Bootstrapping: **Intrinsic Universality** in tile assembly

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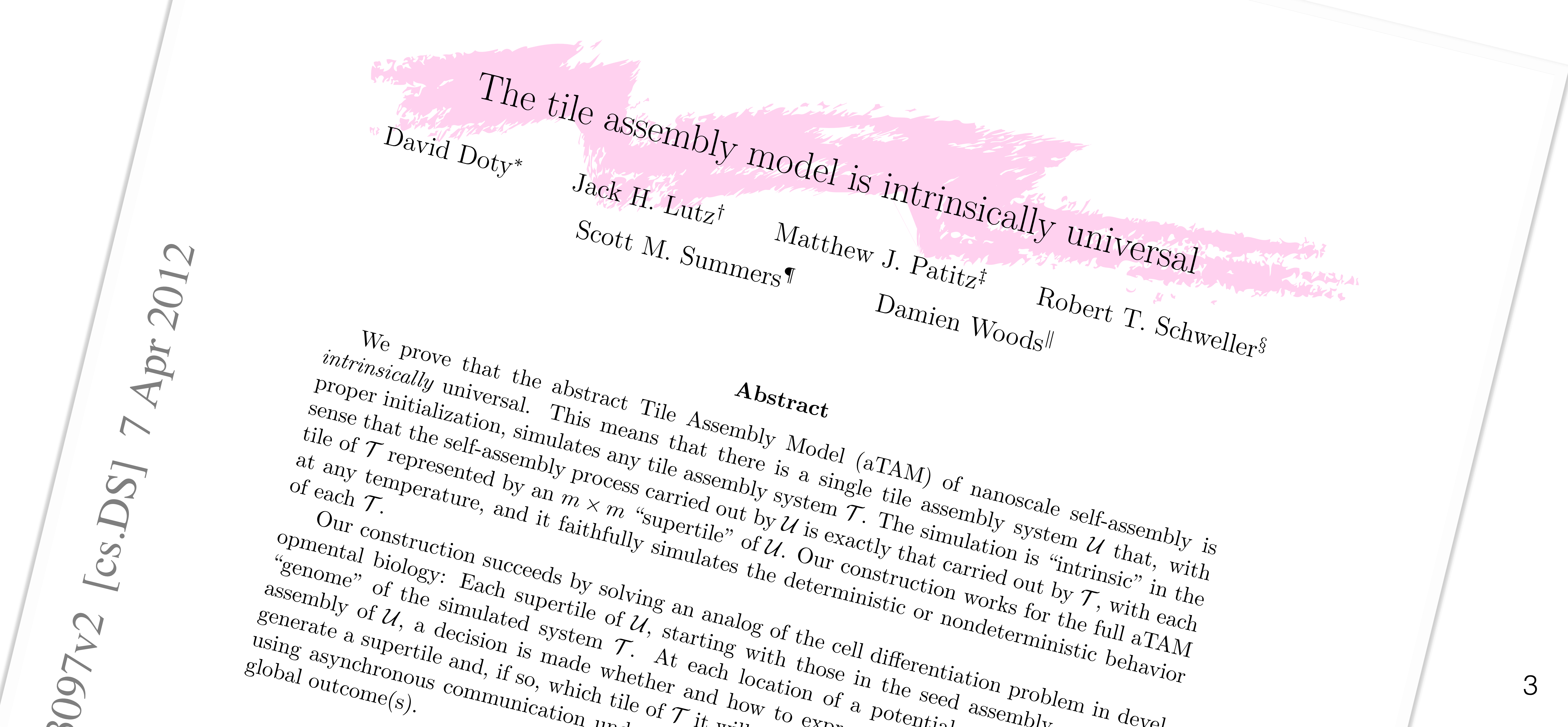
Most slides borrowed from Damien Woods

Universal Tileset at T°2

A universal tile set to build any (assemblable) shape

Is there a universal tileset at T°=2 ?

- Yes! Up to rescaling



The tile assembly model is intrinsically universal

David Doty* Jack H. Lutz† Matthew J. Patitz‡ Robert T. Schweller§ Scott M. Summers¶ Damien Woods‖

Abstract

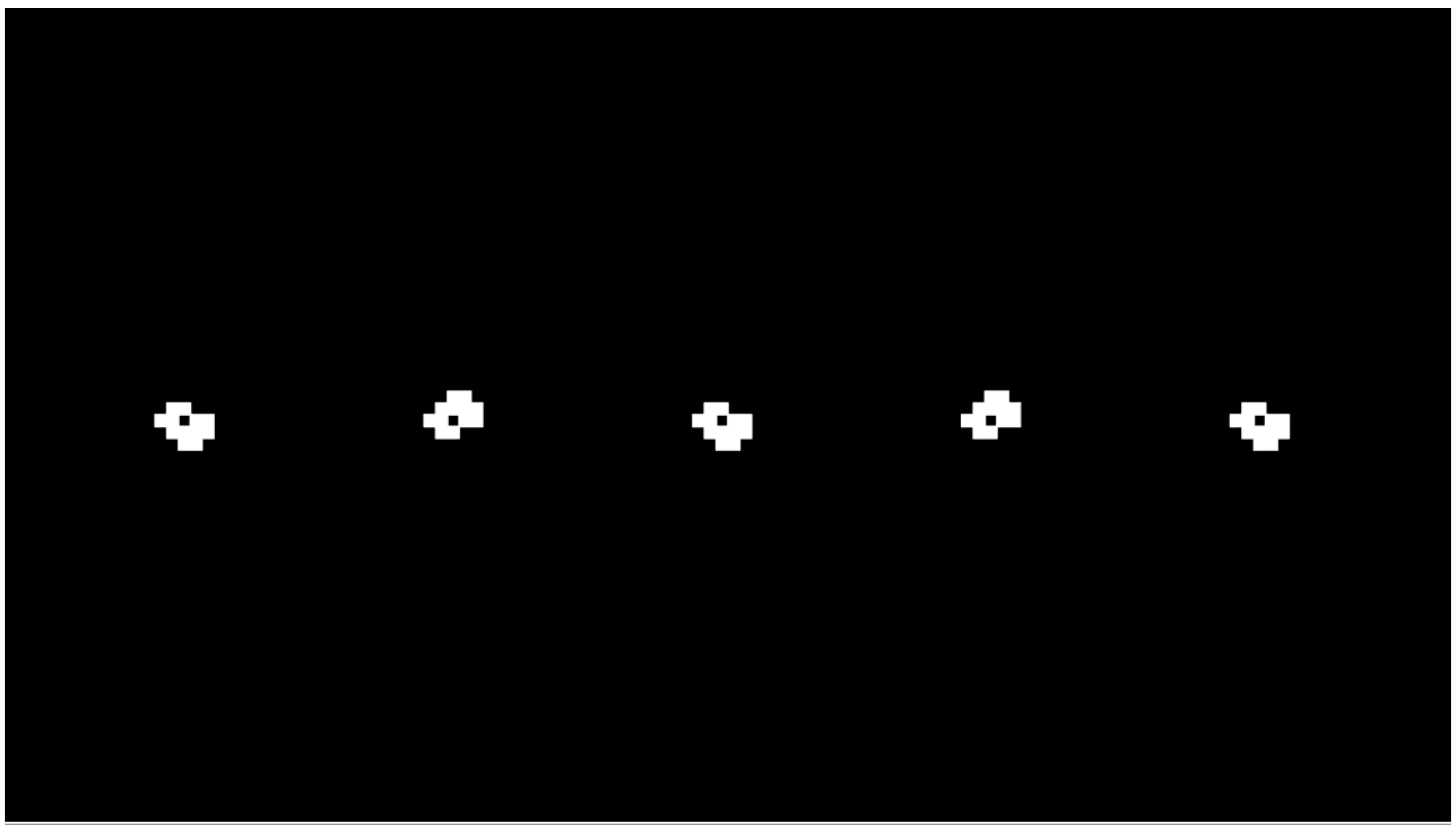
We prove that the abstract Tile Assembly Model (aTAM) of nanoscale self-assembly is *intrinsically* universal. This means that there is a single tile assembly system $\mathcal{U}$ that, with proper initialization, simulates any tile assembly system $\mathcal{T}$. The simulation is "intrinsic" in the sense that the self-assembly process carried out by $\mathcal{U}$ is exactly that carried out by $\mathcal{T}$, with each tile of $\mathcal{T}$ represented by an $m \times m$ "supertile" of $\mathcal{U}$. Our construction works for the full aTAM at any temperature, and it faithfully simulates the deterministic or nondeterministic behavior of each $\mathcal{T}$.

Our construction succeeds by solving an analog of the cell differentiation problem in devel opmental biology: Each supertile of $\mathcal{U}$, starting with those in the seed assembly "genome" of the simulated system $\mathcal{T}$. At each location of a potential assembly of $\mathcal{U}$, a decision is made whether and how to expr generate a supertile and, if so, which tile of $\mathcal{T}$ it wil using asynchronous communication un global outcome(s).

097v2 [cs.DS] 7 Apr 2012

Is there a universal tileset at T°=2 ?

- Rescaling : ***intrinsic*** *simulation*

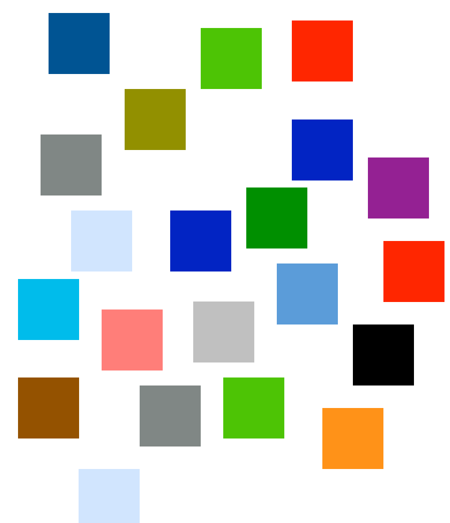


Brice Due 2006

The Game of Life self-simulating itself intrinsically:
Smaller cells simulate macro-cells

Comparing tile assembly models

Is there a set of **intrinsically universal tiles** that can **simulate any tile set**?

