HdR:

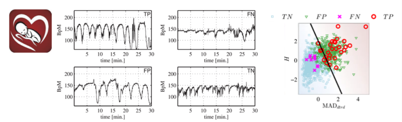
Approches variationnelles régularisées pour la résolution de problèmes inverses et pour l'apprentissage machine : de la modélisation aux algorithmes à grande échelle

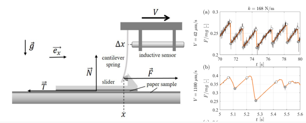
Nelly Pustelnik

ENS de Lyon – 30 septembre 2021

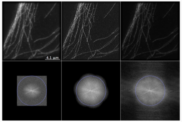
Context

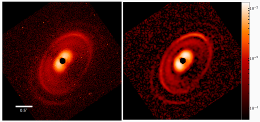
- **Signal processing**

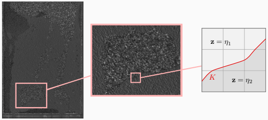




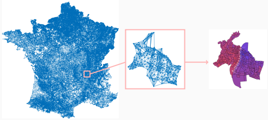
- **Image processing**

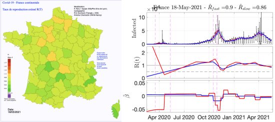






- **Graph processing**





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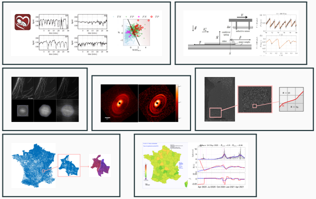

➞ **Three quantities of interest**:

- $\mathrm{z} \in \mathbb{R}^{\overline{N}}$: Data/measures,
- $\overline{\mathrm{x}} \in \mathbb{R}^{N}$: True (unknown) parameters,
- $\widehat{\mathrm{x}} \in \mathbb{R}^{N}$: Estimated parameters.

➞ **Questions to address**:

- Global procedure rather than successive steps.
- Handle with large dimensionality.
- Parameter free.
- Stability of the result.

Context

Data: z

↓

Direct/acquisition model

↓

Design a cost function

↓

Minimization algorithm

↓

Hyperparameter tuning

↓

Estimated parameters: $\widehat{\mathrm{x}}(\mathrm{z}; \widehat{\lambda})$

$$\mathrm{z} = \mathcal{D}(\mathrm{A}\overline{\mathrm{x}})$$

↓

$$\widehat{\mathrm{x}}(\mathrm{z}; \lambda) \in \underset{\mathrm{x}}{\mathrm{Argmin}}\ \boldsymbol{\psi}(\mathrm{x}; \mathrm{z}) + \lambda \boldsymbol{\varphi}(\mathrm{x})$$

↓

Sequence $\mathrm{x}^{[k+1]} = \boldsymbol{\Phi}\mathrm{x}^{[k]}$

↓

$$\widehat{\lambda} \in \underset{\lambda}{\mathrm{Argmin}} \|\overline{\mathrm{x}} - \widehat{\mathrm{x}}(\mathrm{z}; \lambda)\|_2^2$$

Context

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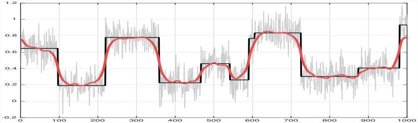
$$\widehat{\lambda} \in \underset{\lambda}{\mathrm{Argmin}} \|\overline{\mathrm{x}} - \widehat{\mathrm{x}}(\mathrm{z}; \lambda)\|_2^2$$

Piecewise constant denoising: $z = \bar{x} + n$ with $n = \mathcal{N}(0, \sigma^2 \mathrm{Id})$

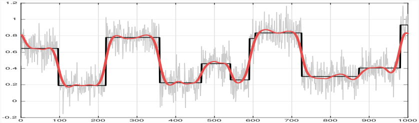
➡ Minimization problem

$$\widehat{x}(z; \widehat{\lambda}) = \arg\min_{x \in \mathbb{R}^N} \frac{1}{2}\|x - z\|_2^2 + \lambda \|Dx\|_\bullet \quad \text{where} \quad \begin{cases} Dx = d * x \\ \lambda > 0 \end{cases}$$

➡ Linear denoising



$d = \begin{bmatrix} 1 & -1 \end{bmatrix}$; $\|\cdot\|_\bullet = \|\cdot\|_2^2$; Large λ



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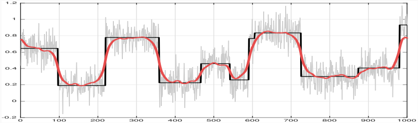

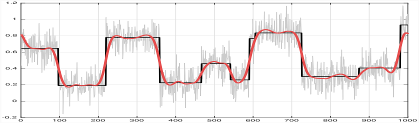

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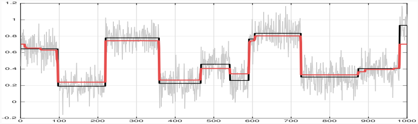
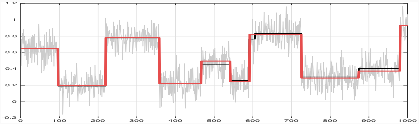
➡ Non-linear denoising.





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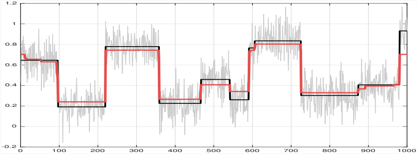
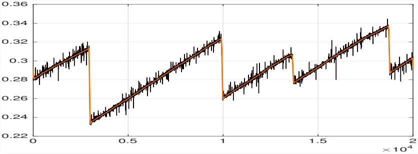
➡ **Non-linear denoising: piecewise constant/linear**





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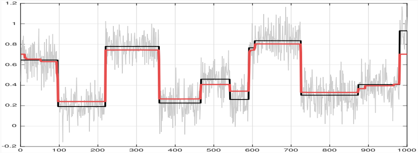

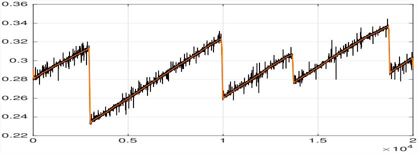

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➡ In general, more complex objective function:

$$\widehat{\mathrm{x}} \in \underset{\mathrm{x}}{\mathrm{Argmin}}\ \psi(\mathrm{x}; \mathrm{z}) + \lambda \varphi(\mathrm{x})$$

Iterative scheme

➡ Minimization problem

$$\widehat{\mathrm{x}} \in \underset{\mathrm{x}\in\mathbb{R}^N}{\mathrm{Argmin}}\ f(\mathrm{x})$$

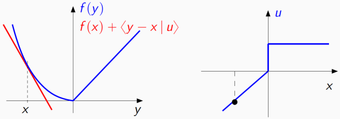
- f differentiable: (explicit) **gradient method**:

$$(\forall k \in \mathbb{N}) \qquad \mathrm{x}^{[k+1]} = \mathrm{x}^{[k]} - \tau_k \nabla f(\mathrm{x}^{[k]})$$

- f non-differentiable: (explicit) **subgradient method**:

$$(\forall k \in \mathbb{N}) \qquad \mathrm{x}^{[k+1]} = \mathrm{x}^{[k]} - \tau_k \mathrm{u}^{[k]} \quad \text{with} \quad \mathrm{u}^{[k]} \in \partial f(\mathrm{x}^{[k]})$$

$$\partial f(x) = \{u \in \mathcal{H} \mid (\forall y \in \mathcal{H})\ \langle y - x | u\rangle + f(x) \leq f(y)\}$$



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$$= \text{prox}_{\tau_k f}(x^{[k]})$$

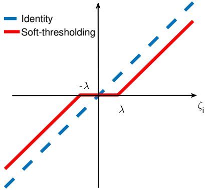
Proximity operator

<u>Definition</u> [Moreau,1965] Let $f : \mathcal{H} \to]-\infty, +\infty]$ be a convex, l.s.c., and proper function. The proximity operator of f at point $\mathrm{x} \in \mathcal{H}$ is the **unique point** denoted by $\mathrm{prox}_f \mathrm{x}$ such that

$$(\forall \mathrm{x} \in \mathcal{H}) \qquad \mathrm{prox}_f \mathrm{x} = \arg\min_{v \in \mathcal{H}} \frac{1}{2} \|v - \mathrm{x}\|^2 + f(v)$$

➡ **Existing many closed form expressions**

- $\mathrm{prox}_{\lambda\|\cdot\|_1}$: **soft-thresholding** with a fixed threshold $\lambda > 0$.
- exhaustive list: **PROX Repository**



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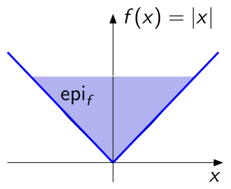
- exhaustive list: **PROX Repository**

➡ **More complicated task**: $\mathrm{prox}_{\iota_{\mathrm{epi}\, f}}$, $\mathrm{prox}_{f_1+f_2}$, $\mathrm{prox}_{f \circ \mathrm{D}}$.

