Broken book decompositions and Birkhoff sections for Reeb vector fields of SHS

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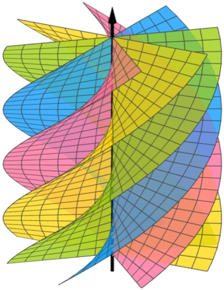
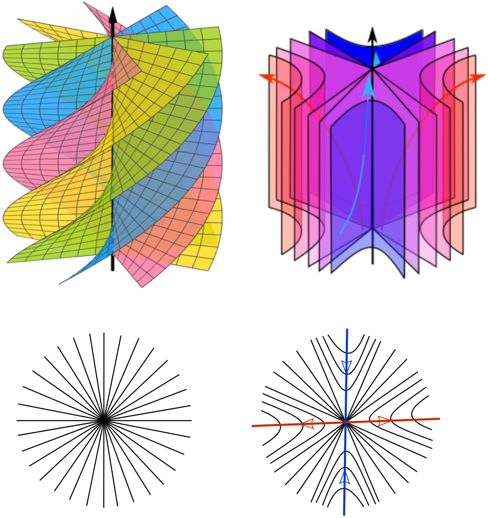
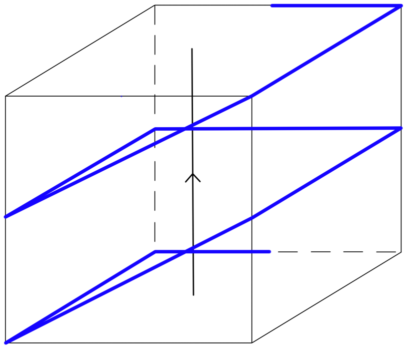
Bindings of broken books



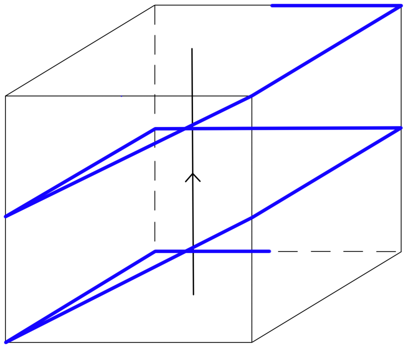
Bindings of broken books

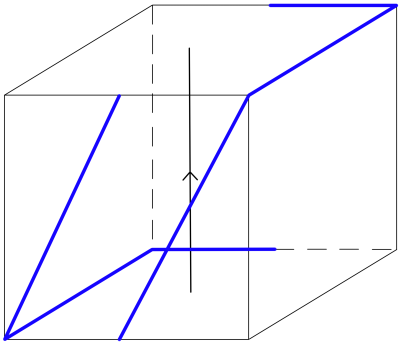


Helicoidal boxes 1

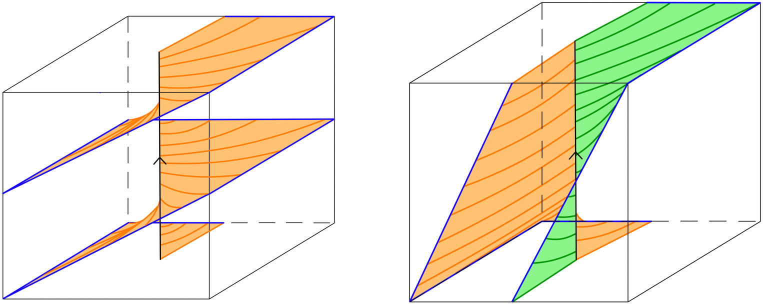


Helicoidal boxes 1

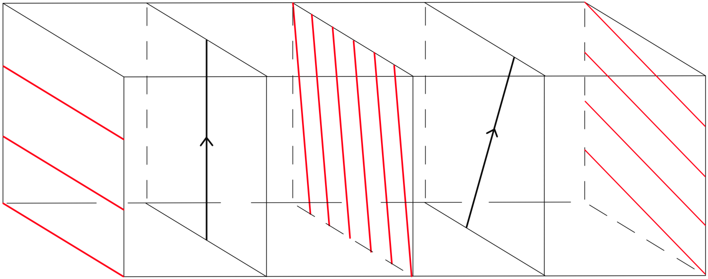




Helicoidal boxes 2



Helicoidal boxes 3



Finite number of periodic orbits

Theorem 1

Let X be an aperiodic Reeb vector field of a SHS (λ, ω) on a closed 3-manifold M. Then one of the following holds:

1. *$M = T^3$ or a positive parabolic torus bundle over $\mathbb{S}^1$ and the flow is a suspension;*
2. *M is a hyperbolic torus bundle over $\mathbb{S}^1$, and the flow is a suspension on $M \setminus T^2$.*

Finite number of periodic orbits

Theorem 2

Let (λ, ω) be a contact non-degenerate SHS on a closed 3-manifold M with at least one periodic orbit and finitely many periodic orbits. Then either

- *The flow is the suspension of a symplectomorphism of a surface Σ_g with finitely many periodic points.*
- *The ambient manifold M is the 3-sphere or a lens space and there are exactly two periodic orbits.*