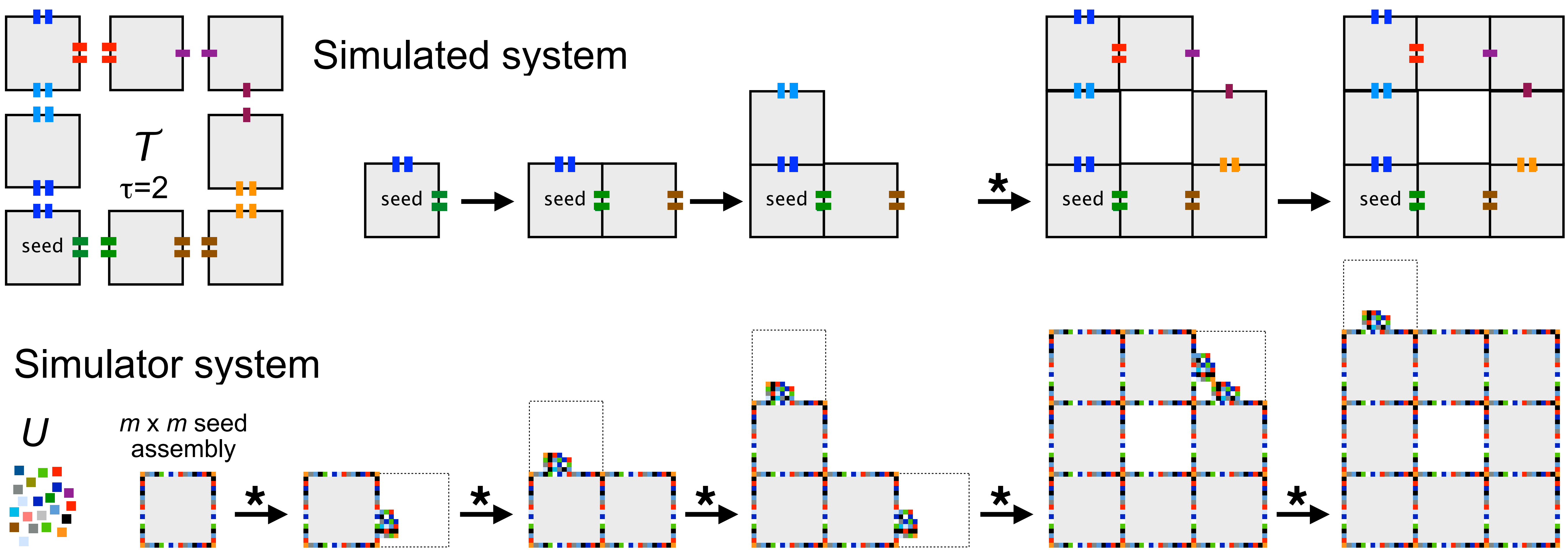


What does “act like” mean?

- What is it that tile assembly systems do?
  - Make shapes and patterns
  - Carry out a crystal-like growth process (dynamics)
- Let define **simulate** using these criteria that are intrinsic to the model

Simulation

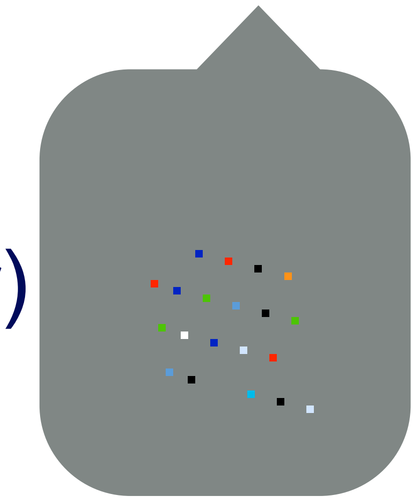
- For (any) simulated tile assembly system $\mathcal{T}$
  - $\mathcal{T}$ = (tileset T, seed assembly σ, temperature τ)
- Tile assembly system $\mathcal{U}$ simulates $\mathcal{T}$ if:
  - **Tiles** from $\mathcal{T}$ are represented by ***m* x *m* supertiles** in $\mathcal{U}$
  - Assemblies produced by $\mathcal{U}$ **represent exactly** assemblies produced by $\mathcal{T}$ (via a representation function R : Blocks of tiles from *U* -> tiles from T)
  - **Dynamics are equivalent** in $\mathcal{U}$ and $\mathcal{T}$, ignoring *m* x *m* scaling



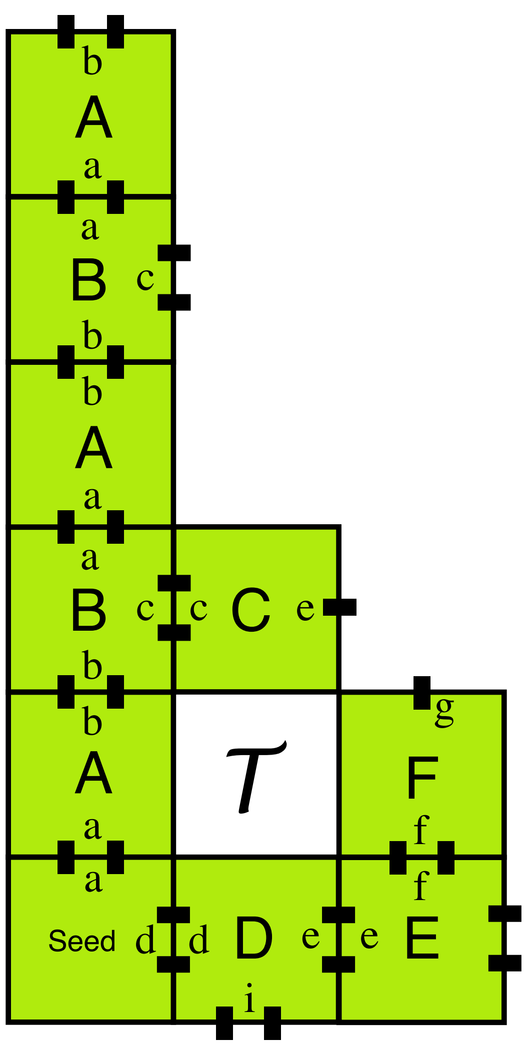
Simulation definition

Ignoring *m* x *m* scaling, production & dynamics are equivalent in the simulated system and simulator

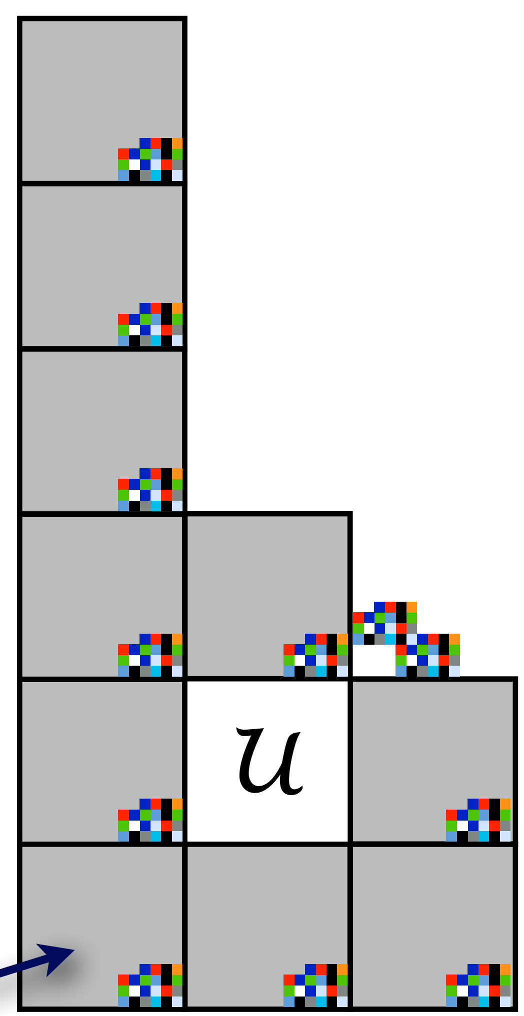
Universal (simulator) tile set

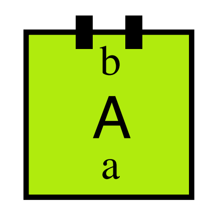


Temperature = 2

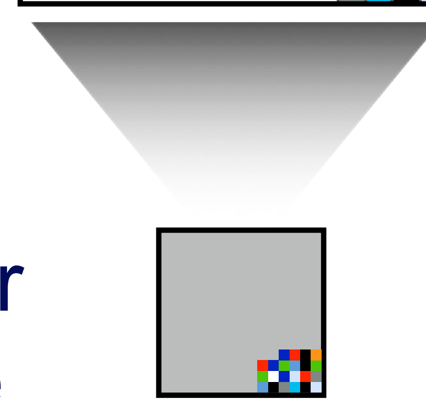


Preassembled seed structure (encodes simulated TAS)





