Small Independent Sets versus Small Separator in Geometric Intersection Graphs.

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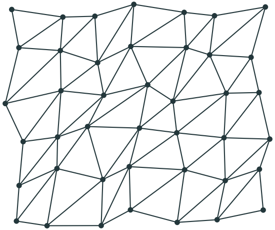
Introduction

Subexponential time

Theorem (Lipton and Tarjan, '79)
Finding a maximum independent set in planar graphs can be solved in $2^{O(\sqrt{n})}$ *time.*



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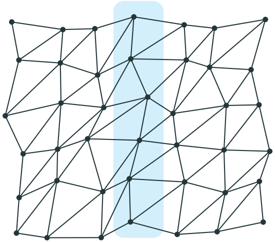

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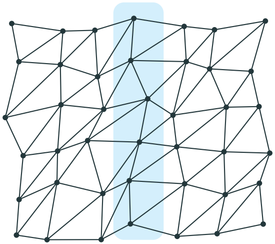

Remarks:

- tight under the ETH ;
- motivation for treewidth.

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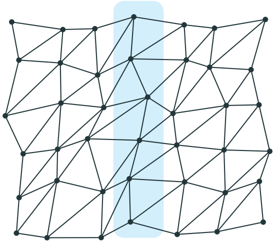

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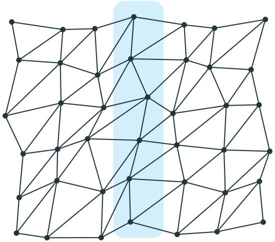

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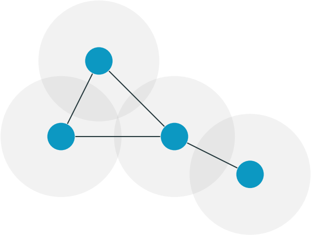
square-root phenomenon

Unit disk graphs

Definition

A *unit disk graph* is the intersection graph of unit disks on the plane.

Example



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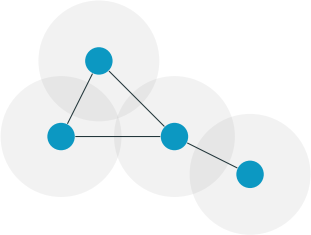

Definition
A *unit ball graph in* $\mathbb{R}^d$ is the intersection graph of unit balls in $\mathbb{R}^d$.

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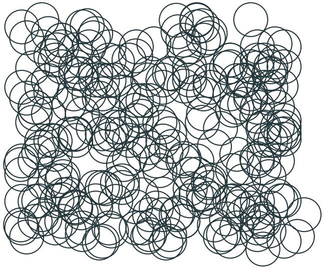

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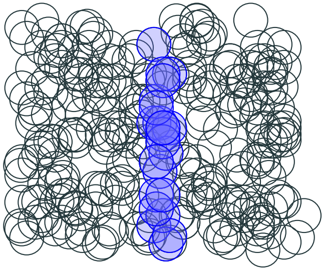

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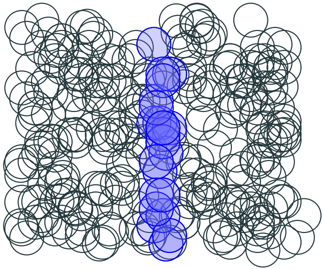

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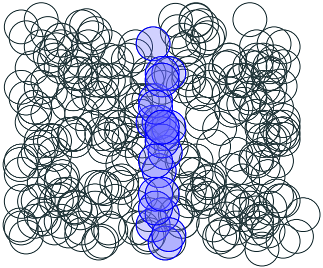

Motivates dense analog of treewidth:

- $\mathcal{P}$-flattened treewidth (De Berg et al, '18);
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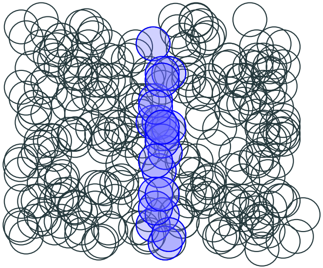

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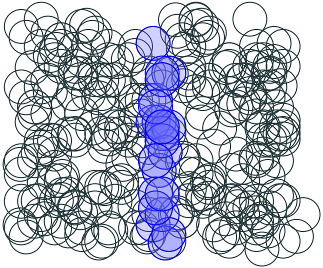

