

HW1 Molecular Programming

M2ICACR16 2025.11.28 - Due on Fri. 12.05 before 15:45



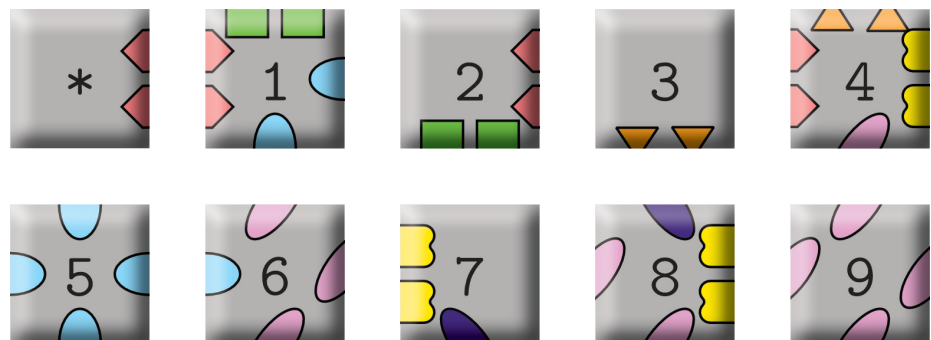
You are asked to complete the exercise marked with a [★] and to send me your solutions to:

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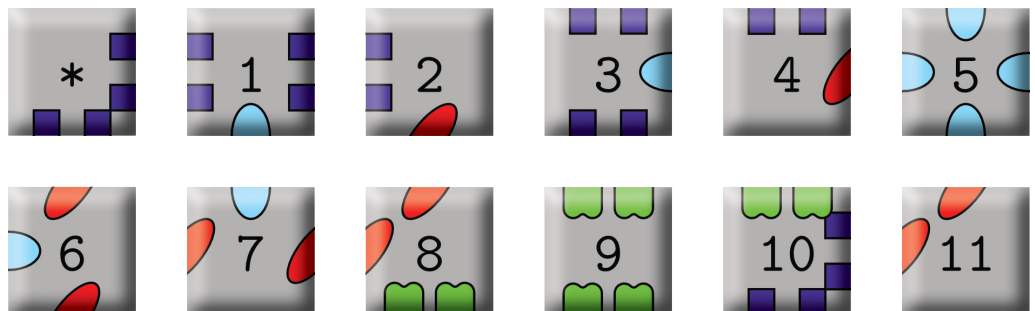
as a PDF file named **HW1-Lastname.pdf** on Fri. 12.05 before 15:45.

■ **Exercise 1 (Algorithmic Self-Assembly).** Recall that the self-assembly process consists in, given a finite tileset (with infinitely many tiles of each type), starting from the seed tile (marked with a ★), glueing tiles with matching colors to the current aggregate so that each new tile is attached by at least two links to the aggregate (either on the same border or on two borders). Recall that a shape is *final* if no tiles can be attached to it anymore.

► **Question 1.1)** What is the exact family of final shapes self-assembled by the following tile-set? (No proof nor justification is asked.) Indicate the local order of assembly by drawing arrows over the tiles of a generic final shape. Which are the two competing tiles that decide the size of the resulting final shape?

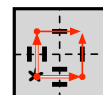


[★] **Exercise 2.** What is the family of shapes built by this tileset at temperature $T^o = 2$? (no justification asked)



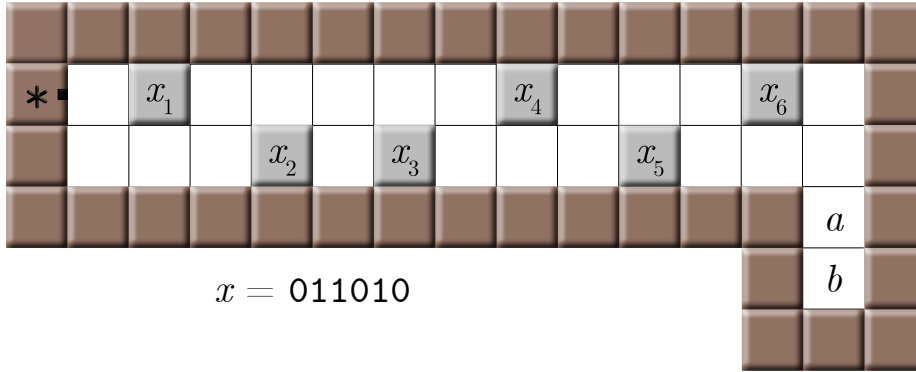
Indicate the assembly order with arrows⁽¹⁾ on a generic production. Is this a well-ordered tileset?