Simulated tile

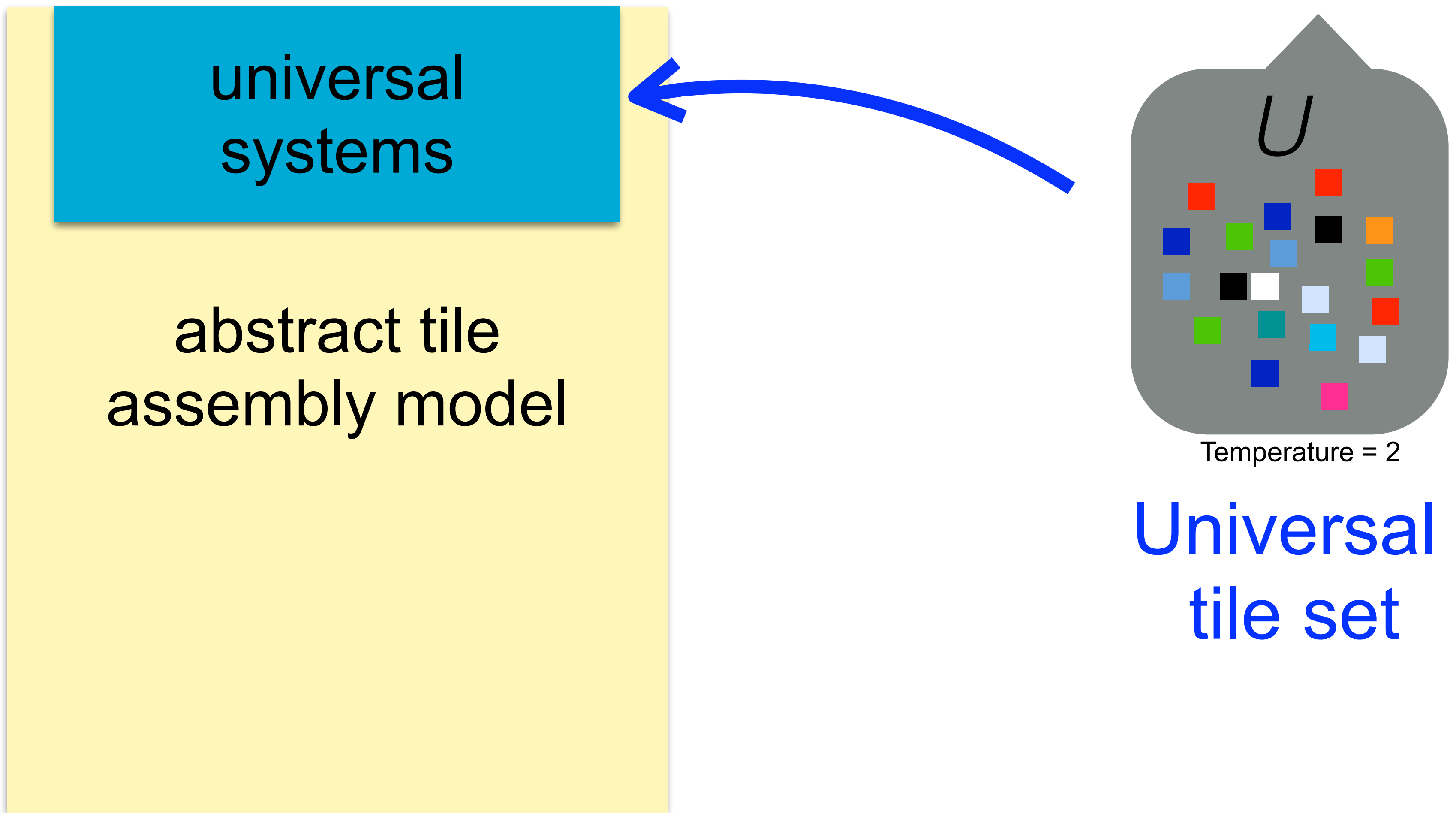


Simulator supertile

- Green tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

etc.

Is the abstract tile assembly model intrinsically universal? **Yes!**



Theorem: There is a single intrinsically universal tile set *U* that simulates *any* tile assembly system

Doty, Lutz, Patitz, Schweller, Summers, Woods. FOCS 2012

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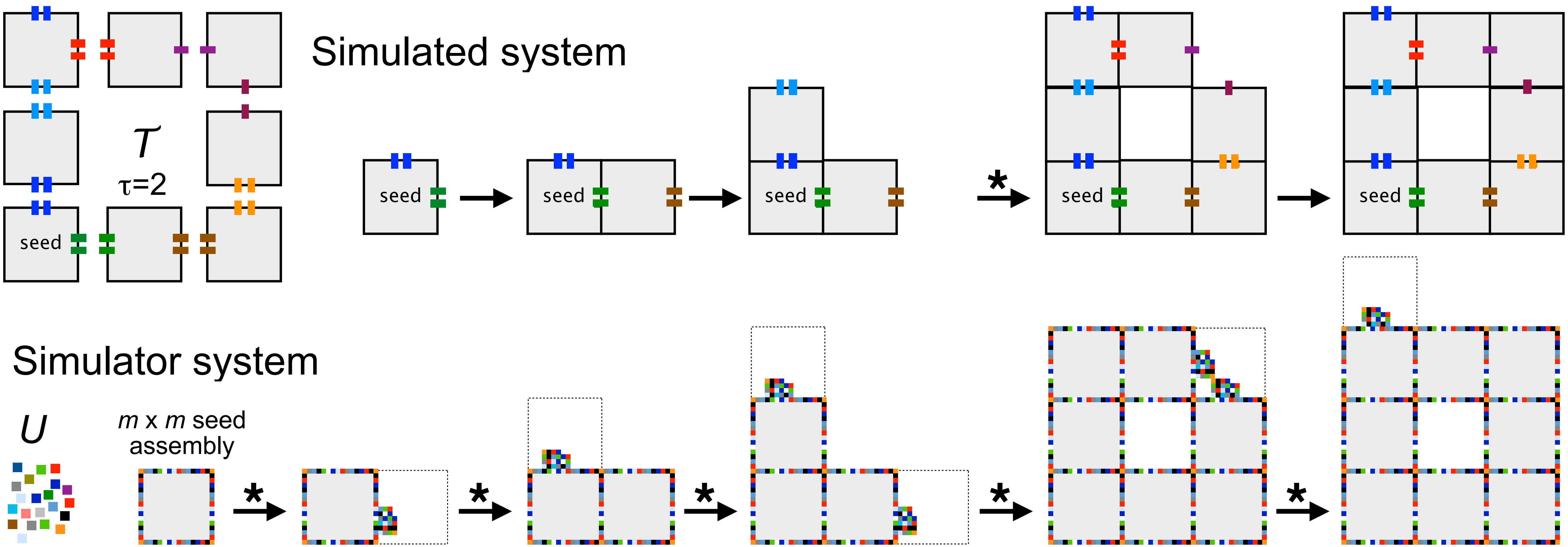
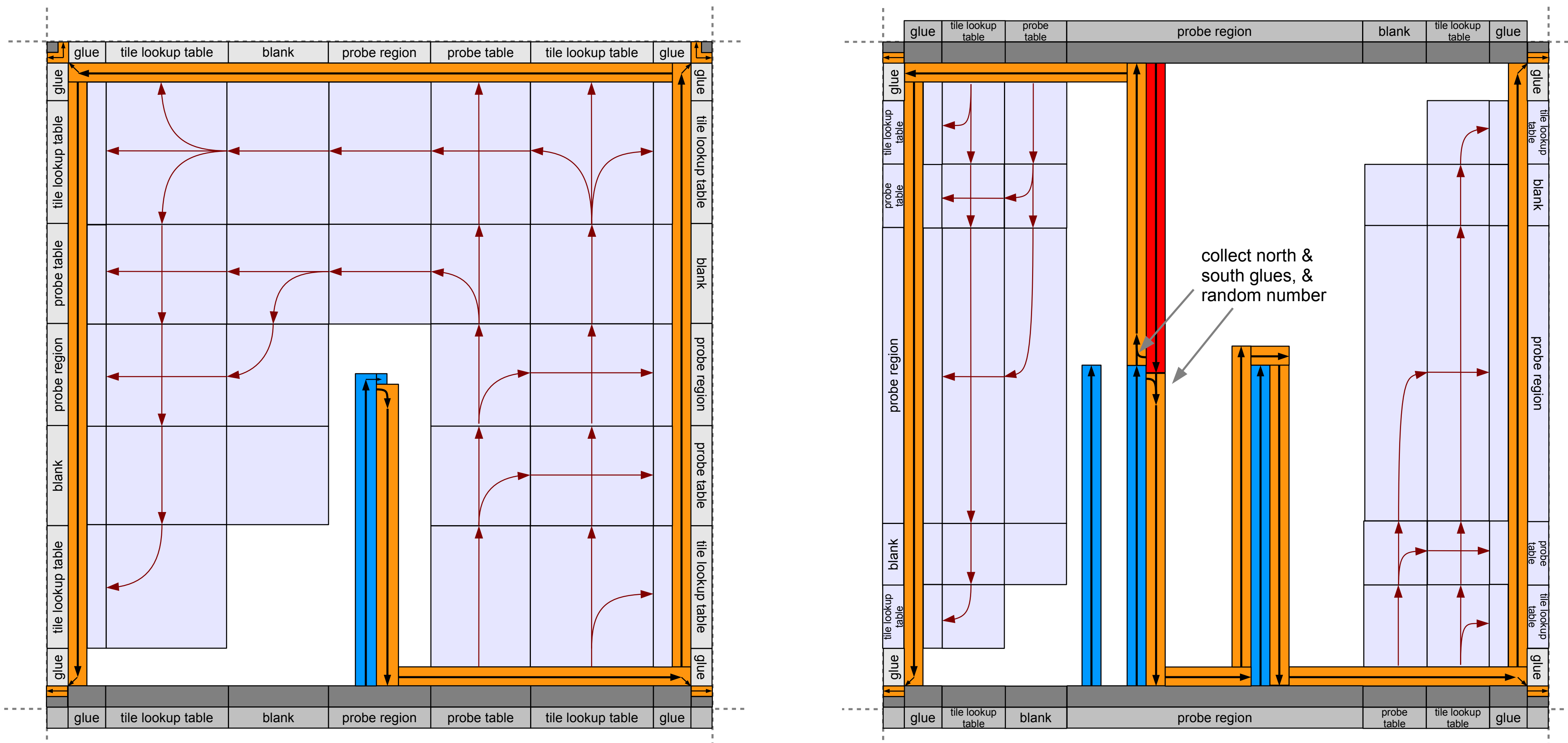


Is there a universal tileset at T°=2 ?

- Rescaling : *intrinsic simulation*

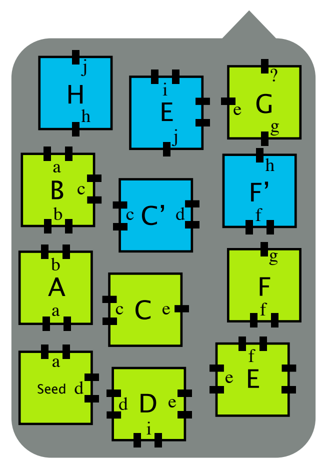


Doty Lutz Patitz Schweller Summers Woods 2012

Examples of macro-tiles

Superside

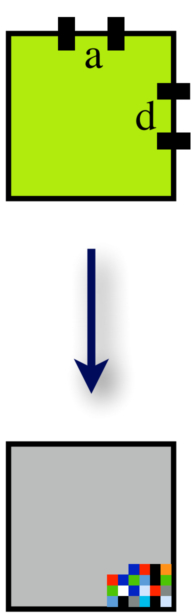
Encoding of the entire simulated **tile assembly system** written down using tiles from the simulator *U*