Maximum Indep. Set, Dominating Set, Hamiltonian Path	$2^{O(n^{1-1/d})}$
2-Subcoloring	$2^{O(n^{1-1/(d+1)})}$
Maximum Clique, Clique Cover	$2^{O(n)}$

2-**subcoloring**

Definitions :

- A *2-subcoloring* of a graph G is a partition of $V(G)$ into V_1, V_2 such that $G[V_i]$ is a disjoint union of cliques for all $i = 1, 2$

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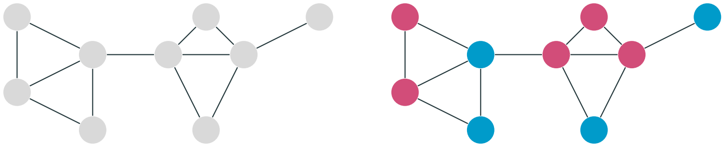

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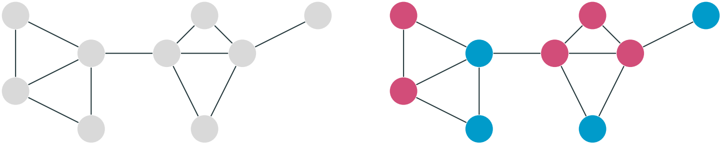

Proposition
A graph G is a disjoint union of cliques if and only if it is P_3-free.

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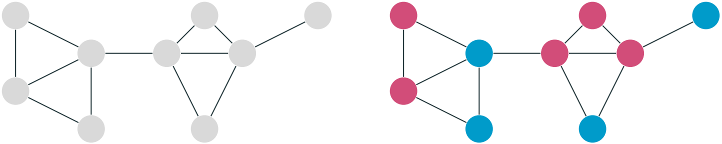
- A 2-*subcoloring* of a graph G is a partition of $V(G)$ into V_1, V_2 such that $G[V_i]$ is a disjoint union of cliques for all $i = 1, 2$
- 2-Subcoloring : Given a graph G, does G have a 2-subcoloring ?

Example



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- M. and Watrigant '25

  $\boxed{\text{2-Subcoloring remains NP-hard on unit disk graphs.}}$

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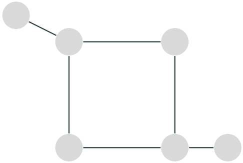
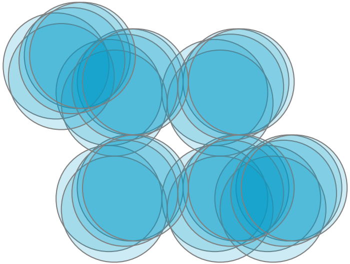


Planar graph



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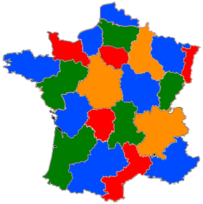
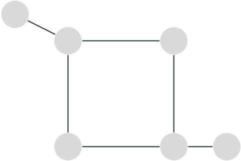
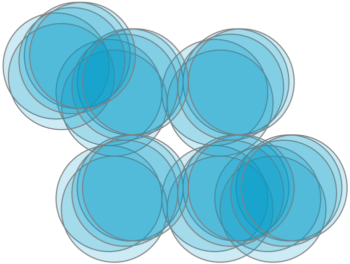




Planar graph

$\chi \leq 4$



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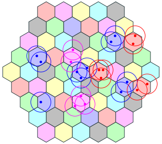

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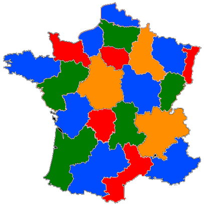

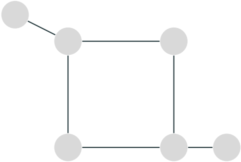

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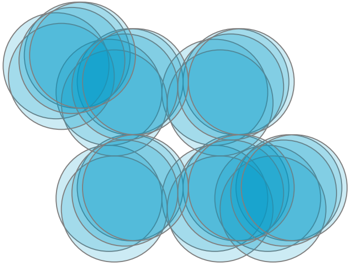

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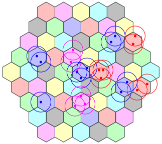

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Question: Does 2-SUBCOLORING problem exhibit the square-root phenomenon on unit disk graphs?