Proximity operators: projection onto epigraphs

Definition : The **epigraph** of f is

$$\operatorname{epi} f = \{(x,\zeta) \in \operatorname{dom} f \times \mathbb{R} \mid f(x) \leq \zeta\}$$



➡ **Examples**:

- Astronomy: Stokes parameters constraint $\sqrt{Q^2 + U^2} \leq I$
- Projection onto ℓ_1-ball: $\sum_n |x_n| \leq \eta \Leftrightarrow \begin{cases} |x_n| \leq \zeta_n \\ \sum_n \zeta_n \leq \eta \end{cases}$

➡ **New result**: **[Chierchia, P., Pesquet, Pesquet-Popescu, 2015]**

$$P_{\operatorname{epi} f}(x,\zeta) = (p,\theta) \quad \text{where} \quad \begin{cases} p = \operatorname{prox}_{\frac{1}{2}(\max\{f-\zeta,0\})^2}(x), \\ \theta = \max\{f(p),\zeta\}. \end{cases}$$

Iterative scheme

➡ **Minimization problem** :

$$\widehat{\mathrm{x}} \in \underset{\mathrm{x}}{\mathrm{Argmin}}\ f_1(\mathrm{x}) + f_2(\mathrm{x})$$

→ Requires the computation of $\nabla f_1 + \nabla f_2$ or $\mathrm{prox}_{f_1+f_2}$

➡ **New results**

- **[Pustelnik, Condat]** $\mathrm{prox}_{f_1+f_2} = \mathrm{prox}_{f_2} \circ \mathrm{prox}_{f_1}$ if $f_2(\mathrm{x}) = \sum_{n\in\Omega} h(x_n)$ where $h \in \Gamma_0(\mathbb{R})$ $f_1(\mathrm{x}) = \sum_{(m,m')\in\mathbb{E}} \sigma_{C_{m,m'}}(x_{m'} - x_m)$ and σ_C is a support function on a real closed interval.
- **[Foare, Pustelnik, Condat, 2019]** Closed form expression if $f_1(\mathrm{x}) = \|(1-\mathrm{x}) \odot \mathrm{Du}\|^2$ with u fixed, $f_2(\mathrm{x}) = \sum_n h_n(x_n)$ where $h_n \in \Gamma_0(\mathbb{R})$.

➡ **Few closed from expressions in the literature**.

Iterative scheme

➡ **Minimization problem** :

$$\widehat{x} \in \underset{x}{\operatorname{Argmin}}\ f_1(x) + f_2(x)$$

➡ **Design of a recursive sequence of the form**

$$(\forall k \in \mathbb{N}) \qquad x^{[k+1]} = \boldsymbol{\Phi} x^{[k]},$$

Gradient descent	$\boldsymbol{\Phi} = \mathrm{Id} - \tau(\nabla f_1 + \nabla f_2)$
Proximal point algorithm	$\boldsymbol{\Phi} = \mathrm{prox}_{\tau(f_1+f_2)}$
Forward-Backward	$\boldsymbol{\Phi} = \mathrm{prox}_{\tau f_2}(\mathrm{Id} - \tau \nabla f_1)$
Peaceman-Rachford	$\boldsymbol{\Phi} = (2\mathrm{prox}_{\tau f_2} - \mathrm{Id}) \circ (2\mathrm{prox}_{\tau f_1} - \mathrm{Id})$
Douglas-Rachford	$\boldsymbol{\Phi} = \mathrm{prox}_{\tau f_2}(2\mathrm{prox}_{\tau f_1} - \mathrm{Id}) + \mathrm{Id} - \mathrm{prox}_{\tau f_1}$

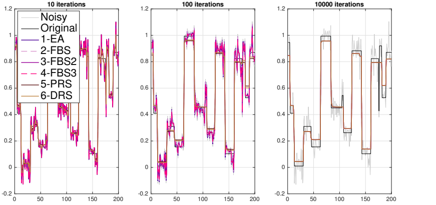
Iterative schemes: prox versus grad

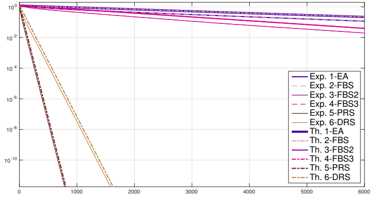
➡ **Minimization problem** :

$$\widehat{\mathrm{x}} \in \underset{\mathrm{x}}{\mathrm{Argmin}}\ f_1(\mathrm{x}) + f_2(\mathrm{x})$$

➡ **Convergence of the sequence** $\left(\mathrm{x}^{[k+1]}\right)_{k\in\mathbb{N}}$ already derived in the literature under specific assumptions for each algorithms.

➡ **New result: Convergence rate and comparisons**: requires **strong convexity** (f-$\frac{\rho}{2}\|\cdot\|^2$ convex). **[Briceño-Arias, P., sub. 2021]**





Iterative scheme

➡ **Minimization problem** :

$$\widehat{\mathrm{x}} \in \underset{\mathrm{x}}{\mathrm{Argmin}}\ f_1(\mathrm{x}) + h_2(\mathrm{D}\mathrm{x})$$

- **Require the computation of** $\mathrm{prox}_{h_2(\mathrm{D}\cdot)}$. **Few closed form.**
- **Reformulation in the dual**: $\min_{\mathrm{w}\in\mathcal{G}} f_1^*(-\mathrm{D}^*\mathrm{w}) + h_2^*(\mathrm{w})$,
- **Primal-dual algorithms**: $\min_{\mathrm{x}} f_1(\mathrm{x}) + f_2(\mathrm{x}) + h_2(\mathrm{D}\mathrm{x})$,
  [Condat,2013][Vũ,2013] [Chambolle-Pock,2011]

Hyperparameters setting: $\tau > 0$, $\gamma > 0$, such that $\frac{1}{\tau} - \gamma\|\mathrm{D}\|^2 > \frac{\beta}{2}$
For $k = 0, 1, \ldots$

$$\left\lfloor \begin{array}{l} \mathrm{w}^{[k+1]} = \mathrm{prox}_{\tau f_2}\left(\mathrm{w}^{[k]} - \tau\nabla f_1(\mathrm{w}^{[k]}) - \tau\mathrm{D}^*\mathrm{x}^{[k]}\right) \\ \mathrm{x}^{[k+1]} = \mathrm{prox}_{\gamma h_2^*}\left(\mathrm{x}^{[k]} + \gamma\mathrm{D}(2\mathrm{w}^{[k+1]} - \mathrm{w}^{[k]})\right) \end{array} \right.$$

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  [Condat,2013][Vũ,2013] [Chambolle-Pock,2011]

  $\rightarrow$ Acceleration with f_1 **strongly convex**.