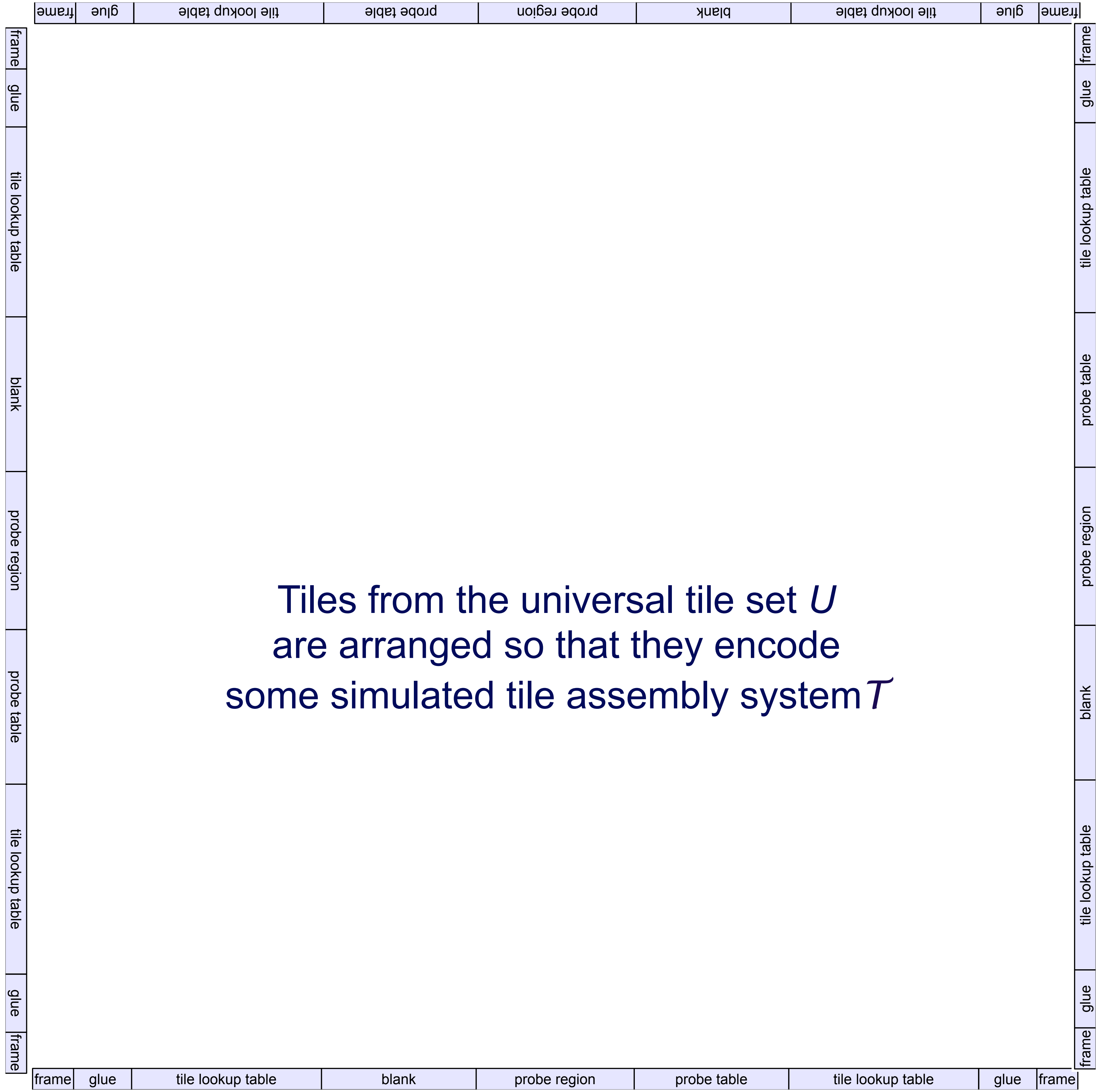
Temperature = 2

frame	glue	tile lookup table	blank	probe region	probe table	tile lookup table	glue	frame
4	$O(\log \lvert T \rvert)$	$O(\lvert T \rvert^4 \log \lvert T \rvert)$	$O(\lvert T \rvert^2)$	$O(\lvert T \rvert^2)$	$O(\lvert T \rvert^2)$	$O(\lvert T \rvert^4 \log \lvert T \rvert)$	$O(\log \lvert T \rvert)$	4

$|T|$ is number of tiles in the simulated tileset T.

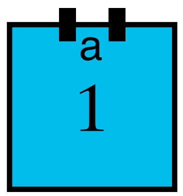


Preassembled seed supertile



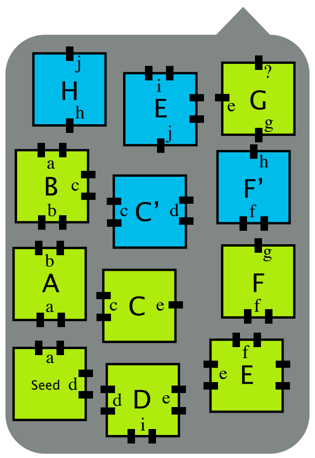
Tiles from the universal tile set *U* are arranged so that they encode some simulated tile assembly system *T*

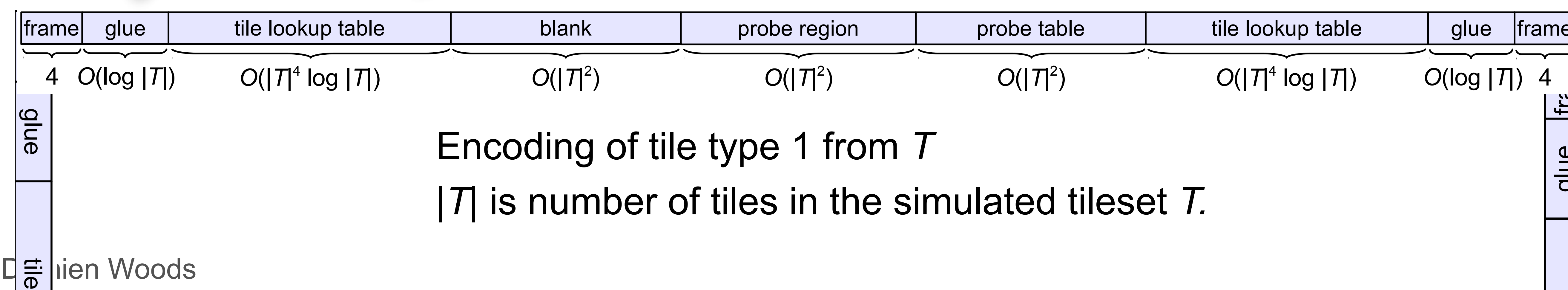
Superside



Encoded **glue** of **this** superside (e.g. "a")

Encoding of the entire simulated **tile assembly system** written down using tiles from the simulator *U*





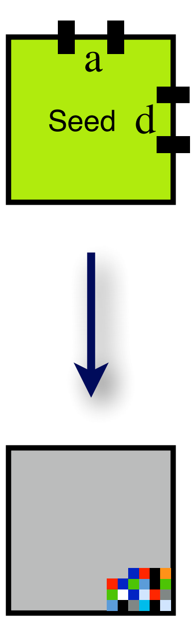
Encoding of tile type 1 from *T*

$|T|$ is number of tiles in the simulated tileset *T*.

seed supertile

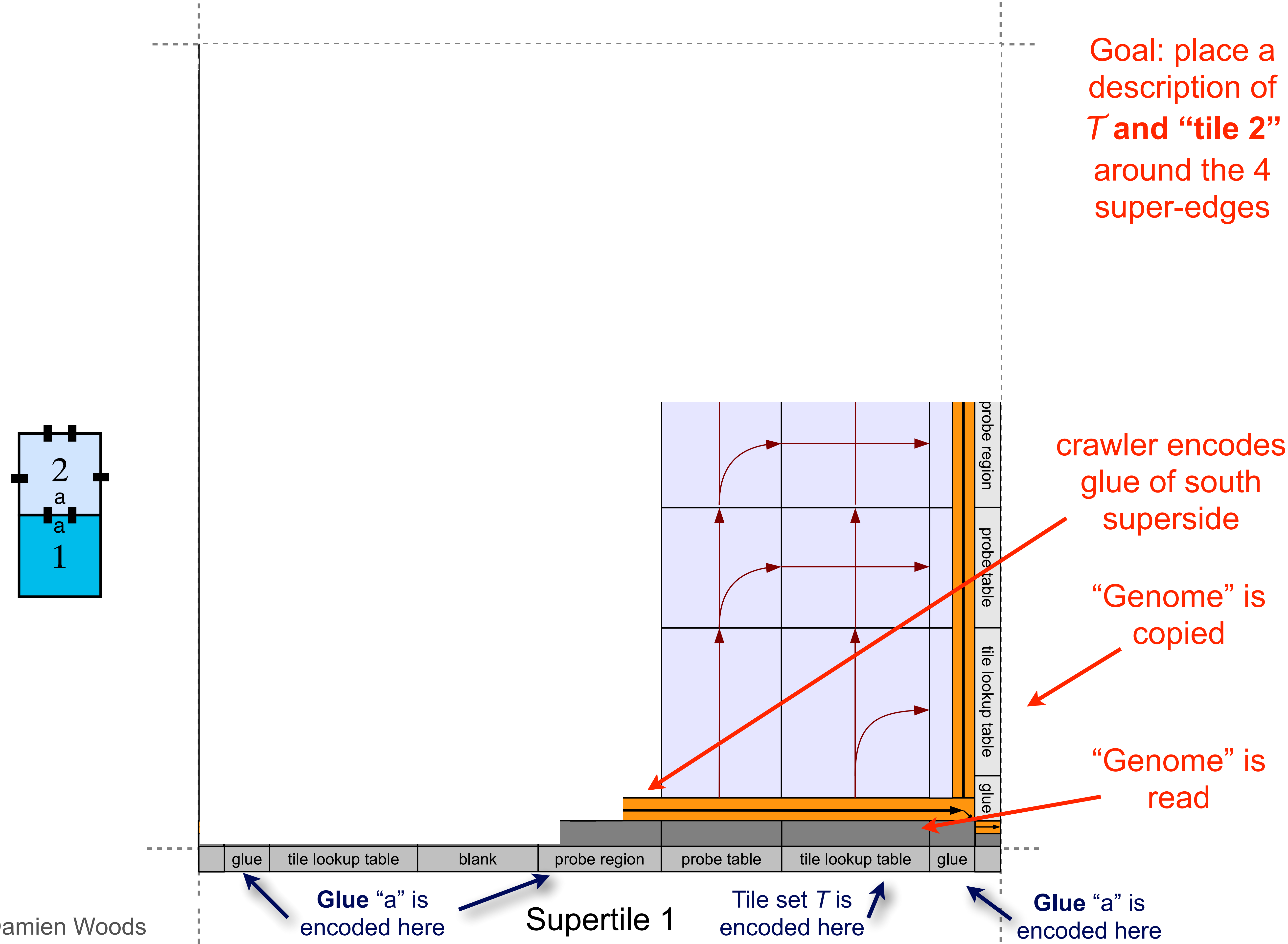
Growth begins from here!

