ERRATA FOR MY ARTICLES

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Contents

1. Représentations p-adiques et équations différentielles

Example 2.8.1. Replace $\mathbf{A}_{\max}^{+}$ by $\mathbf{A}_{\max}$.

Sections 3.3, 5.5. Kedlaya has completely changed his article [34], so that most references to it are now incorrect.

Theorem 4.10. Theorem 4.10 is actually due to Forster, see : O. Forster, Zur Theorie der Steinschen Algebren und Moduln, Math. Zeitschrift, 97, p. 376ff, 1967.

Proposition 2.24. The log map is not defined for $x = 0$. In addition, I only define it on $\tilde{\mathbf{A}}^{+}$ but later, I use it on $\tilde{\mathbf{A}}^{\dagger}$ (for example : $\log(\pi_K)$). The (easy) extension to $\tilde{\mathbf{A}}^{\dagger}$ is done by Colmez in [Col08].

Proof of lemma 5.27. Replace $\mathrm{GL}_d(\mathbf{A}^{\dagger,r}, K)$ by $\mathrm{GL}_d(\mathbf{A}_K^{\dagger,r})$ and $\mathrm{M}_d(\mathbf{A}^{\dagger,r}, K)$ by $\mathrm{M}_d(\mathbf{A}_K^{\dagger,r})$.

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Matrices. I wrote all matrices "the wrong way". For example, if f and g are two semilinear maps, then in my notation, $\mathrm{Mat}(fg) = f(\mathrm{Mat}(g))\,\mathrm{Mat}(f)$. To recover the usual notation, one needs to transpose everything (this is done in my other articles).

Proof of proposition 5.15. It is not true that $\iota_n(N_s) = K_n[\![t]\!] \otimes_K \mathrm{D}_{\mathrm{dR}}(V)$. What is true is that the image of ι_n is dense for the t-adic topology. This is what is proved and used in the rest of the proof.

Page 229, line 3. Replace $\tilde{\mathbf{A}}^+$ by $\tilde{\mathbf{A}}$.

The ring $\mathbf{B}_K^\dagger$. I say that the ring $\mathbf{B}_K^\dagger$ is a ring of power series with coefficients in F, but that is not always the case. It is a ring of power series with coefficients in K_0', the maximal unramified extension of F inside K_∞, which may be larger than F. Since it is true that $(\mathbf{B}_K^\dagger)^{\Gamma_K} = F$, this does not affect the results of the paper, and most proofs go through unchanged.

As I am regularly asked for an example where $K_0' \neq F$, here is one: the field $\mathbf{Q}_p(\zeta_p)$ has a quadratic subfield, for example $\mathbf{Q}_p(\sqrt{p^*})$ where $p^* = (-1)^{(p-1)/2} \cdot p$. Let y be an element of $\mathbf{Z}_p^\times$ whose reduction mod p is not a square, so that $\mathbf{Q}_p(\sqrt{y})$ is quadratic unramified over $\mathbf{Q}_p$. The field $K = \mathbf{Q}_p(\sqrt{y \cdot p^*})$ is totally ramified over $F = \mathbf{Q}_p$ but $K(\zeta_p)$ and hence K_0' contains $\mathbf{Q}_p(\sqrt{y})$.

Monodromy. In order to recover the (φ, N)-module $\mathrm{D}_{\mathrm{st}}(\cdot)$, one should take $N(\log(\pi)) = -p/(p-1)$ instead of $N(\log(\pi)) = -1$.

Diagram on page 271. In the diagram at the top of the page, replace ∇_M by the connexion attached to ∂_M.

Lemma 2.7. Replace $k \gg 0$ by $k \gg -\infty$ in $\sum_{k \gg 0} p^k [x_k]$.

Propositions 2.11 and 2.12. These are only true if I is such that $[\tilde{p}]/p-1$ or $[\tilde{p}^{p^n}]/p-1$ belong to $\tilde{\mathbf{A}}_I$. Otherwise, replace $\tilde{\mathbf{A}}_I$ by $\widetilde{\mathbf{B}}_I$.