Hyperparameters setting: $\tau > 0$, $\gamma > 0$, such that $\frac{1}{\tau} - \gamma\|D\|^2 > \frac{\beta}{2}$

For $k = 0, 1, \ldots$

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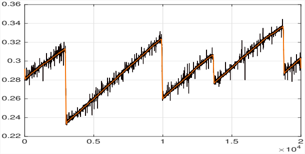
Parameter-free and fast piecewise linear denoising

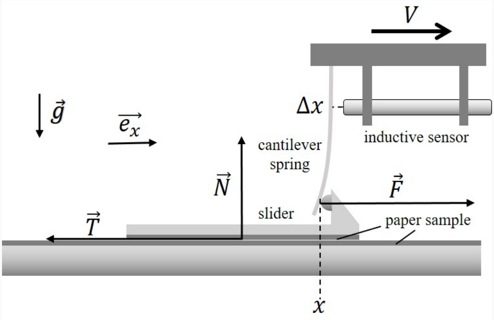
(Parameter-free and) fast piecewise linear denoising

➡ **Motivation: Stick-slip denoising** [Defi Imag'In CNRS 2017]

$$\widehat{u}(z;\lambda) = \arg\min_{u\in\mathbb{R}^N} \frac{1}{2}\|u - z\|_2^2 + \lambda\|Du\|_1$$



- LPENSL experiment



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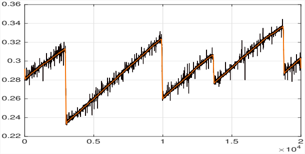
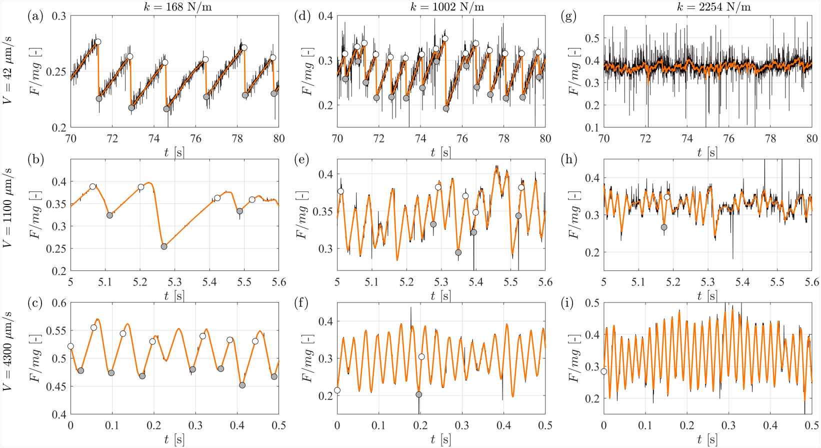


- Variability in the signals and large dataset.
- Minimization performed with Chambolle-Pock with strong convexity of the data-term.
- Selection of the hyperarameter λ by the expert.

  [Colas, Pustelnik, Oliver, Geminard, Vidal, 2019]



k = 168 N/m
k = 1002 N/m
k = 2254 N/m
V = 42 μm/s
V = 1100 μm/s
V = 4300 μm/s
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
F/mg [-]
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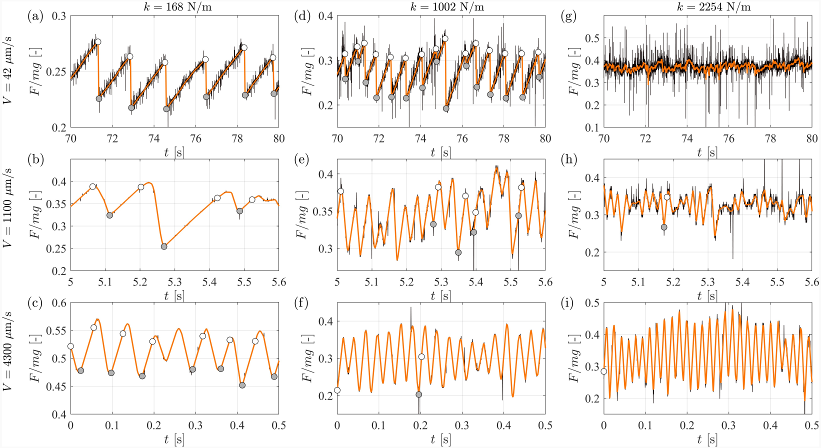
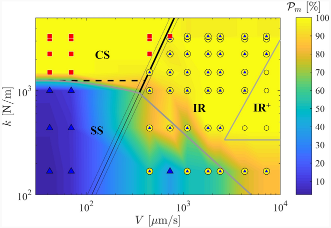


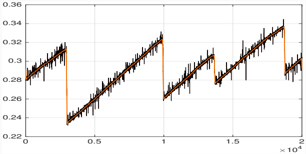
➡ **Phase diagram:**



Parameter-free and fast piecewise linear denoising

➡ **Minimization problem**

$$\widehat{\mathrm{u}}(\mathrm{z};\lambda) = \arg\min_{\mathrm{u}\in\mathbb{R}^N} \frac{1}{2}\|\mathrm{u}-\mathrm{z}\|_2^2 + \lambda\|\mathrm{D}\mathrm{u}\|_1$$



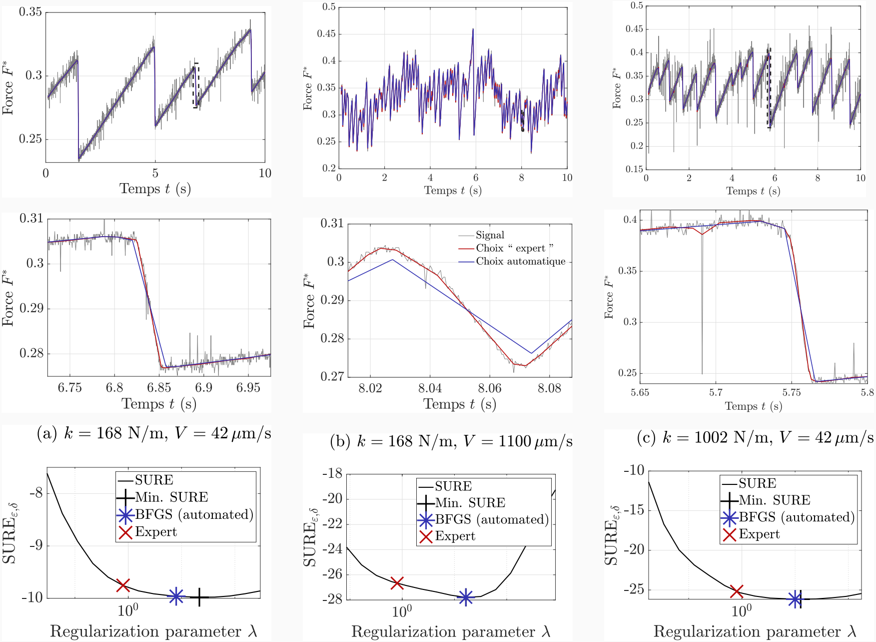
- Selection of the hyperarameter by the expert ⇒ **Automatic tuning with FDMC SURE and efficient research using FDMC SUGAR** [Deledalle et al., 2014]

  [Pascal, Pustelnik, Abry, Geminard, Vidal, 2019]

$$\mathbb{E}\|\widehat{\mathrm{u}}(\mathrm{z};\lambda) - \overline{\mathrm{u}}\|^2 = \lim_{\nu\to 0} \mathbb{E}_{\epsilon,\varepsilon} \widehat{R}_{\nu,\varepsilon}(\mathrm{z};\lambda|\sigma).$$

with

$$\widehat{R}_{\nu,\varepsilon}(\mathrm{z};\lambda|\sigma) = \|\widehat{\mathrm{u}}(\mathrm{z};\lambda)-\mathrm{z}\|_2^2 + \frac{2\sigma^2}{\nu}\left\langle\left(\widehat{\mathrm{u}}(\mathrm{z}+\nu\varepsilon;\lambda)-\widehat{\mathrm{u}}(\mathrm{z};\lambda)\right),\varepsilon\right\rangle - \sigma^2 N,$$