Subexponential algorithm for 2-subcoloring

The case of 2-Subcoloring

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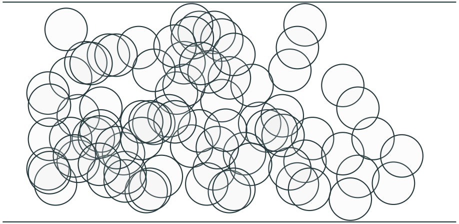

- 2-Subcoloring is NP-complete on unit disk graphs of bounded height.
- If the framework of (de Berg et al '18) applies, it should be polynomial.

A win/win theorem

Two informations :

1. If $\alpha(G) \leqslant k$, then 2-Subcoloring can be solved in time $2^{O(k)} \cdot n^{O(1)}$ (Kanj et al, '18).
2. If $\mathrm{tw}(G) \leqslant k$, then 2-Subcoloring can be solved in time $2^{O(k)} \cdot n^{O(1)}$ (Fiala et al, '03).

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Can we find a win-win ?

Theorem (M., Watrigant '26)
For any n-vertex unit disk graph G, there exists a set $X \subseteq V(G)$ such that:

1. $|X| = O(n^{2/3})$;
2. *for any connected component C of $G - X$, $\alpha(G[C]) = O(n^{2/3})$.*

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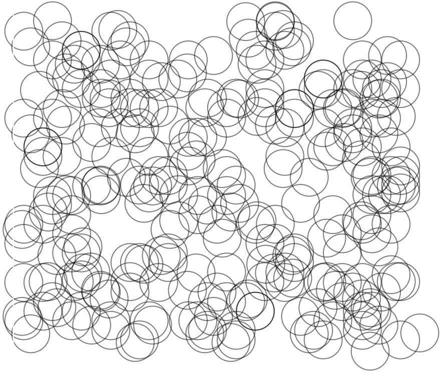
Theorem (M., Watrigant '26)
For any n-vertex intersection graph of similarly-sized β-fat objects in $\mathbb{R}^d$, there exists a set $X \subseteq V(G)$ such that:

1. $|X| = O(n^{1-1/(d+1)})$;
2. *for any connected component C of $G - X$,* $\alpha(G[C]) = O(n^{1-1/(d+1)})$.

Proof

Proof. Let $\mathcal{D} = \{(x_v, y_v)\}_{v \in V(G)}$ the centers and fix $p \in \mathbb{N}$



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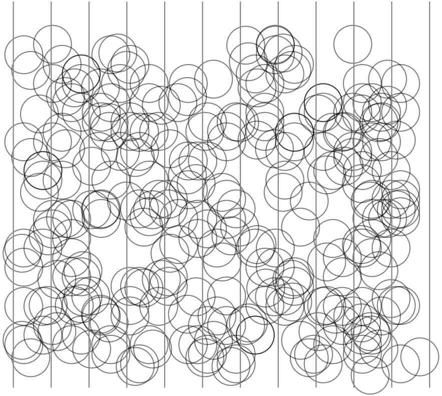

- Let $X_i = \{v \in V(G) \mid \lfloor x_v/p \rfloor = i \mod p\}$ for $i \in \{0, ..., p-1\}$

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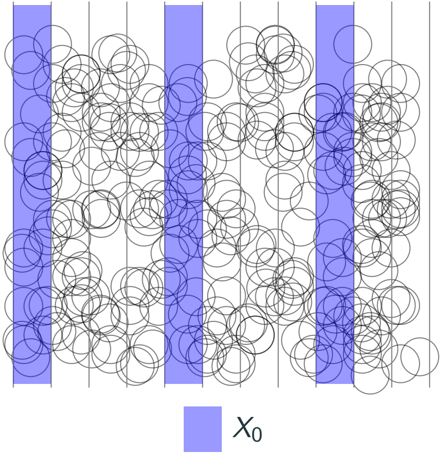