¹Draw an arrow $(i, j) \rightarrow (i', j')$ iff tile (i, j) is attached before tile (i', j') : for instance,



■ **Exercise 3 (Bit-cave).** We want to design a tiling that computes at temperature $T^\circ = 1$ the parity of a number x written in binary $x_1 \dots x_n$: $\text{parity}(x) = (x_1 + \dots + x_n) \bmod 2$. Remember that *glue mismatch* are allowed: a tile can attach to the aggregate as soon as there is at least on side of the tile that agrees with a side of the current aggregate (even if the other sides do not agree).

Consider the "cave" seed (in brown below) where the i -th bit of x is encoded in the $(2i + 1)$ -th column by a (grey) tile placed either on the top row if $x_i = 0$ or in the bottom row if $x_i = 1$.



► **Question 3.1)** Give a (absolute constant size) tiling that at temperature $T^\circ = 1$, in presence of the seed above:

- places a tile at position a and none at position b if and only if $\text{parity}(x) = 0$; and
- places a tile at position b if and only if $\text{parity}(x) = 1$.

Give a generic assembly (run your tiling on the example above) indicating the order of assembly. How many tile types do you have? No mathematical justification asked however a simple explanation of the role of each tile type is necessary.

■ **Exercise 4.** Assume a random Poisson model where the random time X between two consecutive appearances of a tile of a given type τ at a given empty location follows an exponential law: $p(x) = c \cdot e^{-cx}$ where $c > 1$ is the concentration of the tiles of type τ . We want to prove the following theorem:

Theorem 1 (Adleman et al, 2001). Consider an ordered tile system \mathcal{T} that assembles deterministically a single shape S . Let \prec be the partial order of the assembly, i.e. such that $(i, j) \prec (k, l)$ if the tile at position (i, j) is attached before the tile at (k, l) by \mathcal{T} . With very high probability, the assembly time of a shape S by \mathcal{T} is:

$$O(\gamma \times \text{rank}(S))$$

where γ only depends on the concentrations and $\text{rank}(S)$ is the highest rank in the shape S (i.e. the length of the longest path in \prec).

► **Question 4.1)** Let X be an exponential random variable such that $p(X = x) = ce^{-cx}$ for all real $x \geq 0$, for some $c > 0$. Show that X is memoryless, i.e. for all $u, t \geq 0$,

$$p(X = t + u | X \geq u) = p(X = t)$$

Let T be the assembly time of the shape S , i.e. the time at which the last tile of shape S is attached. Let $X_{i,j}$ be the independent exponential random variable for the time between two consecutive appearances of the tile to be attached at position (i, j) in S . We denote by $w(P)$ the random variable for the weight of a \prec -path P , defined as: $w(P) = \sum_{(i,j) \in P} X_{i,j}$.

► **Question 4.2)** Show that:

$$T = \max_{\prec\text{-path } P} w(P)$$

▷ *Hint.* Proceed by recurrence on the rank of the tiles and show that for all tile (i, j) , its assembly time is the random variable $T_{ij} = \max_{\prec\text{-path } P \text{ from } (0,0) \text{ to } (i,j)} w(P)$.

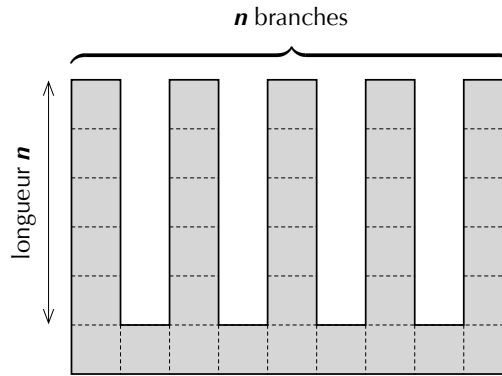
► **Question 4.3)** Let X_1, \dots, X_ℓ be ℓ independent exponential variables s.t. $p(X_i = x) = c_i e^{-c_i x}$ with $c_i > 1$. Show that there is γ which depends only of $\min_i c_i$ such that: for all $n \geq \ell$,

$$\Pr\{X_1 + \dots + X_\ell \geq \gamma \cdot n\} \leq 1/4^\ell \cdot e^{-\gamma(n-\ell)}$$

▷ *Hint.* Note that $\mathbb{E}[e^{X_i}] < \infty$ and apply Markov inequality to $Z = e^{X_1 + \dots + X_\ell}$.

► **Question 4.4)** Conclude.

■ **Exercise 5.** Propose a staged assembly scheme at temperature $T^\circ = 1$ of the shape family E of candelabrams with n branches of length n .



Describe the tiles, glues, their number, the number of stages and the number of different bechers needed. Give an illustration of the stages to build a generic production.