(a) $k = 168$ N/m, $V = 42\,\mu$m/s

(b) $k = 168$ N/m, $V = 1100\,\mu$m/s

(c) $k = 1002$ N/m, $V = 42\,\mu$m/s

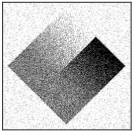
Parameter-free and fast piecewise-smooth denoising/restoration

(Parameter-free and) fast piecewise-smooth denoising

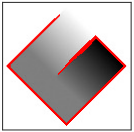
➡ Mumford-Shah functional

$$\underset{u,K}{\text{minimize}} \; \underbrace{\frac{1}{2}\int_{\Omega}(u-z)^2 dxdy}_{\text{fidelity}} + \underbrace{\beta \int_{\Omega\setminus K} |\nabla u|^2 dxdy}_{\text{smoothness}} + \underbrace{\gamma \mathcal{H}^1(K\cap\Omega)}_{\text{length of contours}}$$

- Ω: image domain,
- $z = \bar{u} + n$ with $n = \mathcal{N}(0, \sigma^2 \mathrm{Id})$
- $u \in W^{1,2}(\Omega)$: piecewise smooth approx. of z,
- K: set of discontinuities,
- $\mathcal{H}^1$: Hausdorff measure,
- $\beta > 0$, $\gamma > 0$: regularization parameters.



z



$\widehat{u}$ and $\widehat{K}$ (red)

(Parameter-free and) fast piecewise-smooth denoising

➡ **Mumford-Shah (1989) minimization problem**

$$\underset{u,K}{\text{minimize}}\ \frac{1}{2}\int_\Omega (u-z)^2 dxdy + \beta \int_{\Omega\setminus K} |\nabla u|^2 dxdy + \gamma \mathcal{H}^1(K\cap\Omega)$$

➡ **Discrete state-of-the-art formulations**

- Ambrosio-Tortorelli (1990) / Foare-Lachaud-Talbot (2016)

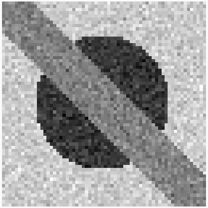
$$(\widehat{u},\widehat{e}) = \underset{u,e}{\text{argmin}}\ \frac{1}{2}\|u-z\|_2^2 + \beta\|(1-e)\odot Du\|^2 + \gamma\left(\varepsilon\|\tilde{D}e\|_2^2 + \frac{1}{4\varepsilon}\|e\|_2^2\right)$$

- $\widehat{e}$ extracted from $\widehat{u} = \underset{u}{\text{argmin}}\ \frac{1}{2}\|u-z\|_2^2 + \lambda\, h(Du)$:

Potts model (1952)	$h(\cdot) = \|\cdot\|_0$
Blake-Zisserman model (1987)	$h(\cdot) = \sum_i \min(\lvert(\cdot)_i\rvert^q, \alpha^q)$
TV denoising - ROF (1992)	$h(\cdot) = \|\cdot\|_{2,1}$
→ Thresholded-ROF (T-ROF)	

(Parameter-free and) fast piecewise-smooth denoising

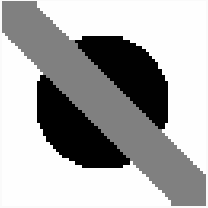
Original



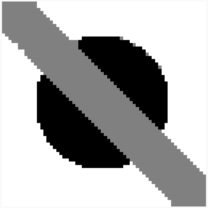
Noisy

Chan-Vese

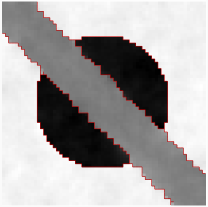
TV



Potts

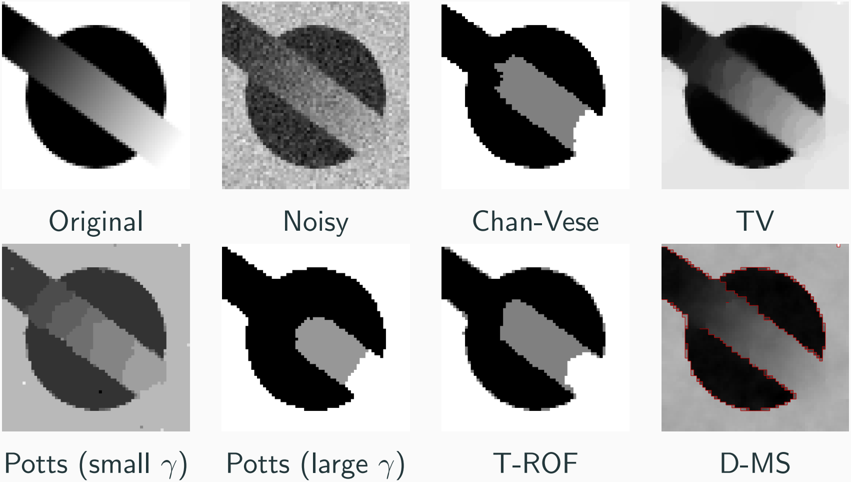


T-ROF



D-MS

(Parameter-free and) fast piecewise-smooth denoising



(Parameter-free and) fast piecewise-smooth restoration

➡ **Minimization problem** **[ANR JCJC 2019]**

$$\underset{\mathrm{u},\mathrm{e}}{\text{minimize}}\ \boldsymbol{\psi}(\mathrm{u};\mathrm{z}) + \underbrace{\beta\|(1-\mathrm{e})\odot D\mathrm{u}\|^2}_{\phi(\mathrm{u},\mathrm{e})} + \boldsymbol{\varphi}(\mathrm{e})$$

➡ **PALM** [Bolte et al., 2013]

For $k \in \mathbb{N}$

Set $\gamma > 1$ and $c_k = \gamma\chi(\mathrm{e}^{[k]})$

$$\mathrm{u}^{[k+1]} = \mathrm{prox}_{\frac{1}{c_k}\boldsymbol{\psi}}\Big(\mathrm{Id} - \tfrac{1}{c_k}\nabla_{\mathrm{u}}\phi\big(\cdot,\mathrm{e}^{[k]}\big)\Big)\mathrm{u}^{[k]}$$

Set $\delta > 1$ and $d_k = \delta\nu(\mathbf{u}^{[k+1]})$

$$\mathrm{e}^{[k+1]} = \mathrm{prox}_{\frac{1}{d_k}\boldsymbol{\varphi}}\Big(\mathrm{Id} - \tfrac{1}{d_k}\nabla_{\mathrm{e}}\phi\big(\mathrm{u}^{[k+1]},\cdot\big)\Big)\mathrm{e}^{[k]}$$

$\rightarrow$ Under technical assumptions, the sequence $(\mathrm{u}^{[k]},\mathrm{e}^{[k]})_{\ell\in\mathbb{N}}$ converges to a critical point.