Corollary 2.20. The condition on r should be that $r \leqslant s, t$ or in other words that $[s;t] \subset [r;+\infty[$.

Diagram on page 280. Technically not a mistake, but in the lower right of the diagram, $\overline{k}$ can be replaced by $\mathcal{O}_{\mathbf{C}_p}/p$.

Page 233. After the proof of lemma 2.5: $\widetilde{\mathbf{B}}_I = \cap_{[r;s] \subset I} \widetilde{\mathbf{B}}_{[r;s]}$. The intersection should be taken over a nested (increasing) family of intervals whose union is I, not all intervals.

2. Bloch and Kato's exponential map: three explicit formulas

Introduction. Not a mistake, but an incomplete attribution: Fontaine actually defined (φ, Γ)-modules in order to study Perrin-Riou's exponential map.

3. Limites de représentations cristallines

Definition III.4.1. The definition of a Wach module over $\mathbf{B}_F^+$ is incomplete, as p is a unit in $\mathbf{B}_F^+$. One needs for example to add the condition that $\mathbf{B}_F \otimes_{\mathbf{B}_F^+} \mathrm{N}$ is an étale (φ, Γ)-module over $\mathbf{B}_F$.

Theorem III.4.4. There is an argument missing from the end of the proof. In the last two lines, we take $y \in \mathrm{D}_{\mathrm{cris}}(V)$ whose image is in $\mathrm{Fil}^i \mathrm{N}(V)/\pi \mathrm{N}(V)$, and implicitely claim that $y \in \mathrm{Fil}^i \mathrm{N}(V)$. However, we only have $y \in \mathrm{Fil}^i \mathrm{N}(V) + \pi \cdot \mathrm{N}(V)$ a priori. Take $\gamma \in \Gamma$ nontorsion. If $j \geqslant 1$, then $\gamma - \chi(\gamma)^j$ sends y to a nonzero scalar multiple of itself, and $\mathrm{Fil}^i \mathrm{N}(V) + \pi^j \cdot \mathrm{N}(V)$ to $\mathrm{Fil}^i \mathrm{N}(V) + \pi^{j+1} \cdot \mathrm{N}(V)$. Using this with $j = 1, \cdots, i-1$ shows that $y \in \mathrm{Fil}^i \mathrm{N}(V)$.

4. Construction de (φ, Γ)-modules : représentations p-adiques et B-paires

Lemma 1.1.11. The proof is incomplete and incorrect (the map $x \mapsto \sum_{i=0}^{h-1} \varphi^i(\omega) \otimes \varphi^i(x)$ does not even have values in $\mathbf{Q}_{p^h} \otimes_{\mathbf{Q}_p} \mathbf{B}_{\mathrm{e}}$). Here is a correct proof. The group $\mathrm{Gal}(\mathbf{Q}_{p^h}/\mathbf{Q}_p)$ acts $\mathbf{Q}_{p^h}$-semi-linearly on $\mathbf{B}_{\mathrm{cris}}^{\varphi^h=1}$ via φ, and the fact that $\mathbf{B}_{\mathrm{cris}}^{\varphi^h=1} = \mathbf{Q}_{p^h} \otimes_{\mathbf{Q}_p} \mathbf{B}_{\mathrm{cris}}^{\varphi=1}$ then follows from Galois descent (Speiser's lemma).

Proposition 3.3.5. In the definition of W (statement of proposition 3.3.5), replace $\mathbf{B}_{\mathrm{dR}}^+ \otimes_{K_\infty} D_\infty$ by $\mathrm{Fil}^0(\mathbf{B}_{\mathrm{dR}} \otimes_{K_\infty} D_\infty)$.

Proposition 3.3.10. There is an argument missing from the proof, namely the following lemma: let D be an F-vector space with an action of G_F, such that $P(\nabla) = 0$ where $P(X) \wedge P(X+j) = 1$ for all $j \in \mathbf{Z}_{\geqslant 1}$, and let W be a $\mathbf{B}_{\mathrm{dR}}^+$-lattice of $\mathbf{B}_{\mathrm{dR}} \otimes_F D$ that is stable under G_F. If we set $\mathrm{Fil}^i D = D \cap t^i \cdot W$, then $W = \mathrm{Fil}^0(\mathbf{B}_{\mathrm{dR}} \otimes_F D)$.