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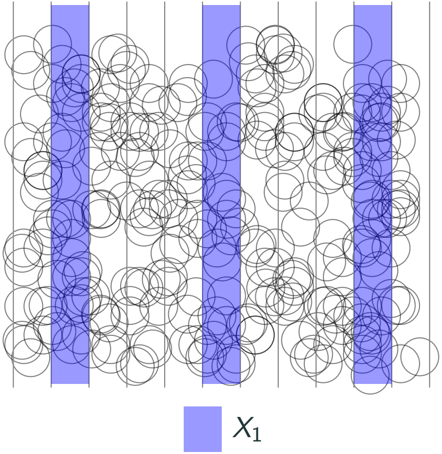

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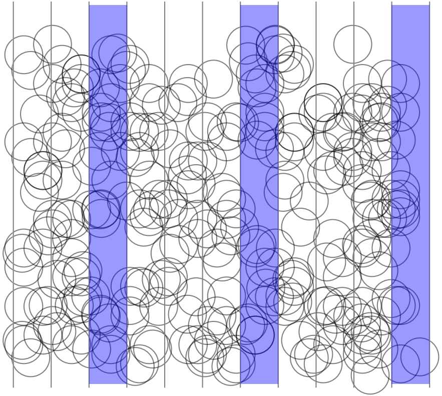

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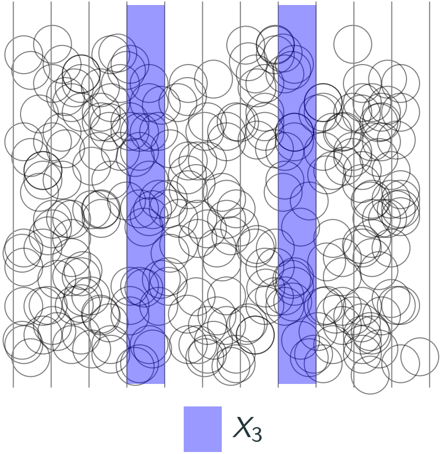

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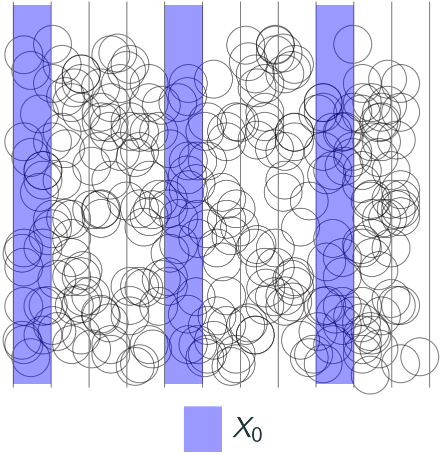

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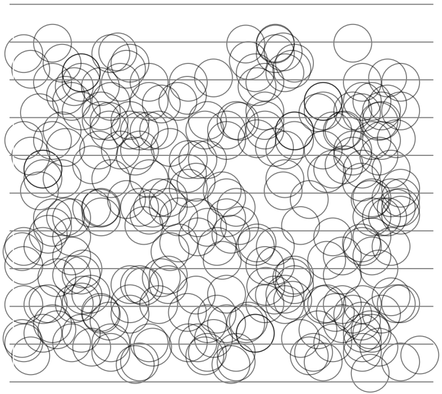

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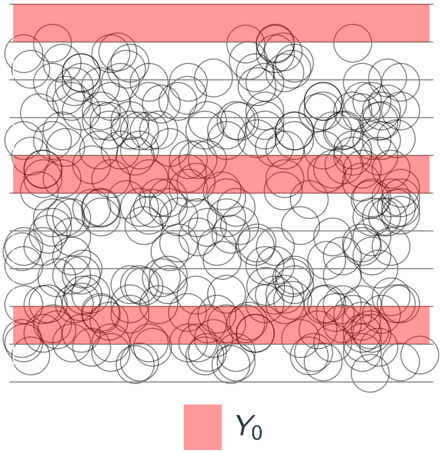

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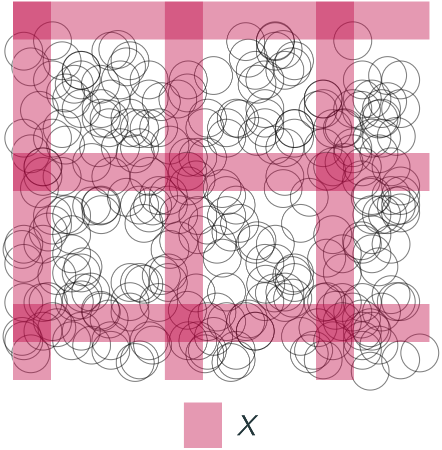