(Parameter-free and) fast piecewise-smooth restoration

➡ Minimization problem

$$\underset{\mathrm{u},\mathrm{e}}{\text{minimize}}\; \boldsymbol{\psi}(\mathrm{u};\mathrm{z}) + \underbrace{\beta\|(1-\mathrm{e})\odot D\mathrm{u}\|^2}_{\phi(\mathrm{u},\mathrm{e})} + \boldsymbol{\varphi}(\mathrm{e})$$

➡ Proposed Semi Linearized PAM (SL-PAM)

For $k \in \mathbb{N}$

Set $\gamma > 1$ and $c_k = \gamma\chi(\mathrm{e}^{[k]})$

$$\mathrm{u}^{[k+1]} = \mathrm{prox}_{\frac{1}{c_k}\boldsymbol{\psi}}\Big(\mathrm{Id} - \tfrac{1}{c_k}\nabla_{\mathrm{u}}\phi\big(\cdot,\mathrm{e}^{[k]}\big)\Big)\mathrm{u}^{[k]}$$

$d_k > 0.$

$$\mathrm{e}^{[k+1]} = \mathrm{prox}_{\frac{1}{d_k}\boldsymbol{\varphi} + \frac{1}{d_k}\phi\left(\mathrm{u}^{[k+1]},\cdot\right)}\mathrm{e}^{[k]}$$

$\rightarrow$ Under technical assumptions, the sequence $(\mathrm{u}^{[k]},\mathrm{e}^{[k]})_{\ell\in\mathbb{N}}$ converges to a critical point. **[Foare, Pustelnik, Condat, 2019]**

(Parameter-free and) fast piecewise-smooth restoration

➡ **Minimization problem**

$$\underset{\mathrm{u,e}}{\mathrm{minimize}}\ \boldsymbol{\psi}(\mathrm{u};\mathrm{z}) + \underbrace{\beta\|(1-\mathrm{e})\odot D\mathrm{u}\|^2}_{\phi(\mathrm{u},\mathrm{e})} + \boldsymbol{\varphi}(\mathrm{e})$$

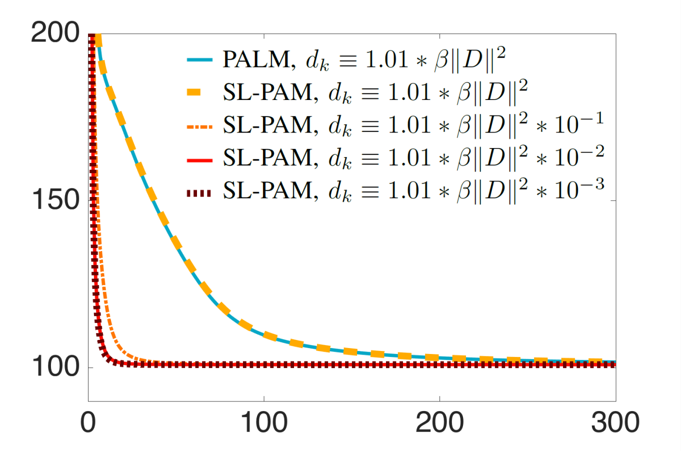
➡ **Proposed Semi Linearized PAM (SL-PAM)**

➡ **Possible choices for penalisation:**

[Foare, Pustelnik, Condat, 2019][Le, Foare, Pustelnik, sub. 2021]

Ambrosio-Tortorelli	$\boldsymbol{\varphi}(\mathrm{e}) = \gamma\varepsilon\|D\mathrm{e}\|_2^2 + \frac{\gamma}{4\varepsilon}\|\mathrm{e}\|_2^2$ with $\varepsilon > 0$
ℓ_1-norm	$\boldsymbol{\varphi}(\mathrm{e}) = \gamma\|\mathrm{e}\|_1$
Ber-Hu	$\boldsymbol{\varphi}(\mathrm{e}) = \sum_{i=1}^{\lvert\mathbb{E}\rvert} \gamma \max\left\{\lvert e_i\rvert, \frac{e_i^2}{4\varepsilon}\right\}$

(Parameter-free and) fast piecewise-smooth restoration



Parameter-free and fast piecewise-smooth denoising

➡ **Minimization problem**

$$\underset{u,e}{\text{minimize}}\ \psi(u;z) + \underbrace{\beta\|(1-e)\odot Du\|^2}_{\phi(u,e)} + \varphi(e)$$

➡ **Semi Linearized PAM (SL-PAM)**

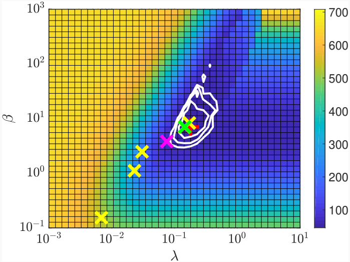
➡ **Several possible choices for penalisation**

➡ **Automatic tuning with FDMC SURE and efficient research using Averaged FDMC SUGAR**

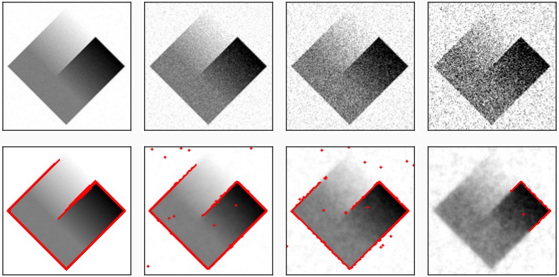
[Lucas, Pascal, Pustelnik, Abry, sub. 2021]

Parameter-free and fast piecewise-smooth denoising

➡ **Search with SUGAR versus Averaged SUGAR**



➡ **Results for increasing (estimated) variance**

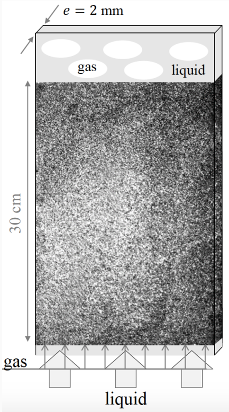


Parameter-free and fast texture segmentation

(Parameter-free and fast) texture segmentation

➡ **LPENSL experiment**: joint gas and liquid flow through a porous medium

- Segment gas/liquid + accurate estimation of the interface.
- Large-scale data



(Parameter-free and fast) texture segmentation

➡ **LPENSL experiment**: joint gas and liquid flow through a porous medium

- Segment gas/liquid + accurate estimation of the interface.
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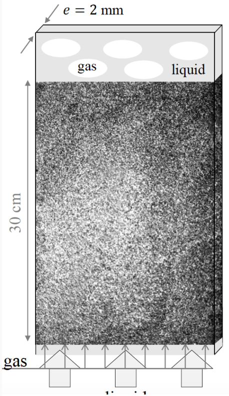
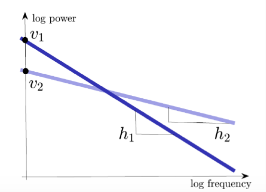


➡ **Texture segmentation**: scale-free descriptors, require to compute the slope at each location.

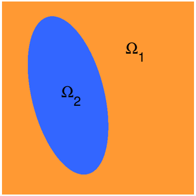
[JCJC GdR ISIS 2014][Defi CNRS Imag'In 2017]



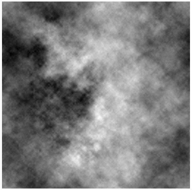
(Parameter-free and fast) texture segmentation

➡ **Local regularity to characterize texture**:
[Jaffard, 2004][Wendt et al., 2009]

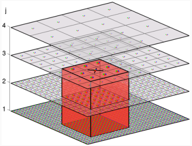
Wavelet coefficients	$\zeta_j = \mathrm{D}_j \mathrm{z}$
Wavelet leaders	$\mathcal{L}_{j,n} = \sup\limits_{\lambda_{j',n'} \subset \Lambda_{j,n}} \lvert \zeta_{j',n'} \rvert$
Behavior through the scales	$\mathcal{L}_{j,n} \simeq s_n 2^{j h_n}$ when $2^j \to 0$
Linear regression across scales	$\widehat{h}_n = \sum_j w_{j,n} \log_2 \mathcal{L}_{j,n}$

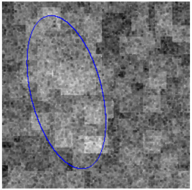


Mask



Original z





Estimate $\widehat{\mathbf{h}}$

(Parameter-free and fast) texture segmentation

➡ **Linear regression across scales at each location**:

$$\boxed{\widehat{h}_n = \sum_j w_{j,n} \log_2 \mathcal{L}_{j,n}} \quad \text{with} \quad \mathrm{w}_n \in C = \left\{ \sum_j w_{j,n} \equiv 0 \ , \ \sum_j j w_{j,n} \equiv 1 \right\}$$

➡ **TV denoising: piecewise constant estimate**:

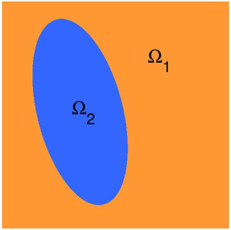
$$\widehat{\mathrm{h}}_{\mathrm{TV}} = \arg\min_{\mathrm{h}} \frac{1}{2} \| \mathrm{h} - \underbrace{\sum_j w_j \log_2 \mathcal{L}_j}_{\widehat{\mathbf{h}}} \|_2^2 + \lambda \| \mathrm{D}\mathrm{h} \|_{2,1}$$