Page 117. In the first paragraph of page 117, P and Q seem to be exchanged at various points.

5. Equations différentielles p-adiques et (φ, N)-modules filtrés

Theorem I.3.3. In item (2), it is better to require that $\mathbf{B}_{\mathrm{rig},K}^{\dagger,pr} \otimes_{\mathbf{B}_{\mathrm{rig},K}^{\dagger,r}} \mathrm{D}_r$ is the $\mathbf{B}_{\mathrm{rig},K}^{\dagger,pr}$-module generated by $\varphi(\mathrm{D}_r)$. This implies that $\mathrm{Mat}(\varphi) \in \mathrm{GL}_d(\mathbf{B}_{\mathrm{rig},K}^{\dagger,pr})$, which is used later in the proof.

Theorem III.2.3. There is an argument missing from the proof, namely the following lemma, which is now lemma 7.6 of [Ber13]: Let D be an F-vector space, and let W be a $\mathbf{B}_{\mathrm{dR}}^+$-lattice of $\mathbf{B}_{\mathrm{dR}} \otimes_F D$ that is stable under G_F, where G_F acts trivially on D. If we set $\mathrm{Fil}^i D = D \cap t^i \cdot W$, then $W = \mathrm{Fil}^0(\mathbf{B}_{\mathrm{dR}} \otimes_F D)$.

Example IV.2.8. In the first example, $\alpha e + f$ should be replaced by $t^{-1}(\alpha e + f)$.

6. Familles de représentations de de Rham et monodromie p-adique

Lemma 2.1.1. For item (2), one needs to assume that M is finitely presented. In this case, the natural map $M \otimes_S \prod S/\mathfrak{m}_x \to \prod M/\mathfrak{m}_x M$ is a bijection (Bourbaki AC, §I.2, exercise 9), and the rest of the proof works.

Section 2.3. It would be better to define a family of representations as a projective S-module, rather than a free S-module.

7. Sur quelques représentations potentiellement cristallines de $\mathrm{GL}_2(\mathbf{Q}_p)$

Item (v) at the beginning of §2.4. There is no map $\varphi^{-m} : \mathbf{B}_{\mathrm{cris}} \to \mathbf{B}_{\mathrm{dR}}$. However whenever this map is used in the paper, it is used on a space on which it is defined.

8. Représentations potentiellement trianguline de dimension 2

Lemme 3.2. There are extensions of $\mathbf{Q}_2$ with Galois group $\mathrm{GL}_2(\mathbf{F}_3)$, and $\mathrm{GL}_2(\mathbf{F}_3)$ admits an irreducible 2-dimensional representation that is not induced. Lemma 3.2 is therefore incorrect. The problem in the proof is that it is not true that the Galois group of an extension of local fields is always supersolvable (for instance it can be S_4 or $\mathrm{GL}_2(\mathbf{F}_3)$).

The lemma holds for $p \neq 2$. Indeed, in the notation of the lemma, there is a finite Galois extension $K/\mathbf{Q}_p$ such that $V|_{G_K}$ is a direct sum of two characters. If these two characters are not equal, then the proof of lemma 3.2 works. If the characters are equal, then the image H of $G_{\mathbf{Q}_p}$ in $\mathrm{PGL}(V)$ gives rise to a finite subgroup of $\mathrm{PGL}_2(E)$. This subgroup is either cyclic, dihedral, or A_4, S_4 or A_5. It is also a quotient of $\mathrm{Gal}(K/\mathbf{Q}_p)$, which is solvable, so it cannot be A_5. In the A_4 or S_4 cases, we necessarily have $p = 2$ by looking at the inertia subgroup of H. If $p \neq 2$, H is therefore cyclic or dihedral and the conclusion of the lemma is true.

Here is a replacement for lemma 3.2: if V is a 2-dimensional representation of $G_{\mathbf{Q}_p}$ such that there exists a finite extension K of $\mathbf{Q}_p$ such that $V|_{G_K}$ is a sum of two characters, then either V is a sum of two characters, or it is induced, or it is a twist of a potentially trivial representation.