Tiles from the universal tile set *U*
are arranged so that they encode
some simulated tile assembly system $\mathcal{T}$

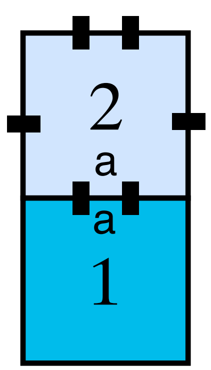


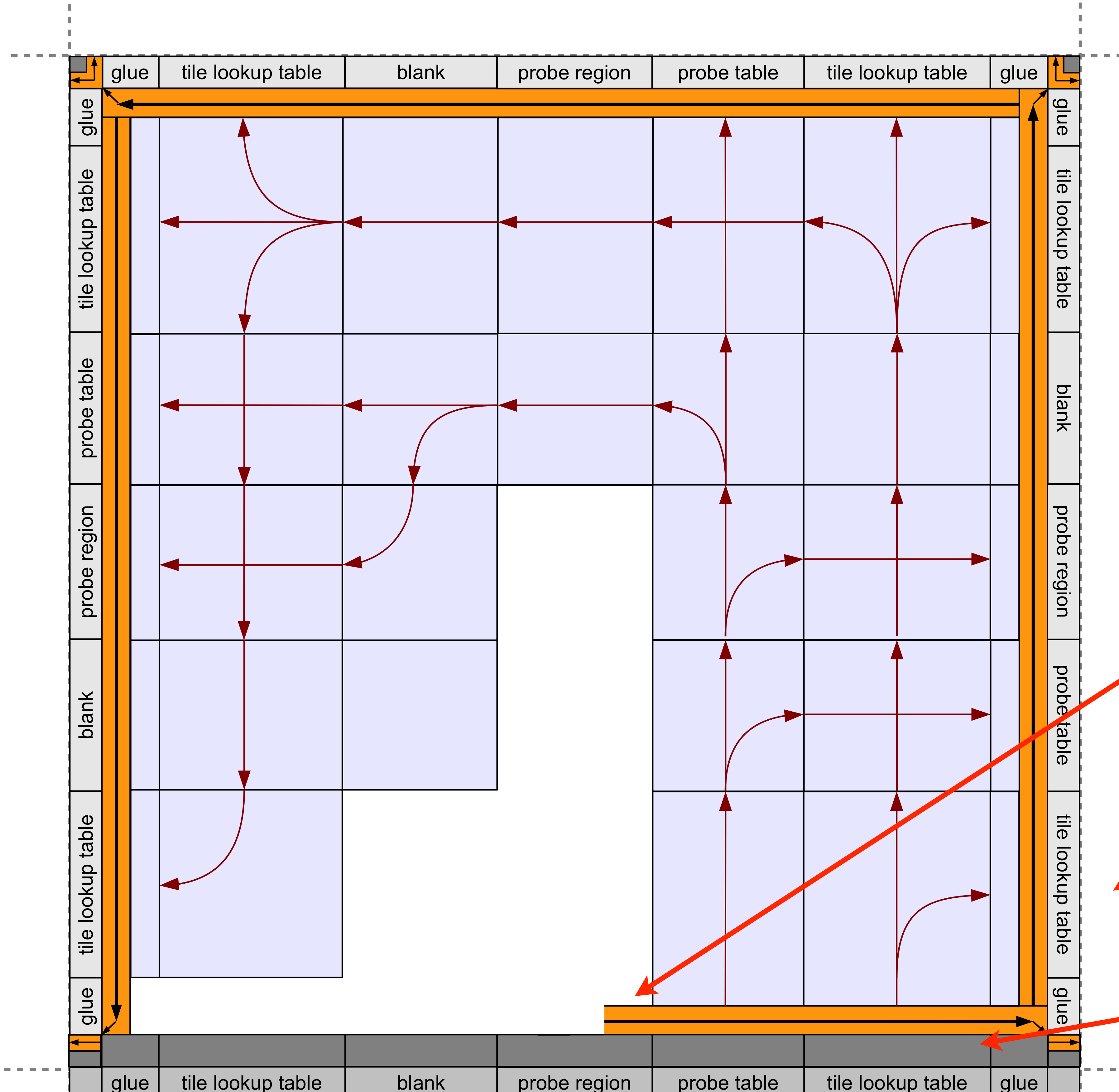
Preassembled
seed supertile

One-sided binding with a single strength-τ south superside



One-sided binding with a single strength-τ south superside





Goal: place a description of *T* **and "tile 2"** around the 4 super-edges

Nondeterminism
Rotations

crawler encodes glue of south superside

"Genome" is copied

"Genome" is read

