

TD01: VECTOR BUNDLES
M1 - DIFFERENTIAL GEOMETRY, 2019-2020

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Exercise 1 (Warm-up). Let M be a smooth manifold. What are the global sections of $M \times \mathbf{R}^k \rightarrow M$, $TM \rightarrow M$ and $\bigwedge^k T^*M \rightarrow M$?

Exercise 2. Recall that a smooth manifold M is said to be *parallelizable* if its tangent bundle is trivial (or, in other words, M admits a smooth global frame). Let M be a smooth m -manifold and $N \subset \mathbf{R}^{n+1}$ an orientable hypersurface. Is the product manifold $M \times N$ parallelizable?

Exercise 3.

- (1) How many vector bundle of rank $r > 0$ over the circle \mathbf{S}^1 do there exist (up to isomorphism)?
- (2) What is the family of line bundles over \mathbf{S}^n up to isomorphism for $n \geq 2$?