➡ **Joint estimation and segmentation**:

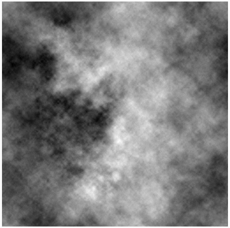
[Pustelnik, Wendt, Abry, Dobigeon, 2016]

$$(\widehat{\mathrm{h}}_{\mathrm{TVW}}, \mathrm{w}) = \arg\min_{\mathrm{h},\mathrm{w}} \frac{1}{2} \| \mathrm{h} - \sum_j w_j \log_2 \mathcal{L}_j \|_2^2 + \lambda \| \mathrm{D}\mathrm{h} \|_{2,1} + \| \mathrm{w} - P_C(\mathrm{w}) \|_2$$

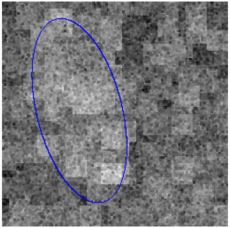
(Parameter-free and fast) texture segmentation



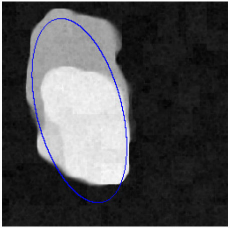
Mask



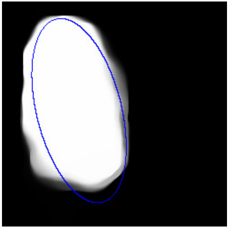
Original z



Estimate $\widehat{\mathbf{h}}$



Estimate $\widehat{\mathbf{h}}_{\mathrm{TV}}$



Estimate $\widehat{\mathbf{h}}_{\mathrm{TVW}}$

(Parameter-free and fast) texture segmentation

➡ **Previous work** [Pustelnik, Wendt, Abry, Dobigeon, 2016]

$$(\widehat{\mathrm{h}}_{\mathrm{TVW}}, \mathrm{w}) = \arg\min_{\mathrm{h},\mathrm{w}} \frac{1}{2}\|\mathrm{h} - \sum_j w_j \log_2 \mathcal{L}_j\|_2^2 + \lambda\|\mathrm{D}\mathrm{h}\|_{2,1} + \|\mathrm{w} - P_C(\mathrm{w})\|_2$$

+ Good texture segmentation performance
+ Convex minimization formulation
+ Combined estimation and segmentation (contrary to $\widehat{\mathbf{h}}_{\mathrm{TV}}$)
− Computational cost. Not adapted for large scale data.

(Parameter-free and fast) texture segmentation

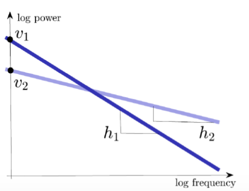
Previous work [Pustelnik, Wendt, Abry, Dobigeon, 2016]

$$(\widehat{\mathrm{h}}_{\mathrm{TVW}}, \mathrm{w}) = \arg\min_{\mathrm{h},\mathrm{w}} \frac{1}{2}\|\mathrm{h} - \sum_j w_j \log_2 \mathcal{L}_j\|_2^2 + \lambda\|\mathrm{Dh}\|_{2,1} + \|\mathrm{w} - P_C(\mathrm{w})\|_2$$

Behavior through the scales

$$\mathcal{L}_{j,\underline{n}} \simeq s_{\underline{n}} 2^{j h_{\underline{n}}} \quad \text{when} \quad 2^j \to 0$$

$$\log_2 \mathcal{L}_{j,\underline{k}} \simeq \underbrace{\log_2 s_{\underline{n}}}_{v_{\underline{n}}} + j h_{\underline{n}}$$



New objective function [Pascal, Pustelnik, Abry, 2021]

$$(\widehat{\mathrm{h}}, \widehat{\mathrm{v}}) \in \operatorname*{Argmin}_{\mathrm{h},\mathrm{v}} \sum_j \|\mathrm{v} + j\mathrm{h} - \log_2 \mathcal{L}_j\|_2^2 + \lambda \left\| [\mathrm{Dh}; \alpha \mathrm{Dv}]^\top \right\|_{2,1}$$

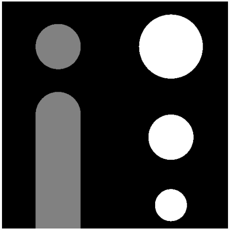
(Parameter-free and) fast texture segmentation

➡ **Minimization problem**

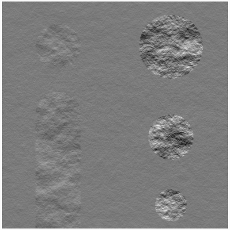
$$(\widehat{\mathrm{h}}, \widehat{\mathrm{v}}) \in \underset{\mathrm{h},\mathrm{v}}{\mathrm{Argmin}} \sum_{j} \|\mathrm{v} + j\mathrm{h} - \log_2 \mathcal{L}_j\|_2^2 + \lambda \left\| \left[\mathrm{Dh}; \alpha \mathrm{Dv}\right]^\top \right\|_{2,1}$$

- Joint estimation of the local variance and local regularity.
- Minimization performed with several algorithmic strategies FB on the dual, FISTA on the dual, Chambolle-Pock, **Chambolle-Pock with strong convexity**.
- Hyperparameter selection by an **expert**.

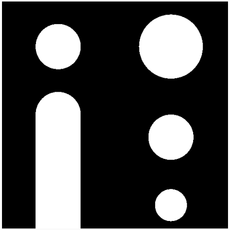
(Parameter-free and) fast texture segmentation



Mask

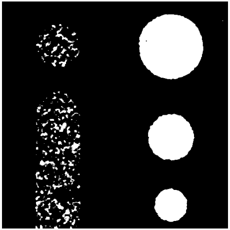


Synthetic texture



Optimal solution

T-ROF
[Cai2013]



Matrix factorization
[Yuan2015]

Proposed
[Pascal2019]

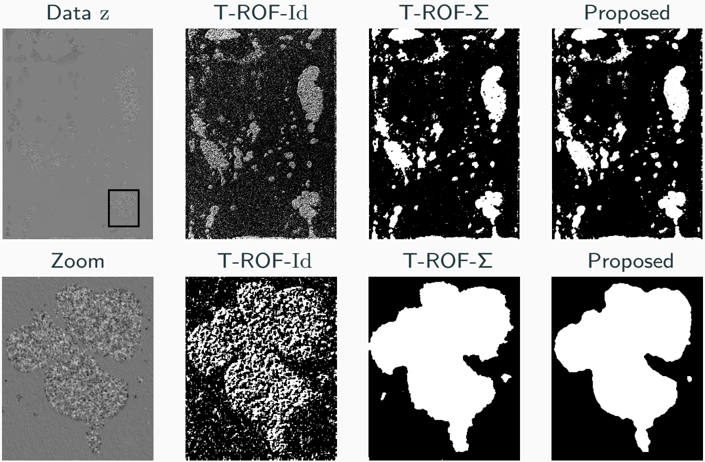
Parameter-free and fast texture segmentation

➡ Minimization problem

$$(\widehat{\mathrm{h}}, \widehat{\mathrm{v}}) \in \underset{\mathrm{h},\mathrm{v}}{\mathrm{Argmin}} \sum_j \|\mathrm{v} + j\mathrm{h} - \log_2 \mathcal{L}_j\|_2^2 + \lambda \left\| \left[\mathrm{Dh}; \alpha \mathrm{Dv}\right]^\top \right\|_{2,1}$$

- Joint estimation of the local variance and local regularity.
- Minimization performed with Chambolle-Pock with strong convexity.
- **Automatic hyperparameter tuning based on FDMC SURE and FDMC SUGAR with correlated noise and full rank matrice.** [Pascal, Vaiter, Pustelnik, Abry, 2021]