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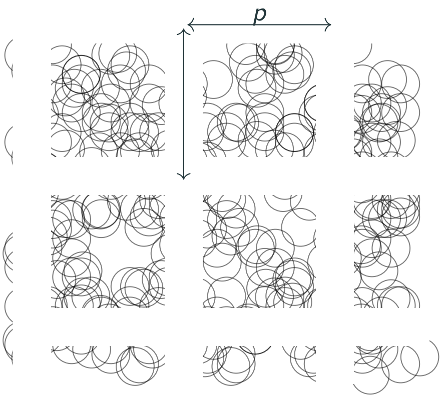

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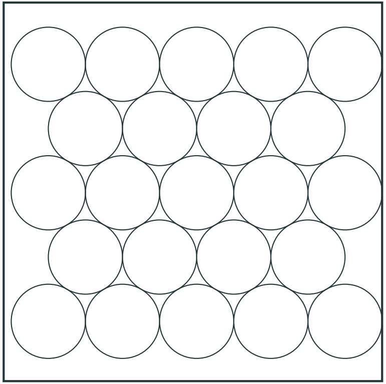

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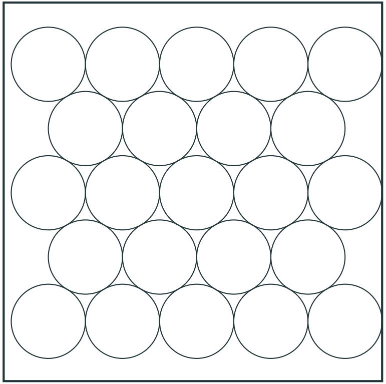

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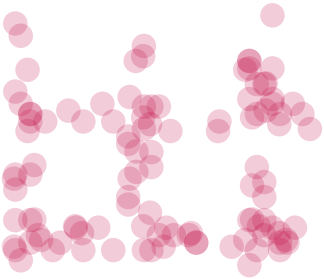
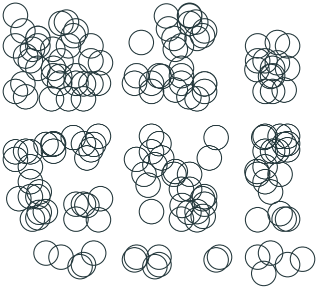


X



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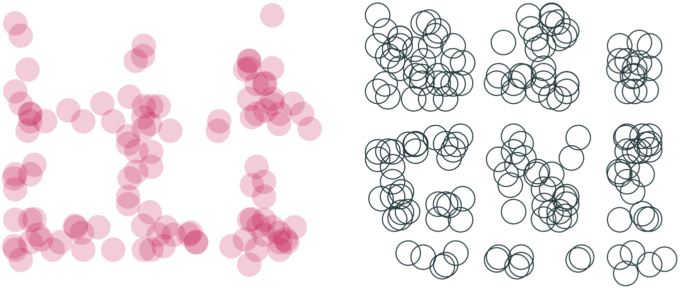

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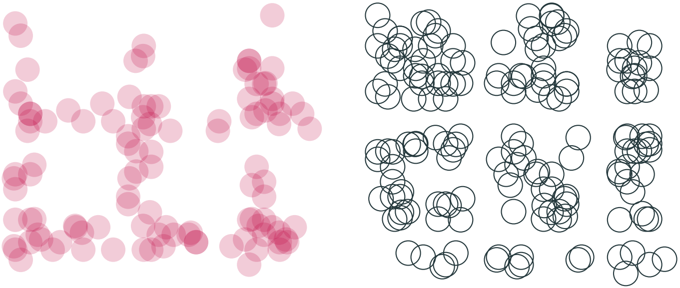

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- $|X| \leqslant 2\frac{n}{p} = 2\frac{n}{n^{1/3}} = 2n^{2/3}$
- For every connected component C of $G - X$, $\alpha(G[C]) = O(p^2) = O(n^{2/3})$

Algorithmic framework

Theorem (M., Watrigant '26)
For any n-vertex unit disk graph G, there exists a set $X \subseteq V(G)$ such that:

