$$\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} = A \qquad \begin{pmatrix} \boxed{0 \quad 1} \\ \boxed{1 \quad 0} \end{pmatrix} = B$$

$$u\,M = \lambda . u$$

$(1\,1)\,B = (1\,1)$

$(1\,1)$ est $\vec{vp}$ pour la vp 1

$$(0 \cdots 0\,1\,0 \cdots 0) \begin{pmatrix} L_1 \\ \vdots \\ \boxed{L_i} \\ \vdots \\ L_n \end{pmatrix} = L_i$$

$\uparrow$ i

$\Rightarrow tr = 0 = 1 + vp \Rightarrow vp = -1$

$(-1\;\;1)\,B = (1\;\;-1)$

$= -(-1, 1)$

$u\,H = 0$

$u . (C_1 \dots C_n) = (\langle u | C_1 \rangle, \dots, \langle u | C_n \rangle)$

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (P)^{-1} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \underbrace{\begin{pmatrix} \boxed{1 \quad 1} \\ \boxed{-1 \quad 1} \end{pmatrix}}_{= P}$$

$A = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$ si A est diagonalisable : ① $\text{tr} A = \underline{\lambda_1 + \lambda_2}$
$= \underline{4}$

② $\det A = \underline{\lambda_1 \cdot \lambda_2} = 3 \cdot 1 - 1 \cdot 1 = \underline{2}$

$\exists u \neq 0$ ~~tq~~ tq $uA = \lambda u$ alors $u(A - \lambda \text{Id}) = 0$

$\Rightarrow \ker(A - \lambda \text{Id}) \neq \{0\}$

$\Rightarrow \det(A - \lambda . \text{Id}) = 0$

$\chi_A = \det(A - X . \text{Id})$ $\quad$ λ est VP si $\chi_A(\lambda) = 0$

$$\chi_A = \det \begin{pmatrix} 1-X & 1 \\ 1 & 3-X \end{pmatrix} = (1-X)(3-X) - 1$$
$$= X^2 - 4X + 2$$
$$= (X-2)^2 - 2$$

$\chi_A = 0 \iff (X-2)^2 - 2 = 0$

$\iff (X-2) = \pm\sqrt{2}$

$\iff X = 2 \pm \sqrt{2}$ $\quad$ deux VP : $2+\sqrt{2}$

et $2-\sqrt{2}$

$uA = (2\pm\sqrt{2}) . u$ $\quad$ $u = (x \ \ y)$

$\iff (x \ \ y) \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} = \left((2\pm\sqrt{2})x, \ (2\pm\sqrt{2})y\right)$

$\iff \begin{cases} x + y = (2\pm\sqrt{2})x \\ x + 3y = (2\pm\sqrt{2})y \end{cases}$ $\quad$ $\boxed{x = 1 \rightarrow y = 1+\sqrt{2}}$

VP pour $2+\sqrt{2}$

$\underbrace{x = 1 \ , \ y = 1-\sqrt{2}}$

$\vec{\text{VP}}$ pour $2-\sqrt{2}$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} = \underline{\begin{pmatrix} & P & \end{pmatrix}^{-1}} \ \underline{\begin{pmatrix} 2+\sqrt{2} & 0 \\ 0 & 2-\sqrt{2} \end{pmatrix}} \ \underbrace{\begin{pmatrix} 1 & 1+\sqrt{2} \\ 1 & 1-\sqrt{2} \end{pmatrix}}_{P}$$

$\text{tr} A = 4 = 2+\sqrt{2}+2-\sqrt{2}$ ✓

$\det A = 3 - 1 = 2 = (2+\sqrt{2})(2-\sqrt{2}) = 2^2 - \sqrt{2}^2 = 4 - 2 = 2$

✓