Proof: there is a finite Galois extension $K/\mathbf{Q}_p$ such that V restricted to G_K is a direct sum of two characters. If these two characters are not equal, then the proof of lemma

3.2 works, so assume that the characters are equal: $V|_{G_K} = \eta \oplus \eta$. The character $\det(V)$ of $G_{\mathbf{Q}_p}$ admits a potential square root (a character δ of $G_{\mathbf{Q}_p}$ such that $\delta^2 = \det(V)$ on an open subroup of $G_{\mathbf{Q}_p}$). If δ is such a potential square root, then $(\eta\delta^{-1})^2 = 1$ on an open subgroup of G_K and hence $\eta = \delta$ on an open subgroup of G_K. The representation $V(\delta^{-1})$ is then potentially trivial.

9. La correspondance de Langlands locale p-adique pour $\mathrm{GL}_2(\mathbf{Q}_p)$

Page 169. In the definition of the parameter spaces, replace the condition "$w(s) \geqslant 1$" by "$w(s) \in \mathbf{Z}_{\geqslant 1}$" (twice).

10. Local constancy for the reduction mod p of 2-dimensional crystalline representations

Theorem B. In the proof of theorem B, we also require that $k' > 3\cdot\mathrm{val}_p(a_p)+\alpha(k'-1)+1$. This can always be achieved by increasing $m(k, a_p)$ if necessary, since $k \mapsto k(1-p/(p-1)^2)$ is an increasing function (for $p \neq 2$ at least!). Hence theorem B is correct as stated. One could also require that $k > 3 \cdot \mathrm{val}_p(a_p) + (k-1)p/(p-1)^2 + 1$, which also gives an explicit lower bound for k in terms of $\mathrm{val}_p(a_p)$, namely $k > 3 \cdot \mathrm{val}_p(a_p)/(1 - p/(p-1)^2) + 1$. Note that if $p = 2$, theorem B is vacuous since $\alpha(k-1) + 1 \geqslant k$.

11. Lifting the field of norms

Question 1.2. It would be better to require that $F_{\mathrm{Id}}(T) = T$. This, however, follows from the other conditions. We have $F_{\mathrm{Id}}(F_{\mathrm{Id}}(T)) = F_{\mathrm{Id}}(T)$ and so $F_{\mathrm{Id}}^{\circ p}(T) = F_{\mathrm{Id}}(T)$. Furthermore, $F_{\mathrm{Id}}(T) \equiv T \bmod \pi$. If $A(T) \equiv T \bmod \pi^r$, then $A^{\circ p}(T) \equiv T \bmod \pi^{r+1}$. So if $F_{\mathrm{Id}}^{\circ p}(T) = F_{\mathrm{Id}}(T)$, then $F_{\mathrm{Id}}(T) = T$.

Proposition 4.2. It is not true in general that $\mathcal{N}(T) = T$. For example if $P(T) = T^2 - a$ and $p = 2$, then $\mathcal{N}(T) = -T - a$ (the equation $\mathcal{N}(T) = T$ does hold if $(-1)^{q-1}P(T)$ is a monic polynomial of degree q and $P(0) = 0$).

What is true is that if $P(T) \in T \cdot \mathcal{O}_E[\![T]\!]$, then $\mathcal{N}(T) \in T \cdot \mathcal{O}_E[\![T]\!]^\times$. Let $n(T)$ denote the power series $n(T) = \mathcal{N}(T) \in T \cdot \mathcal{O}_E[\![T]\!]^\times$ and let $n^{\circ-1}(T) \in T \cdot \mathcal{O}_E[\![T]\!]^\times$ denote its composition inverse. Let $\mathcal{N}'$ be defined by $\mathcal{N}'(f(T)) = n^{\circ-1} \circ \mathcal{N}(f(T))$. We then have $\mathcal{N}'(T) = T$ and the proof of proposition 4.2 works with $\mathcal{N}'$ instead of $\mathcal{N}$: if $k \geqslant 1$, then $\mathcal{N}'(T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E^k \mathbf{A}_K) \subset T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E^{k+1}\mathbf{A}_K$. This implies, by induction on k, that $(T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E \mathbf{A}_K)^{\mathcal{N}'(x)=x} \subset T \cdot \mathcal{O}_E[\![T]\!]^\times$. We have $F_g(T) \in T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E \mathbf{A}_K$ and $\mathcal{N}'(g(T)) = g(T)$ if $g \in \Gamma_K$ and hence $F_g(T) \in (T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E \mathbf{A}_K)^{\mathcal{N}'(x)=x} \subset T \cdot \mathcal{O}_E[\![T]\!]^\times$.