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1. Test every 2-subcoloring of $G[X]$ in time $2^{O(|X|)}$
2. For each of them, use the $2^{\alpha(G[C])} \cdot n^{O(1)}$-time algorithm to extend it to each connected component C of $G - C$

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- *The framework also applies to a problem called Two Sets Cut-Uncut, introduced in (Bentert et al, '2023).*
- *It motivates the definition of a new graph parameter, the α-modulator number.*

α-modulator number

Definition
Given a graph G, the *α-modulator number of G*, denoted α-mod(G) is the minimum integer k such that there exists $X \subseteq V(G)$ such that :

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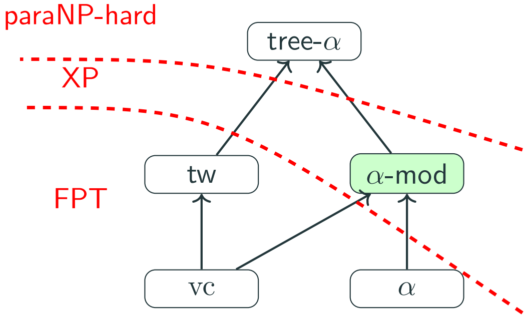

Parameterized complexity of 2-Subcoloring.

Lower Bound

Matching Lower Bound

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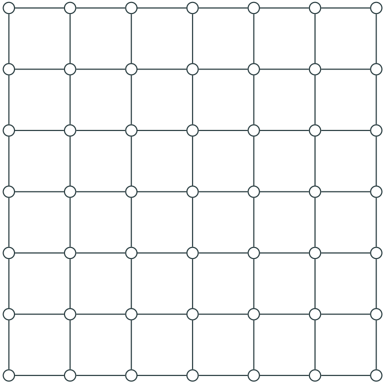
weak square-root phenomenon

Reduction sketch



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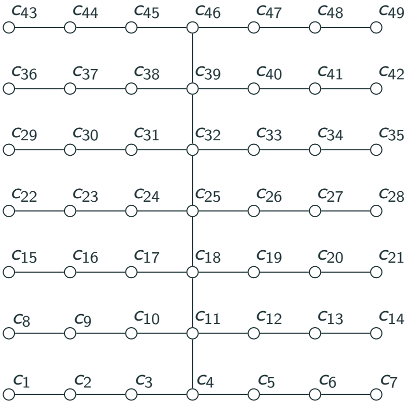

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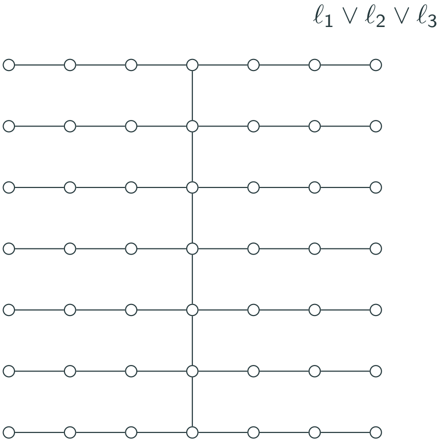

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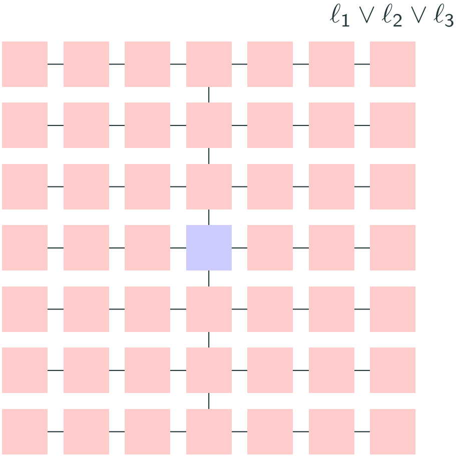

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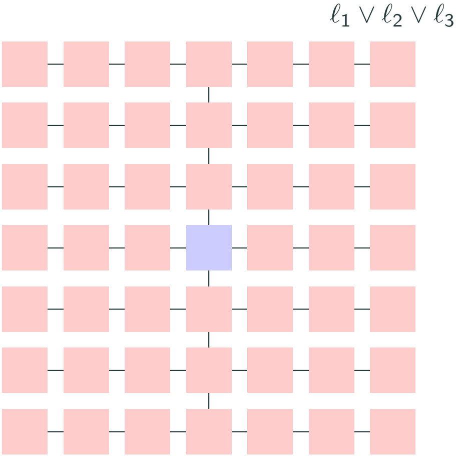

