group and study certain sets $\mathcal{D}$ of elements of $X \cdot \mathcal{O}_K[\![X]\!]$ that commute with each other under composition. This is the object of Lubin's theory of p-adic dynamical systems (see [**Lub94**]).

Let us say that $\mathcal{D} \subset X \cdot \mathcal{O}_K[\![X]\!]$ is a stable p-adic dynamical system of finite height if the elements of $\mathcal{D}$ commute with each other under composition, and if $\mathcal{D}$ contains a stable series f such that $f'(0) \in \mathfrak{m}_K$ and $f(X) \not\equiv 0 \bmod \mathfrak{m}_K$ (i.e. f is of finite height) as well as a stable series u such that $u'(0) \in \mathcal{O}_K^\times$. We can then assume that $\mathcal{D}$ is as large as possible, namely that any power series $g \in X \cdot \mathcal{O}_K[\![X]\!]$ that commutes with the elements of $\mathcal{D}$ belongs to $\mathcal{D}$. For example, if F is a formal group of finite height, then $\mathrm{End}(F)$ is a stable p-adic dynamical system.

Given a stable p-adic dynamical system of finite height $\mathcal{D}$, the set of preperiodic points $\mathrm{Preper}(g)$ is independent of the choice of a stable $g \in \mathcal{D}$ (see §3 of [**Lub94**]). One can then define $\mathrm{Preper}(\mathcal{D})$ as the preperiodic set of any stable element of $\mathcal{D}$. To what extent does $\mathrm{Preper}(\mathcal{D})$ determine a stable p-adic dynamical system of finite height $\mathcal{D}$?

In order to extend our results from formal groups to stable p-adic dynamical systems of finite height, we can ask whether the consequence of corollary 1.2 holds in more generality: for which stable p-adic dynamical systems of finite height $\mathcal{D}$ is there a stable power series $w \in \mathcal{D}$ and an element $\sigma \in \mathrm{Gal}(\overline{K}/K)$ such that $\sigma(z) = w(z)$ for all $z \in \mathrm{Preper}(\mathcal{D})$?

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Laurent Berger