In the statement of prop 4.2, one should therefore assume that $P(T) \in T \cdot \mathcal{O}_E[\![T]\!]$. The only place where this prop is used is lemma 4.5, in which $P(T)$ does belong to $T \cdot \mathcal{O}_E[\![T]\!]$.

12. Théorie de Sen et vecteurs localement analytiques

Section 6. Section 6 starts with "*Dans ce chapitre, nous ne faisons pas d'hypothèse sur le groupe de Lie Γ_K. Comme Γ_K est un groupe de Lie p-adique compact, de dimension finie, il existe (§27 de [26]) un groupe analytique $\mathbb{G}$, défini sur $\mathbf{Q}_p$, tel que l'on ait $\Gamma_K = \mathbb{G}(\mathbf{Z}_p)$.*"

One should rather ask that $\mathbb{G}$ is a rigid analytic group such that Γ_K has an open subgroup that is isomorphic to $\mathbb{G}(\mathbf{Q}_p)$. The existence of such a $\mathbb{G}$ is proved in §3.5 of Lahiri's *Rigid analytic vectors in locally analytic representations*. It can also be deduced from §27 of [26] as well as the theorem on page 116 of [32].

13. Multivariable (φ, Γ)-modules and locally analytic vectors

Proposition 2.10. The elements x_τ from prop 1.1 likely do not belong to $\hat{F}_\infty$. They belong to $\hat{K}_\infty$ if $F^{\mathrm{Gal}} \subset K$. The proof of prop 2.10 therefore only works as written if $F^{\mathrm{Gal}} \subset K$. The general case follows by replacing K by $K' = F^{\mathrm{Gal}} \cdot K$ and then (for the second assertion) using the fact that $\hat{K}_\infty \cap K'_\infty = K_\infty$.

Lemma 4.3. There are arguments missing from the proof, in particular the fact that there exists $C(I)$ such that $V(x, I) \geqslant V(tx, I) - C(I)$ if $x \in \widetilde{\mathbf{B}}^I$. There should also be a sublemma to the effect that if $y \in t_\pi(\widetilde{\mathbf{B}}^I_F)^{F\text{-la}}$ then $\nabla(y) \in t_\pi(\widetilde{\mathbf{B}}^I_F)^{F\text{-la}}$ as well.

Theorem 4.4. The proof should start by saying that we take $x \in \tilde{\mathbf{A}}^{[r;s]}$. In addition, there is a problem with the proof of item (1). Indeed, u^d/π does not belong to $\tilde{\mathbf{A}}^{[0;s]}$ so it is not true that if $x \in \tilde{\mathbf{A}}^{[r;s]}$, then there exists k_n such that $(u^d/\pi)^{k_n} \cdot x \in \tilde{\mathbf{A}}^{[0;s]} + \pi^n \tilde{\mathbf{A}}^{[r;s]}$.

The element u^d/π belongs to $\tilde{\mathbf{A}}^{[0;s]}$ if $r = s$, so that the proof works for $r = s$. Now take $x \in \tilde{\mathbf{A}}^{[r;s]}$. The proof tells us that $x = \varphi_q^{-m}(f(u))$ where $f(Y)$ converges on the annulus corresponding to $[q^m s; q^m s]$. We can therefore write $f(Y) = f^+(Y) + f^-(Y)$ where $f^+(Y)$ is the positive part and converges on $[0; q^m s]$ and $f^-(Y)$ is the negative part and converges and is bounded on $[q^m s; +\infty[$. The element $x^- = \varphi_q^{-m}(f^-(u))$ belongs to both $\widetilde{\mathbf{B}}^{[r;s]}$ (since $x^- = x - x^+$) and to $\widetilde{\mathbf{B}}^{[s;+\infty[}$, so that it belongs to $\widetilde{\mathbf{B}}^{[r;+\infty[}$.