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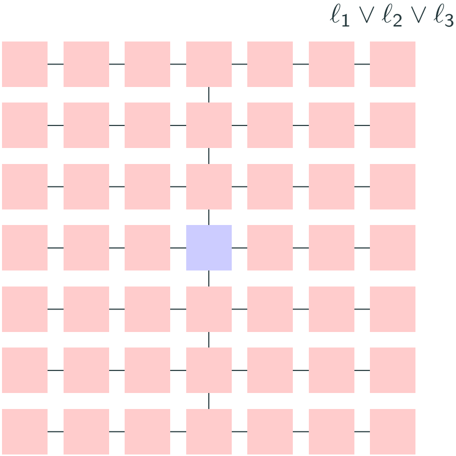

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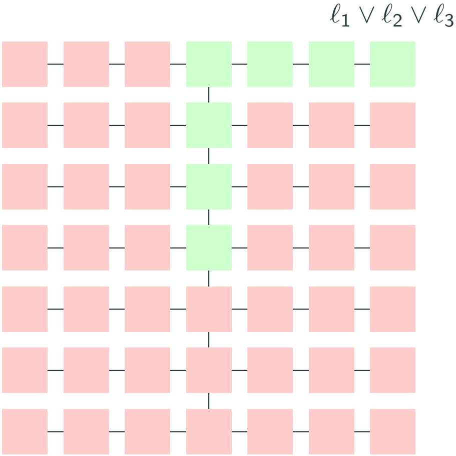

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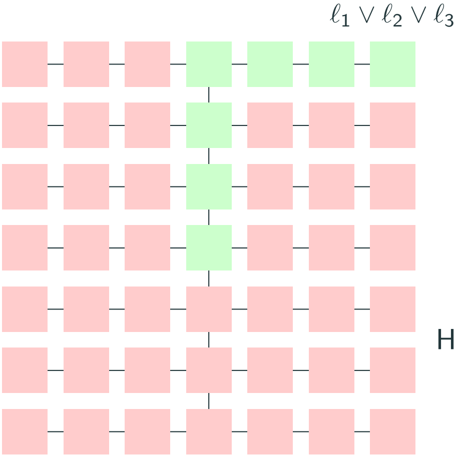

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How many vertices in total :

$$\leq \sum_{\ell \text{ literal}} RL_\ell = O(m \cdot \sqrt{m}) = O(n^{\frac{3}{2}})$$

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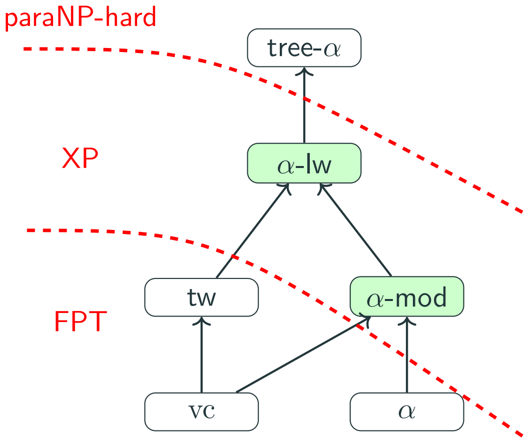

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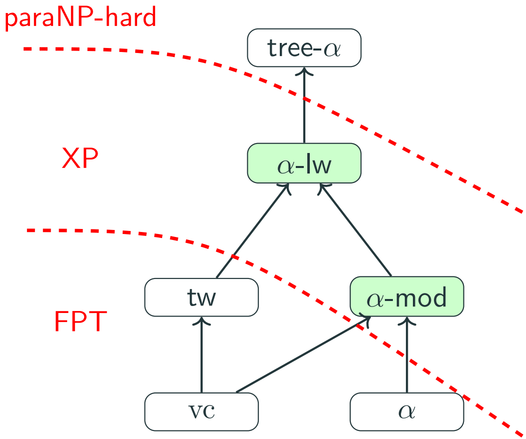

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THANKS!