Parameter-free and fast texture segmentation



Hyperparameters selection in image restoration: from standard approaches to deep learning

Image restoration $\mathrm{z} = \mathcal{D}(\mathrm{A}\overline{\mathrm{x}})$

$$\widehat{\mathrm{u}} \in \underset{\mathrm{u} \in C \subset \mathbb{R}^{\overline{N}}}{\mathrm{Argmin}}\ h_1(\mathrm{Au}, \mathrm{z}) + \lambda h_2(\mathrm{Du})$$

➡ **Study circumstellar environment**

- Convex minimization problem involving epigraphic constraint:

$$\widehat{\mathrm{u}} \in \underset{\mathrm{u}=(I,Q,U)\in E}{\mathrm{Argmin}} \sum_{j=1}^{2}\sum_{k=1}^{K} \left\| \mathrm{z}_{j,k} - \mathrm{T}_{j,k}\mathrm{H}\big(\nu_{j,k,1} I + \nu_{j,k,2} Q + \nu_{j,k,3} U\big) \right\|^2_{\Sigma^{-1}_{j,k}} + \lambda \|\mathrm{Du}\|_{\bullet}$$

where $\mathrm{T}.$ models geometrical transformations, H models blur and coefficients $\nu.$.

[Denneulin, Langlois, Thiebaut, Pustelnik, 2021]

- **SURE based estimation procedure.**

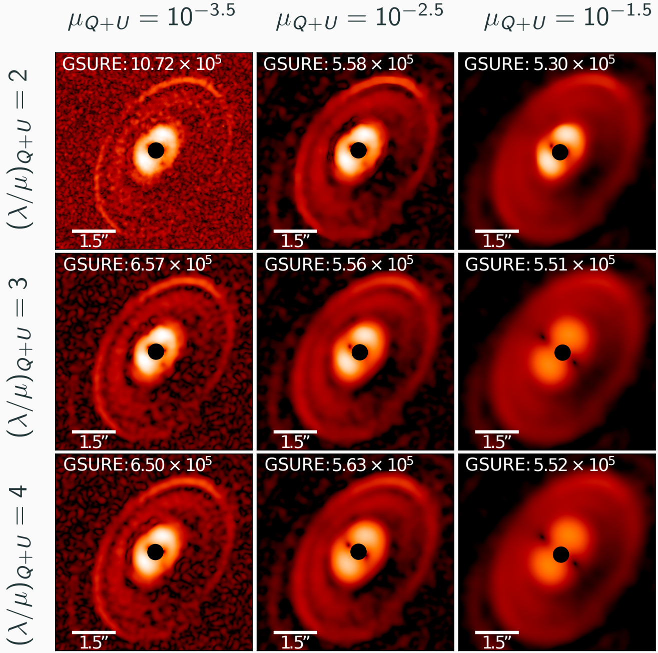

$\mu_{Q+U} = 10^{-3.5}$
$\mu_{Q+U} = 10^{-2.5}$
$\mu_{Q+U} = 10^{-1.5}$
$(\lambda/\mu)_{Q+U} = 2$
$(\lambda/\mu)_{Q+U} = 3$
$(\lambda/\mu)_{Q+U} = 4$
GSURE: 10.72×10^5
GSURE: 5.58×10^5
GSURE: 5.30×10^5
GSURE: 6.57×10^5
GSURE: 5.56×10^5
GSURE: 5.51×10^5
GSURE: 6.50×10^5
GSURE: 5.63×10^5
GSURE: 5.52×10^5
1.5″

Image restoration and constrained formulation

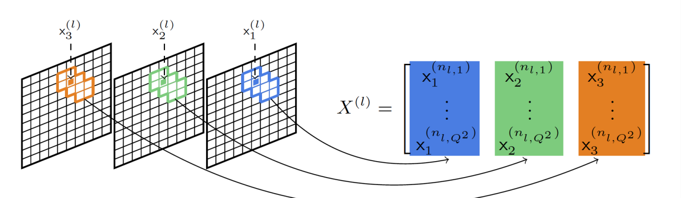
$$\widehat{u} \in \underset{u \in C \subset \mathbb{R}^N}{\text{Argmin}}\ h_1(Au, z) + \lambda h_2(Du)$$

➡ **Tensor based penalization** and **constrained formulation**

[Chierchia, Pustelnik, Pesquet, Pesquet-Popescu, 2014]

- Advanced analysis operator and penalization as a constraint

$$\widehat{u} \in \underset{u \in [0,255]^N}{\text{Argmin}}\ \|Au - z\|^2 \quad \text{s.t.} \quad \sum_{\ell=1}^{L} \|D_\ell B_\ell u\|_q \leq \eta$$



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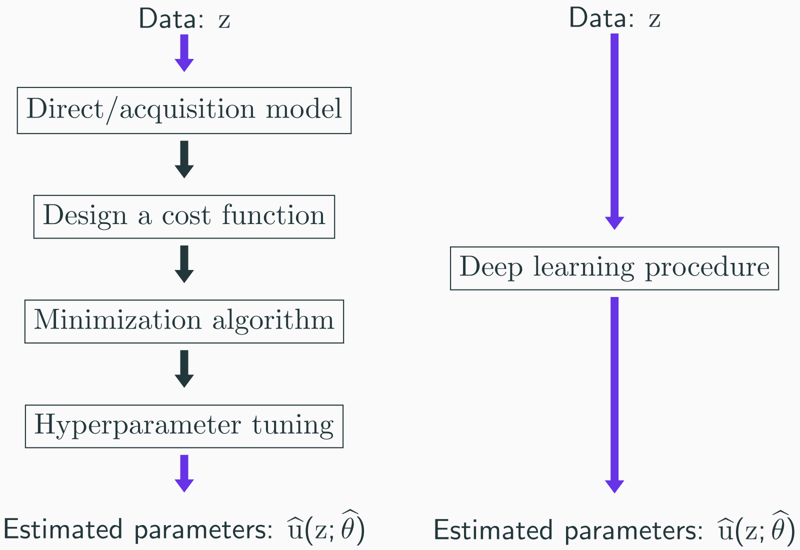
- Solved by epigraphical splitting leading to closed form expression of the involved projection.
- Physical interpretation of the parameter η.

Standard learning and deep learning



Standard learning and deep learning

➡ **Create a database** $\mathcal{S} = \left\{ (\overline{\mathrm{u}}_\ell, \mathrm{z}_\ell) \in \mathcal{H} \times \mathcal{G} \;\middle|\; \ell \in \{1, \ldots, L\} \right\}$

➡ **Learn a prediction function** d_Θ

$$\widehat{\Theta} \in \underset{\Theta}{\mathrm{Argmin}}\ \mathrm{E}(\Theta) := \frac{1}{L} \sum_{\ell=1}^{L} f_1\big(\mathrm{d}_\Theta(\mathrm{z}_\ell), \overline{\mathrm{u}}_\ell\big) + f_2(\Theta)$$

- **Linear model**: $\mathrm{d}_\Theta(\mathrm{z}_\ell) = \Theta^\top \mathrm{z}_\ell$

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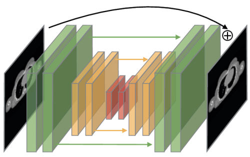
- **Non-linear model**:

$$\boxed{\mathrm{d}_\Theta(\mathrm{z}_\ell) = \eta^{[K]}\big(W^{[K]} \ldots \eta^{[1]}(W^{[1]}\mathrm{z}_\ell + b^{[1]}) \ldots + b^{[K]}\big)}$$

where $\Theta = \{W^{[k]}, b^{[k]}\}_{1 \leq k \leq K}$ with
$W^{[k]}$ denotes a weight matrix,
$b^{[k]}$ is a bias vector,
$\eta^{[k]}$ is the nonlinear activation function.