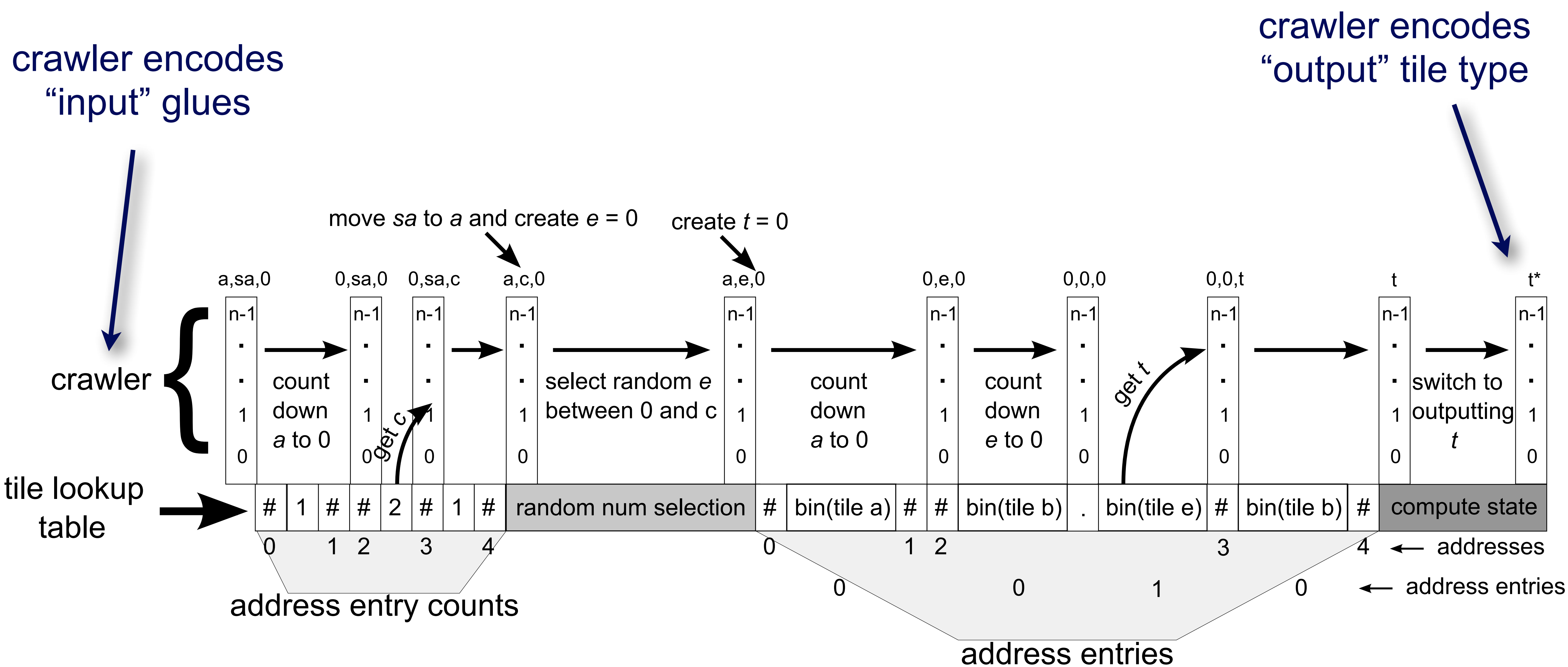
Glue "a" is encoded here

Supertile 1

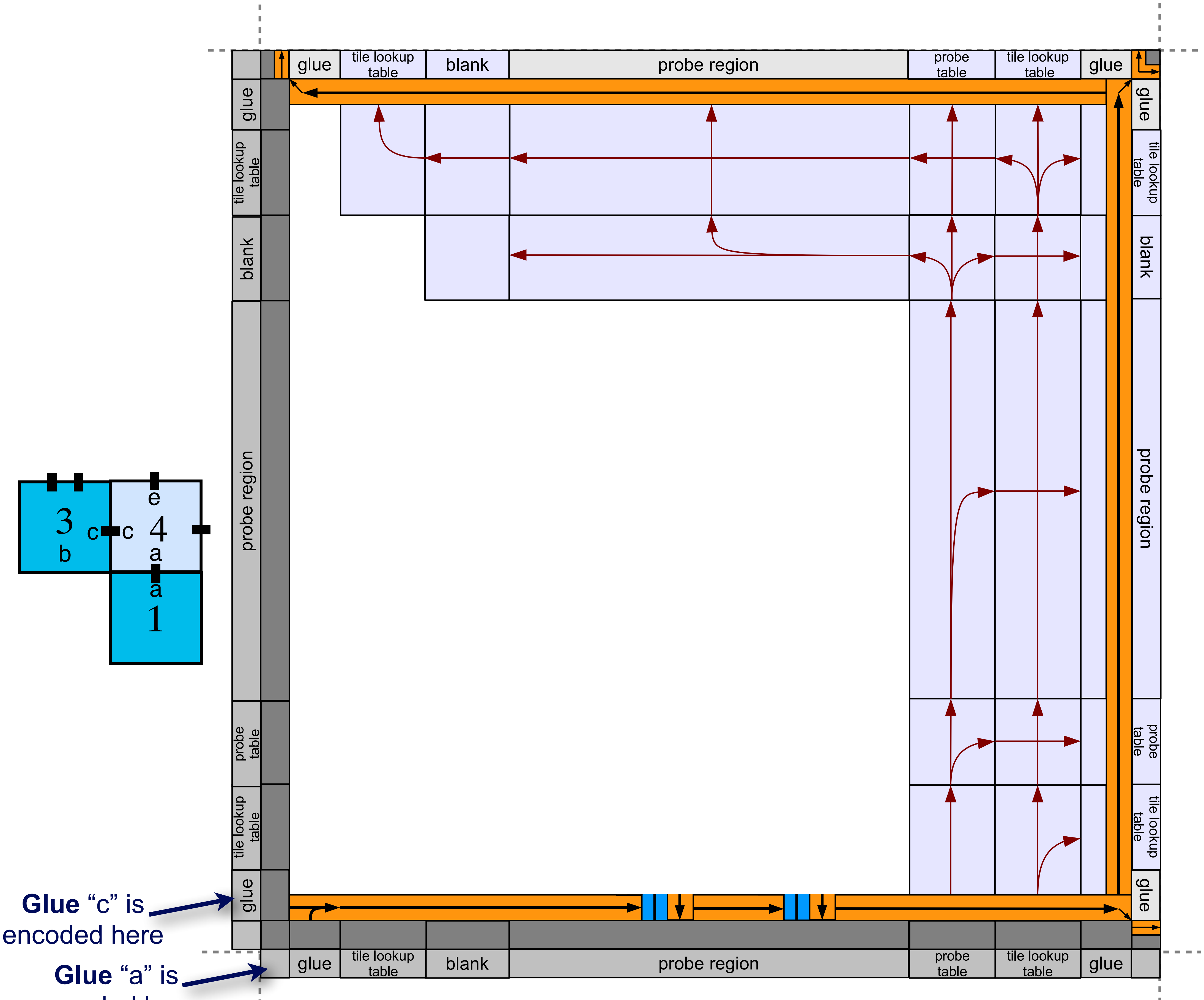
Tile set *T* is encoded here

Glue "a" is encoded here

Crawler doing a tile lookup



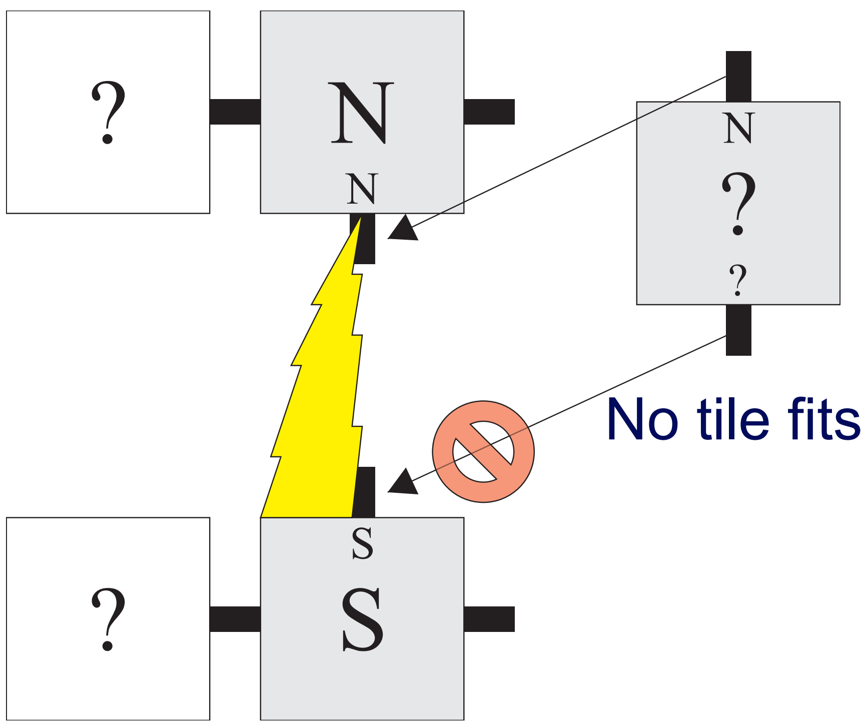
Two-sided binding with adjacent cooperating supersides



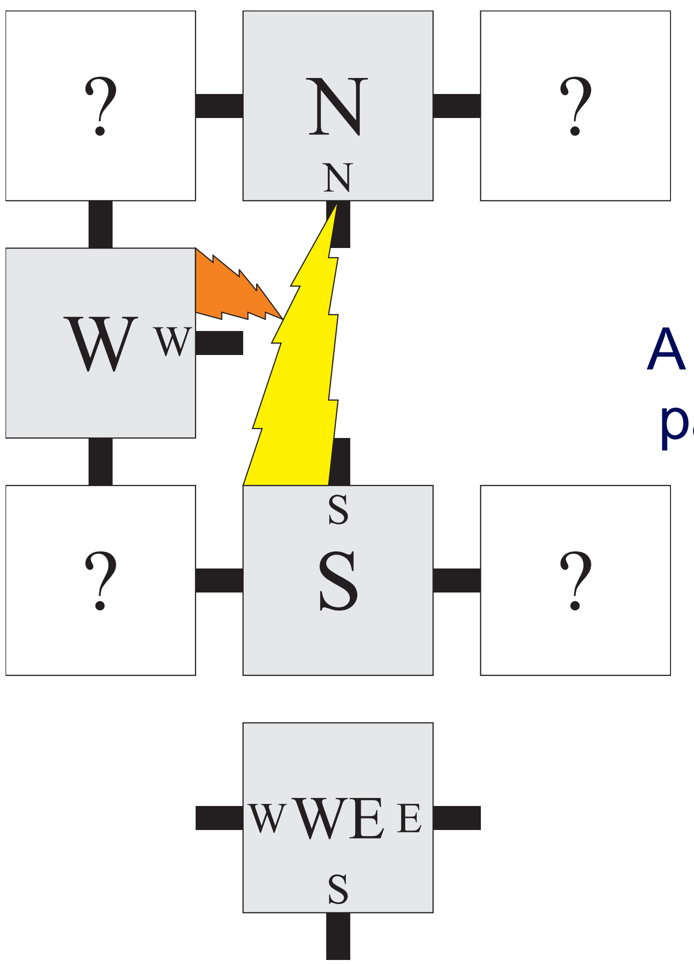
Glue "c" is encoded here

Glue "a" is encoded here

A key problem



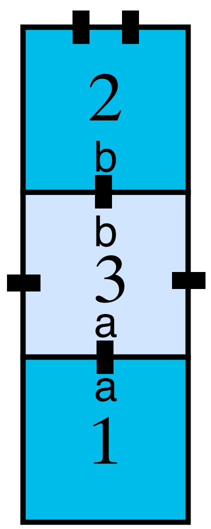
Better luck next time!

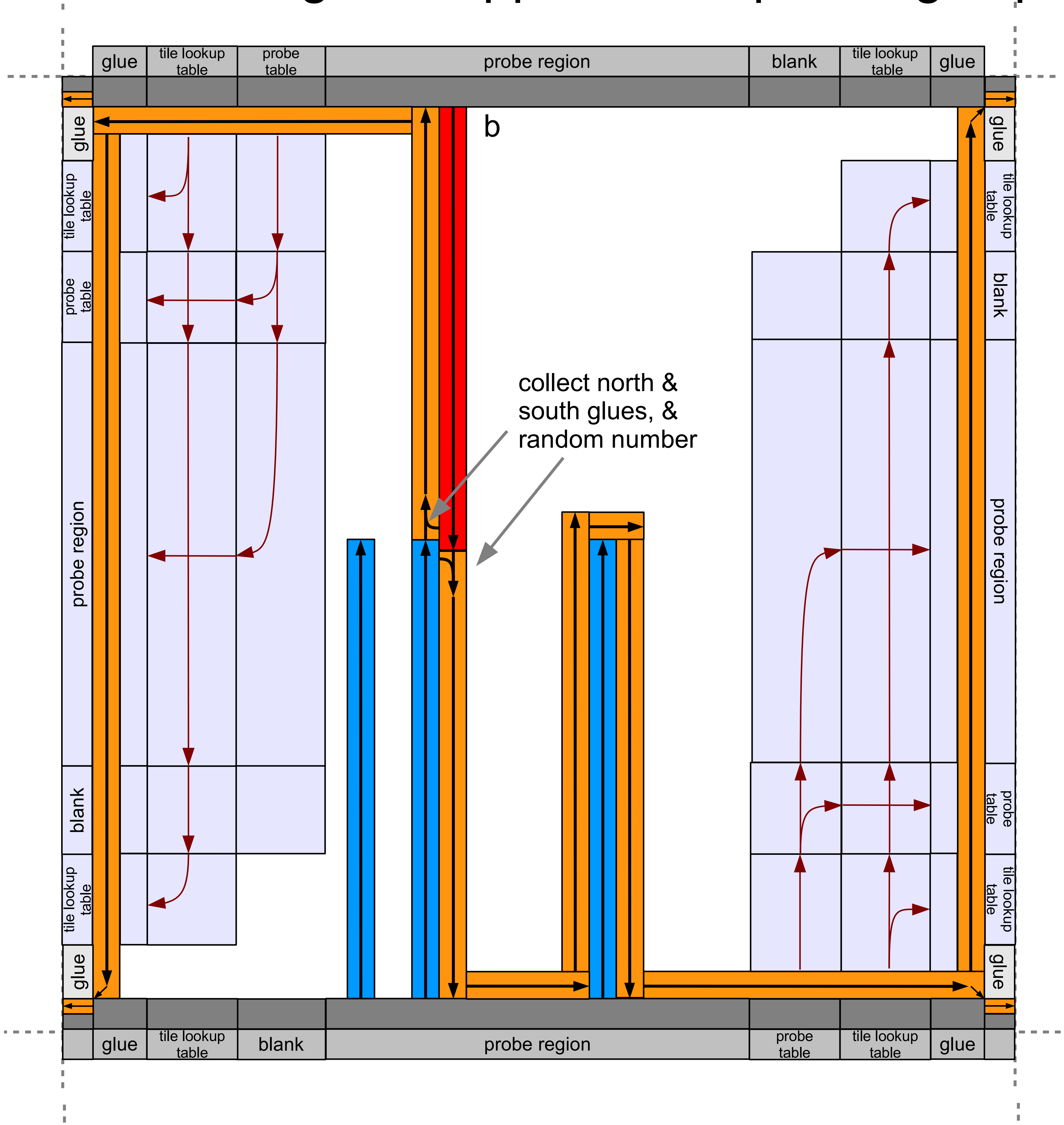


A tile fits, but the path to the right is blocked!