We now claim that if the power series $f^-(Y)$ converges on the annulus $[q^m s; +\infty[$ and if $f^-(u)$ belongs to $\widetilde{\mathbf{B}}^{[q^m r;+\infty[}$, then $f^-(Y)$ converges on $[q^m r; +\infty[$. In the cyclotomic case, this is proved in lemma II.2.2 of [CC98]. The proof in the LT case is analogous. This implies that $f(Y)$ converges on the annulus corresponding to $[q^m r; q^m s]$.

Lemma 5.3. The condition that I does not contain 0 is not necessary. The lemma (and theorem 5.4 and corollary 5.5) is true if 0 belongs to I. The condition is that I does not contain $+\infty$.

14. Lubin's conjecture for full p-adic dynamical systems

Corollary 2.7. It is not true that $\mathrm{L}_{\mathcal{F}}(a) = \mathrm{L}_{\mathcal{F}}(b)$ implies that $a = b$, even if $\mathrm{L}_{\mathcal{F}}(a)$ is in $\mathrm{Fil}^1 \mathbf{B}_{\mathrm{dR}}$. We need a and b themselves to be in $\mathrm{Fil}^1 \mathbf{B}_{\mathrm{dR}}$. The additional argument is the one that is in the proof of lemma 4.1 of [Spe18]. Assume that $\tau = \mathrm{Id}$ (otherwise twist everything by τ). Let $x_n = \theta \circ \varphi_q^n(x)$. We have $g(\mathrm{L}_{\mathcal{F}}(x_n)) = \eta(g) \cdot \mathrm{L}_{\mathcal{F}}(x_n)$ for all $n \geqslant 1$ so that $\mathrm{L}_{\mathcal{F}}(x_n) = 0$ for all $n \geqslant 1$. The x_n are zeroes of $\mathrm{L}_{\mathcal{F}}$ that converge to 0 so that $x_n = 0$ for $n \gg 0$. This implies that $\varphi_q^n(x) \in \mathrm{Fil}^1 \mathbf{B}_{\mathrm{dR}}$ for $n \gg 0$ and the proof of corollary 2.7 now works with $\varphi_q^n(x)$ for $n \gg 0$ instead of x.

15. Iwasawa theory and F-analytic Lubin-Tate (φ, Γ)-modules

Introduction and §1.1. The condition that $F/\mathbf{Q}_p$ is Galois is not necessary, and is used nowhere in the paper. One can assume that F is a finite extension of $\mathbf{Q}_p$.

Theorem 1.3.1. In the proof of the theorem, the operator ∇ is not defined. It is ∇_g for some $g \in \Gamma_K$ close to 1.

Theorem 3.5.3. In the second displayed formula, $\exp^*_{F_n, V^*(1-j)}$ should be replaced by $\exp^*_{F_n, V(\chi_\pi^j)^*(1)}$

References

[Ber13] Laurent Berger, *Multivariable Lubin-Tate (φ, Γ)-modules and filtered φ-modules*, Math. Res. Lett. **20** (2013), no. 3, 409–428. MR 3162836

[CC98] F. Cherbonnier and P. Colmez, *Représentations p-adiques surconvergentes*, Invent. Math. **133** (1998), no. 3, 581–611. MR 1645070

[Col08] Pierre Colmez, *Espaces vectoriels de dimension finie et représentations de de Rham*, Astérisque (2008), no. 319, 117–186, Représentations p-adiques de groupes p-adiques. I. Représentations galoisiennes et (φ, Γ)-modules. MR 2493217

[Spe18] Joel Specter, *The crystalline period of a height one p-adic dynamical system*, Trans. Amer. Math. Soc. **370** (2018), no. 5, 3591–3608. MR 3766859

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