(source click here)

Standard activation functions

➡ **Preliminary remarks** [Combettes, Pesquet,2020]

- **Most of activation functions are proximity operator** :
  ReLU, Unimodal sigmoid, Softmax ...

- For $W^{[k]}$ bounded linear operators and η_k proximity operators, d_θ model allows to derive tight Lipschitz bounds for feedforward neural networks in order to evaluate their **stability**.

➡ **Unroll proximal algorithms (e.g. LISTA)**

Unfolded Condat-Vũ splitting algorithm

➡ **Minimization problem:**

$$\underset{\mathrm{u}}{\text{minimize}}\ \frac{1}{2}\|\mathrm{Au}-\mathrm{z}\|_2^2 + \|\mathrm{Hu}\|_1 \quad \text{where } \mathrm{H} = \lambda \mathrm{D}$$

➡ **Condat-Vũ iterations:**

$$\begin{array}{|ll} \mathrm{w}^{[k+1]} & = \mathrm{w}^{[k]} - \tau \mathrm{A}^*(\mathrm{Aw}^{[k]}) - \mathrm{z}) - \tau \mathrm{H}^* \mathrm{u}^{[k]} \\ \mathrm{u}^{[k+1]} & = \mathrm{prox}_{\gamma\|\cdot\|_1^*}(\mathrm{u}^{[k]} + \gamma \mathrm{H}(2\mathrm{w}^{[k+1]} - \mathrm{w}^{[k]})) \end{array}$$

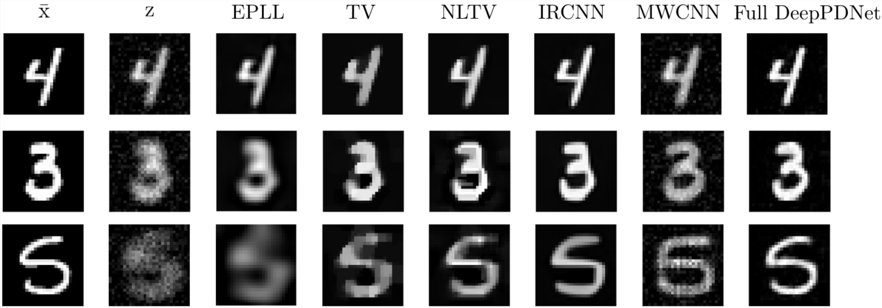
➡ **Proposed architecture:**

[Jiu, Pustelnik, 2021]

$$\mathrm{x}^{[k+1]} = \eta^{[k]}(W^{[k]}\mathrm{x}^{[k]} + b^{[k]}) \quad \text{where} \quad \begin{cases} \mathrm{x}^{[k]} = (\mathrm{w}^{[k]}, \mathrm{u}^{[k]},) \\ W^{[k]} = \begin{pmatrix} \mathrm{Id} - \tau \mathrm{A}^*\mathrm{A} & -\tau \mathrm{H}^* \\ \gamma \mathrm{H}(\mathrm{Id} - 2\tau \mathrm{A}^*\mathrm{A}) & \mathrm{Id} - 2\tau\gamma \mathrm{HH}^* \end{pmatrix} \\ b^{[k]} = \begin{pmatrix} \tau \mathrm{A}^*\mathrm{z} \\ 2\tau\gamma \mathrm{HA}^*\mathrm{z} \end{pmatrix} \\ \eta^{[k]} = \begin{pmatrix} \mathrm{Id} \\ \mathrm{prox}_{\gamma\|\cdot\|_1^*} \end{pmatrix} \end{cases}$$

Parameters to learn: $\Theta = \{\mathrm{H}, \tau, \gamma\} \rightarrow \Theta = \{\mathrm{H}^{[k]}, \tau^{[k]}, \gamma^{[k]}, \}_{1 \leq k \leq K}$

Unfolded Condat-Vũ splitting algorithm



Conclusion and perspectives

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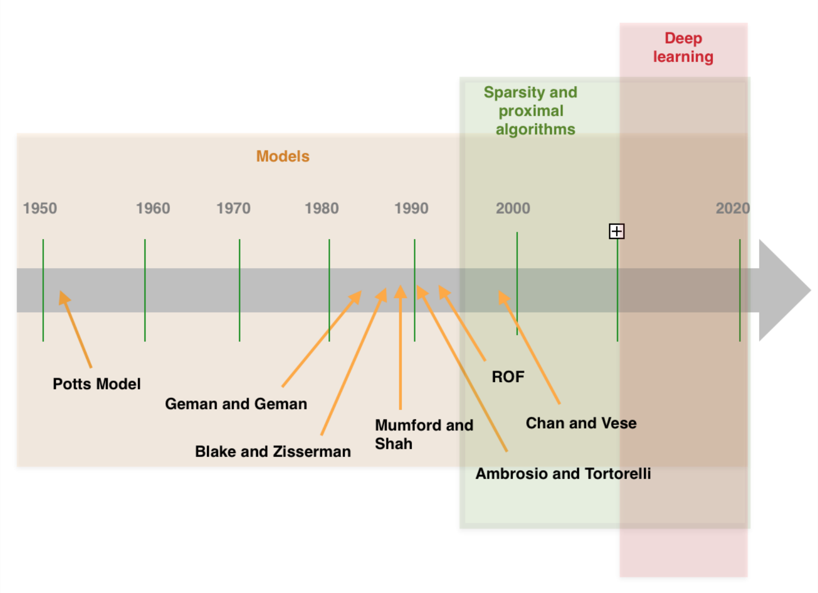
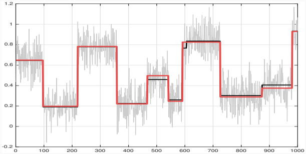


Perspective on signal denoising

➡ **Estimate the optimal λ**

$$(\widehat{u}, \widehat{\sigma}, \widehat{\lambda}) = \underset{u,\sigma,\lambda}{\arg\min} \|u - z\|_q^q + \lambda \|Du\|_0 + \phi(\sigma, \lambda)$$



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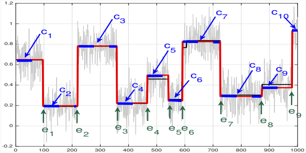

- **Main idea: Design $\phi(\sigma, \lambda)$ from Bayesian arguments**
  [Frecon, Pustelnik, Dobigeon, Wendt, Abry, 2017]
- **Similar framework for 2D based on D-MS model ?**

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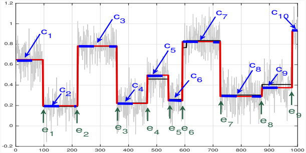

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➡ **Estimate the optimal D** (e.g. Stick-slip,Genome replication)

- Continuous dictionnaries,
- Deep learning.

Perspective on contour detection / texture segmentation

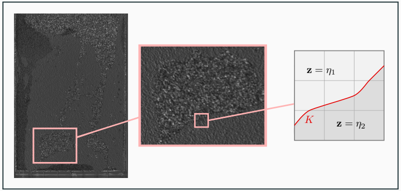
➡ Modelling

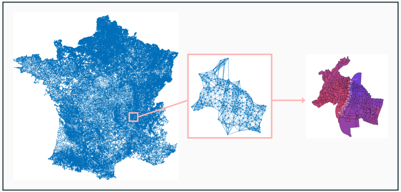
- Additional **texture descriptors** (e.g. orientation, multifractal).
- Multigrid convergence of the objective functions.

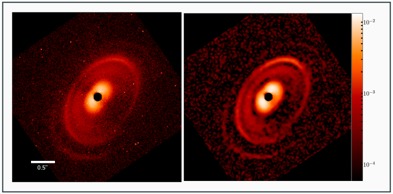
➡ Optimization

- **Stronger convergence guarantees** when A is involved in D-MS.
- **Deeper analysis (rate)** of the algorithms PALM/SLPAM for contour detection and Dual FB/Dual FISTA/CP for texture segmentation.
- Convergence rate in optimization versus Stability in unroll algorithms. Which algorithm is the best ?
- Acceleration without strong convexity.

Applications







Thank you !