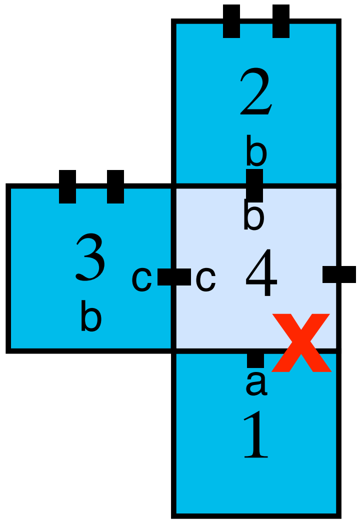
Uh oh!

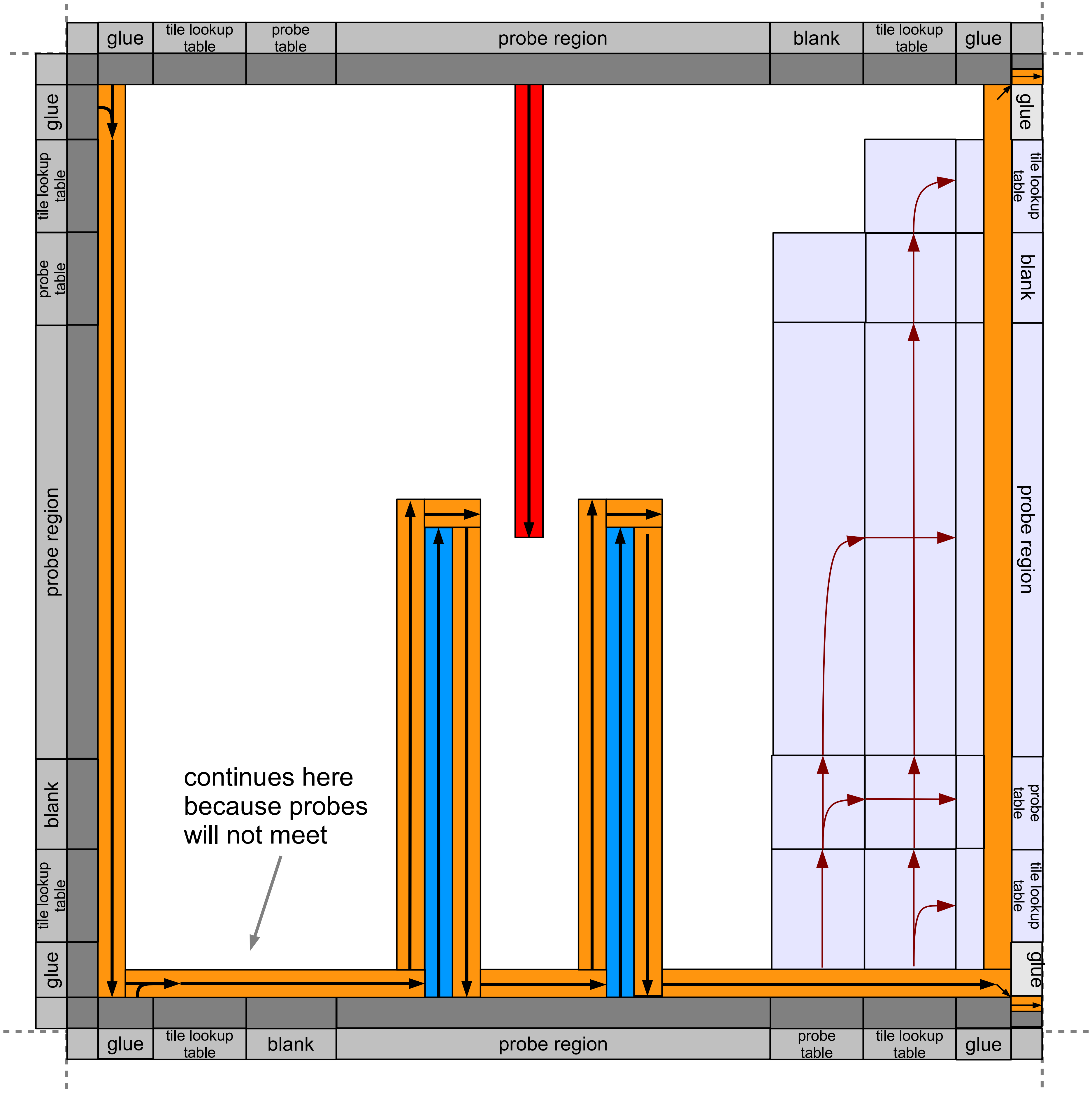
Two-sided binding with opposite cooperating supersides

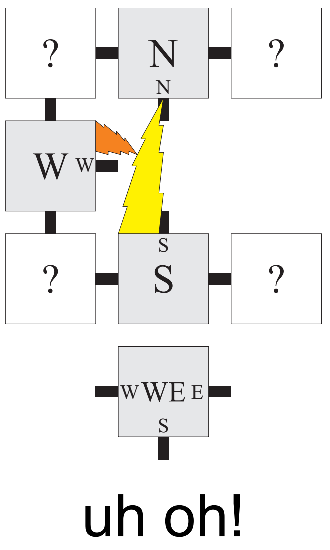




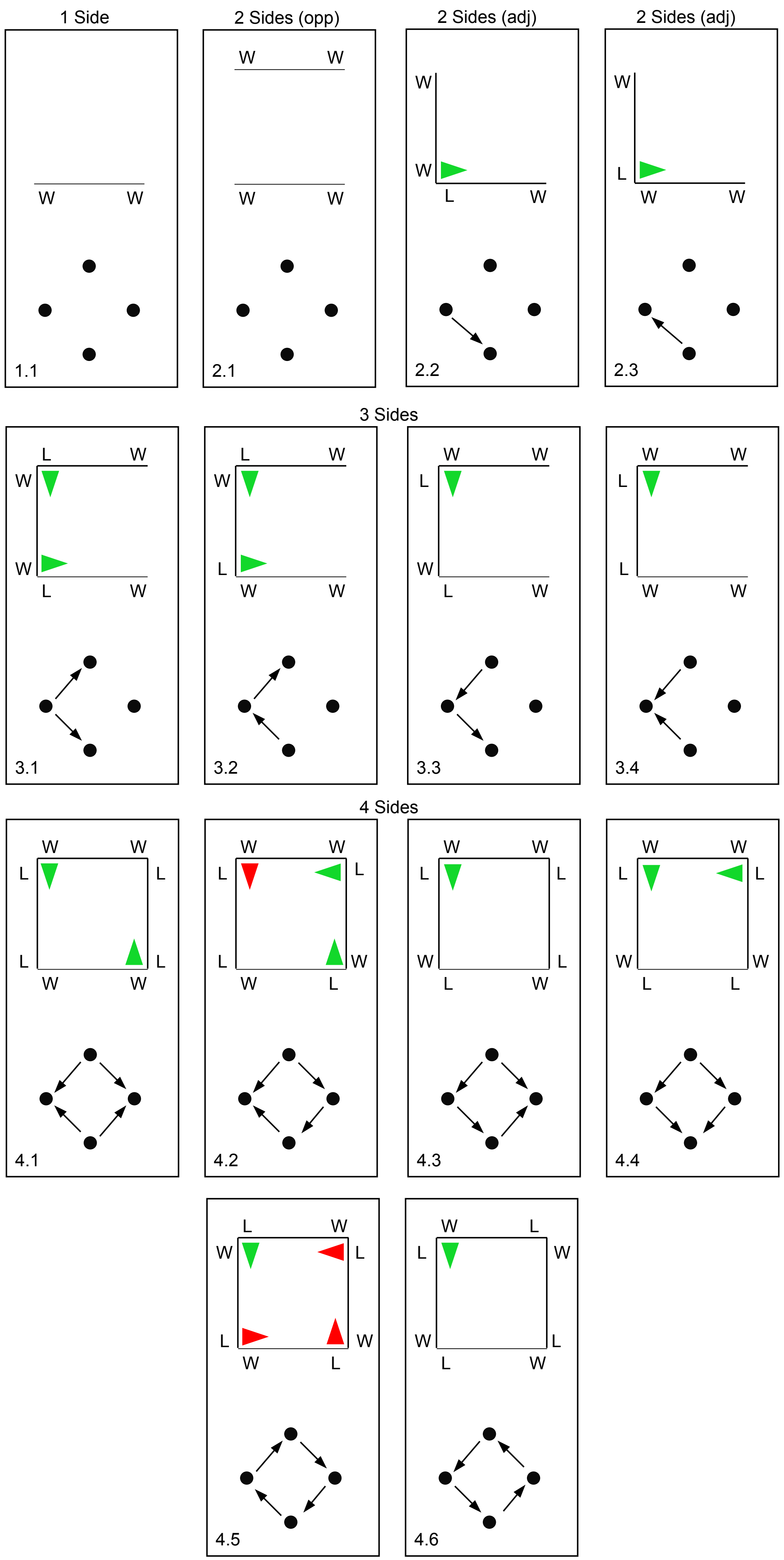
3-sided “uh-oh” example: probes miss each other

