

## CR12

<https://www.lirmm.fr/~ochem/slides1.pdf>

## HW1: $d$ -directed VS critical exponent

- $w$  is  $d$ -directed if for every factor  $f$  of  $w$  with  $|f| = d$ ,  $f^R$  is not a factor of  $w$ .
- $abcab$  is a repetition with period 3 and exponent  $5/3$ .
- $ababa$  is a repetition with period 2 and exponent  $5/2$ .
- $\alpha$ -free: no exponent  $\geq \alpha$ .
- $(\alpha^+)$ -free: no exponent  $> \alpha$ .
- TO DO: find all pairs  $(d, \alpha)$  such that there exists an infinite binary word that is  $d$ -directed and  $(\alpha^+)$ -free.

## HW2: Gangloff and Talon

- <https://www.sciencedirect.com/science/article/abs/pii/S0304397520301572>
- <https://www.lirmm.fr/~ochem/gangloff2020.pdf>
- **TO DO: Thm 7, Thm 9: (minimal) total domination**

## HW3: Languages with small exponential growth

- There are  $2^n$  binary words.
- There are  $n + 1$  binary words avoiding  $01$ .
- There are  $\Theta(\varphi^n)$  binary words avoiding  $11$ .
- There are exponentially many binary words avoiding  $\{11, 000\}$ .
- TO DO: Find the growth rate given a finite set of forbidden factors.
- TO DO: Use that to find the family of exponential languages with least growth rates.

## HW4: Grytczuk and Stankiewicz

- <https://arxiv.org/pdf/2011.12822.pdf>
- Theorem 5:  $X_5 = X_1^R$  but  $|S_1| = 42$  and  $|S_5| = 41$ .
- Problem 1: multiplicativity.

## HW5: Growth rate of abelian square-free words

- `https://www.lirmm.fr/~ochem/samsonov2011.pdf`
- Extendability of (abelian) square-free words.
- TO DO: compute a set of two-way extendable abelian square-free words.
- TO DO: get an upper bound on the growth rate.

## HW6: Additive cubes over $\{-1, 0, 1\}$

- <https://www.lirmm.fr/~ochem/lietard.pdf>
- Thm 8 p. 204/214
- Are additive cubes avoidable over  $\{1, 2, 3, 4\} = \{-3, -1, 1, 3\}$ ?
- Are additive cubes avoidable over  $\{1, 2, 3\} = \{-1, 0, 1\}$ ?

## HW7: Subtypes for *AABB.ABBA*

- `https://www.lirmm.fr/~ochem/morphisms/aabbc.pdf`
- `https://www.lirmm.fr/~ochem/morphisms/main.pdf` (p. 10)
- Are there subtypes for the exponential case?

## HW8: Consecutive additive squares

- $AABB$  is not 2-avoidable but  $AABBCC$  is 2-avoidable.
- Over two letters, "abelian" is the same as "additive".
- Are there infinite binary words containing only finitely many consecutive additive squares?

## HW9: Does 10 have a friend?

- Is there  $n > 10$  such that  $\sigma(n)/n = 9/5$ ?