HW4 Molecular Programming

M2ICA CR16

2026.01.21 - Due on None

You are asked to complete the exercise marked with a [★] and to send me your solutions to:

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as a PDF file named **HW4-Lastname.pdf** on **None**.

■ **Exercise 1 (The power of zigzag assembly).** In this exercise we want to assemble at temperature $T^\circ = 2$, a pyramide P_n of size $(2n+1) \times (n+1)$ with its bottom line as the seed (in darker gray) as illustrated in Fig. 1.

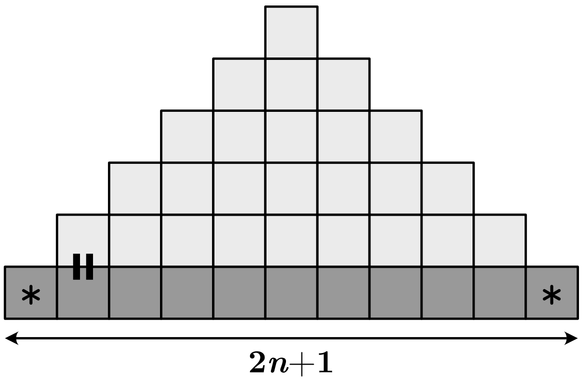


Figure 1: Question 1.1)

► **Question 1.1)** *Provide a fixed tile set (independent of n) that at temperature $T^\circ = 2$, assembles the pyramide P_n, for any n, from its (assumed to be already assembled) bottom row (in darker gray in Fig. 1), with the following constraints:*

- *You are only allowed one glue of strength 2 connecting each pair of rows and the first glue of strength 2 has to be in the left corner of the bottom row seed as illustrated in 1.*
- *Furthermore, you are allowed to distinguish only $o(n)$ tiles in the seed bottom row; all the other tiles must be indistinguishable.*

Provide the tile set as well as a generic assembly indicating the assembly order. Is it ordered? No justification asked.

▷ *Hint. Decide first where you place the glue of strength 2 between each row.*

► **Question 1.2)** *Provide a constant number of dedicated tiles that assemble non-deterministically the seed bottom row required for your tile set for any n, and only these bottom rows.*

Provide a generic assembly. Is it ordered? No justification asked.

▷ *Hint. The only non-determinism must be in the value of n. There must be a single assembly of the seed for every fixed value of n.*

■ **Exercise 2 (Window Movie Lemma).** We investigate the computation power of tile assembly at temperature $T^\circ = 1$. We allow *mismatches*, i.e. a tile can be added to the current aggregate as soon as it is attached by *at least one side* to the current aggregate for which the glues match (the other sides in contact can have mismatching glues). Unless specified explicitly otherwise, all assemblies take place at $T^\circ = 1$ in this exercise.

Let us first consider a (finite) tile set $\mathcal{T}$ which only assembles unidimensional segments of size $1 \times \ell$ for some $\ell \geqslant 1$ starting from its seed tile. Let $\tau = |\mathcal{T}|$ denote the number of tile types in $\mathcal{T}$ in all of the following. Recall that the *final productions* of a tileset $\mathcal{T}$ are the shapes corresponding to every possible assembly of tiles from $\mathcal{T}$ starting from the seed tile of $\mathcal{T}$ and where no more tile can be added.

▶ **Question 2.1)** *Show (and explicit) that there is a constant $k(\tau)$, which depends only on τ, such that if a segment of size $1 \times \ell$ with $\ell \geqslant k(\tau)$ is a final production of $\mathcal{T}$, then there is an integer $1 \leqslant i < k(\tau)$ such that all the segments $1 \times (\ell + n \cdot i)$ are also final productions of $\mathcal{T}$ for all $n \geqslant -1$. If so, we say that the tile set $\mathcal{T}$ is pumpable.*

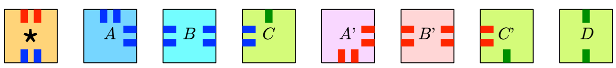
Let us now consider a (finite) tile set $\mathcal{T}$ whose final productions are 2-thick rectangles of size $2 \times \ell$ for some $\ell \geqslant 1$.

▶ **Question 2.2)** *Show (and explicit) that there is a constant $k_2(\tau)$, which depends only on τ, such that if a 2-thick rectangle of size $2 \times \ell$ with $\ell \geqslant k_2(\tau)$ is a final production of $\mathcal{T}$, then $\mathcal{T}$ is pumpable, i.e. that there is an integer $1 \leqslant i < k_2(\tau)$ such that all the 2-thick rectangles $2 \times (\ell + n \cdot i)$ are also final productions of $\mathcal{T}$ for all $n \geqslant -1$.*
▷ *Hint. Pay attention to the order in which the tiles are attached, make sure that the pumped structure can indeed self-assemble.*

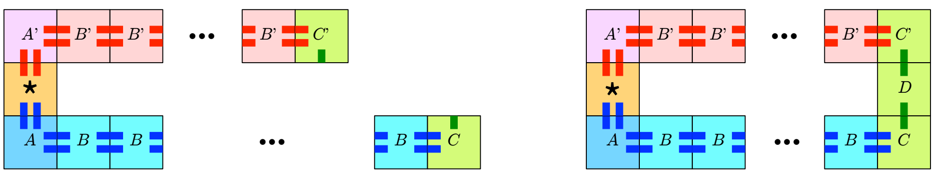
Let us now generalise and consider a (finite) tile set $\mathcal{T}$ whose final productions are q-thick rectangles of size $q \times \ell$ for some $\ell \geqslant 1$.

▶ **Question 2.3)** *Show (and explicit) that there is a constant $k_q(\tau)$, which depends only on τ, such that if a q-thick rectangle of size $q \times \ell$ with $\ell \geqslant k_q(\tau)$ is a final production of $\mathcal{T}$, then $\mathcal{T}$ is pumpable, i.e. that there is an integer $1 \leqslant i < k_q(\tau)$ such that all the q-thick rectangles $q \times (\ell + n \cdot i)$ are also final productions of $\mathcal{T}$ for all $n \geqslant -1$.*

Consider the following tile set $\mathcal{U} = \{\bigstar, A, B, C, A', B', C', D\}$ at $T^\circ = 2$ for which $\bigstar$ is the seed tile:



The final productions of $\mathcal{U}$ at $T^\circ = 2$ consist of two arms which are either 1) of different lengths and then don't touch eachother; or 2) of equal length and then there is a tile D that makes contact between them:



▶ **Question 2.4)** *Show that no tile set can simulate intrinsically at $T^\circ = 1$, the dynamics of $\mathcal{U}$ at $T^\circ = 2$.*
▷ *Hint. As a simplification, consider that in an intrinsic simulation, all megacell corresponding to an empty position in the simulated system must never be filled by more than 30% of tiles, and all megacell corresponding to a non-empty position in the simulated system must be filled at 100% by tiles. If you have time left: how would you waive these assumptions?*