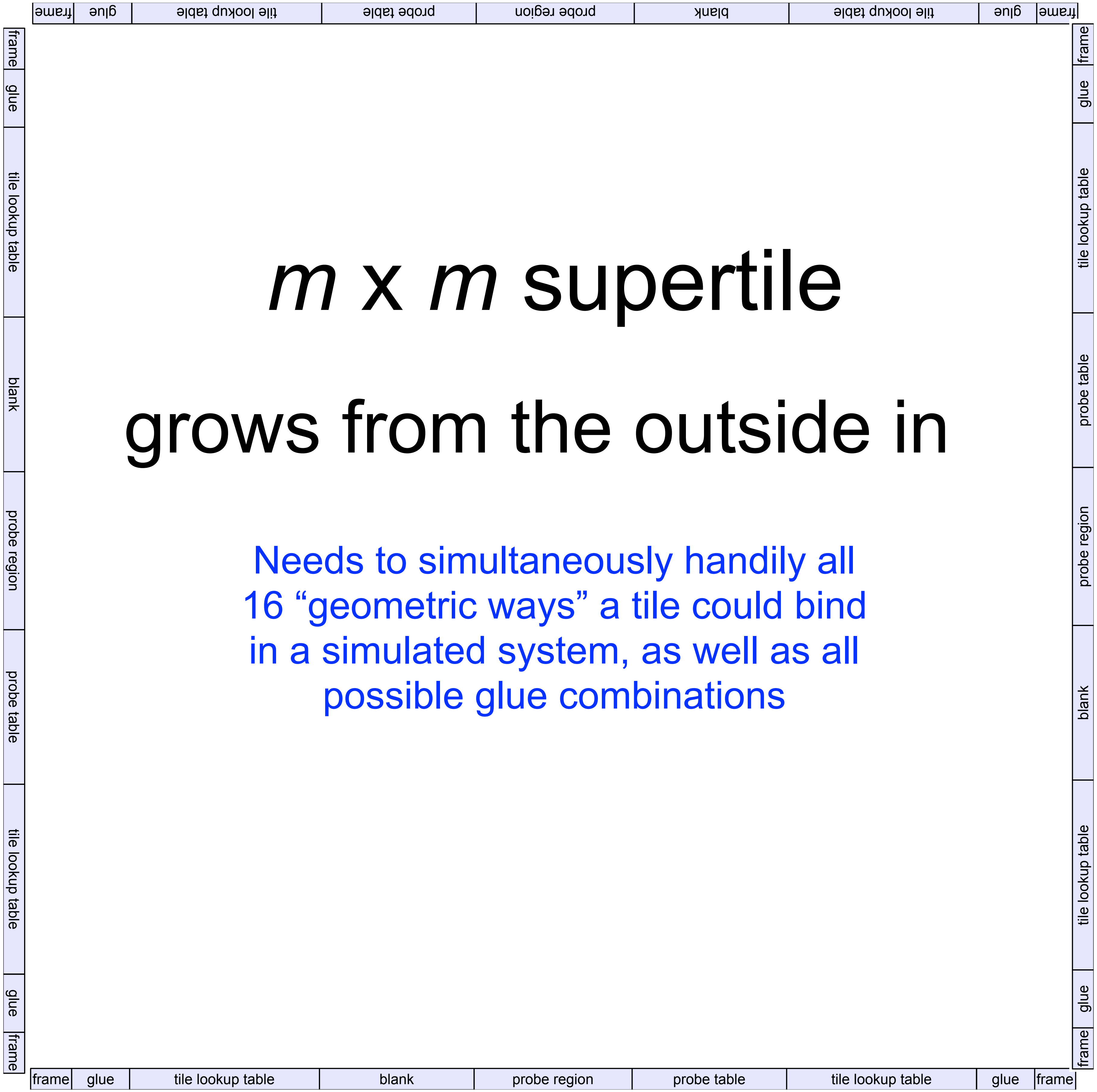






- Variety of cases for different orders of superside arrival
- Superside win/lose configurations and crawler initiation locations (green)
- Proof analogy:
  - Distributed game
  - Computation & geometry
  - Key challenge: make all the tricks work together



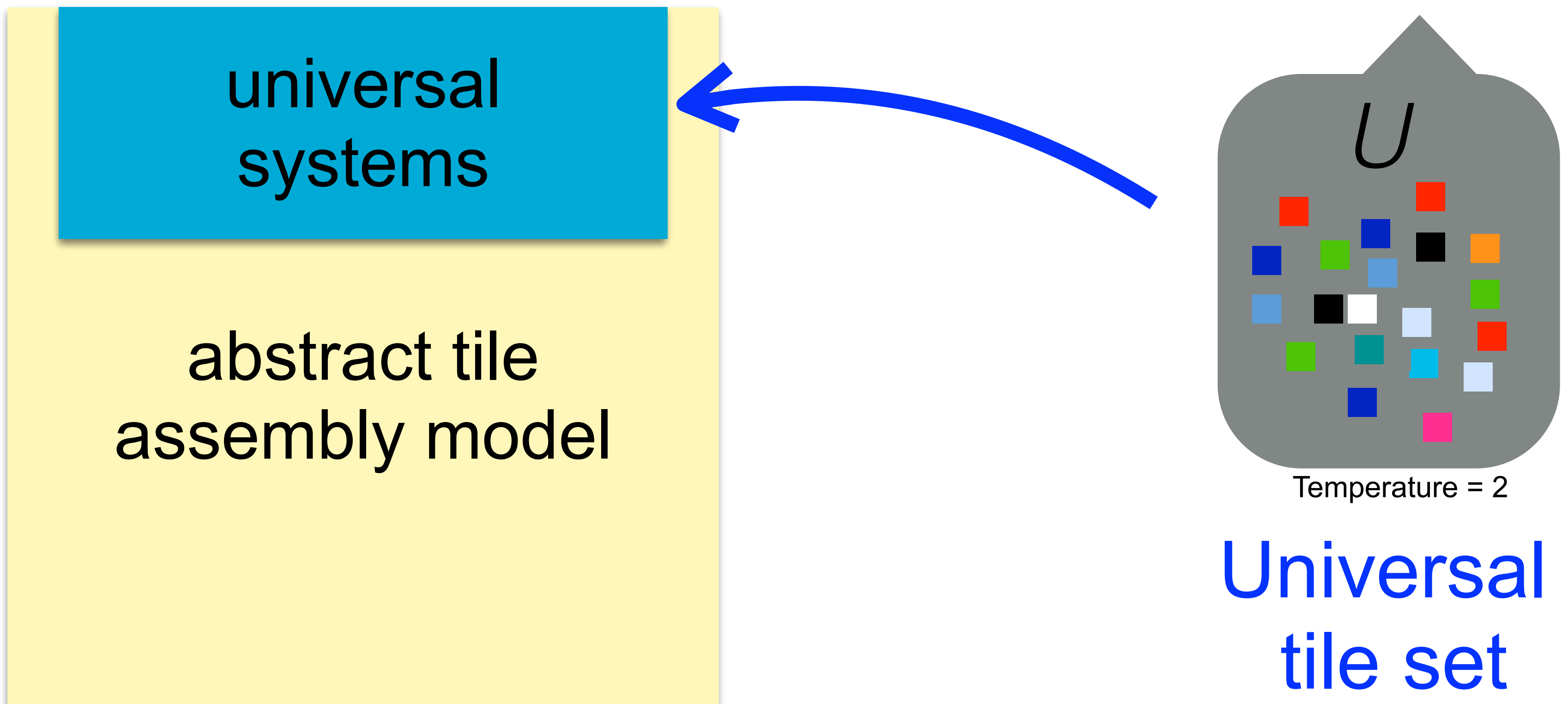


m x *m* supertile

grows from the outside in

Needs to simultaneously handily all 16 “geometric ways” a tile could bind in a simulated system, as well as all possible glue combinations

Is the abstract tile assembly model intrinsically universal? **Yes!**



Theorem: There is a single intrinsically universal tile set *U* that simulates *any* tile assembly system

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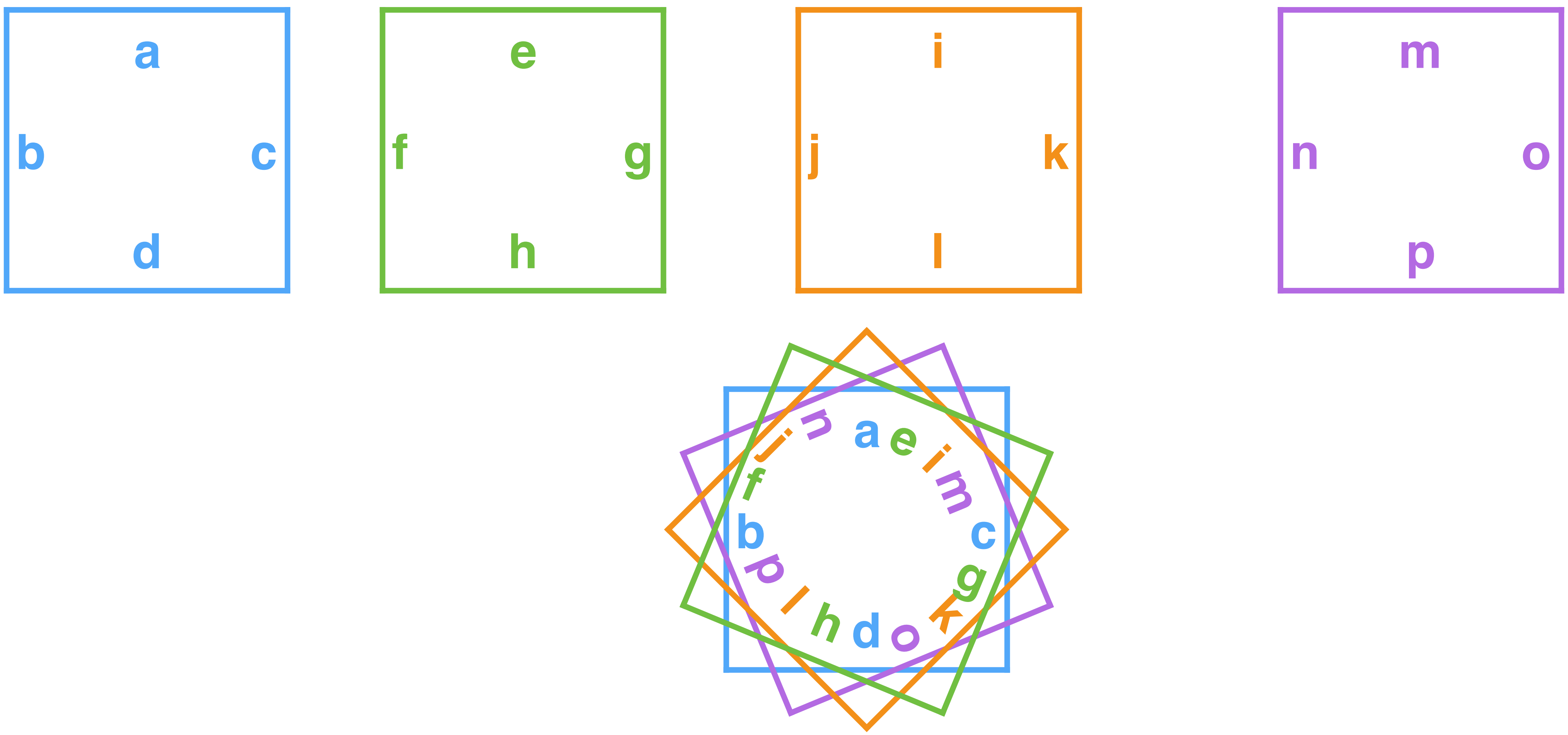
How many tiles do we need ?

- Just ONE with rotation!… What?!?… But a *polygonal* one



Demaine Demaine Fekete Patitz Schweller Winslow Woods 2012

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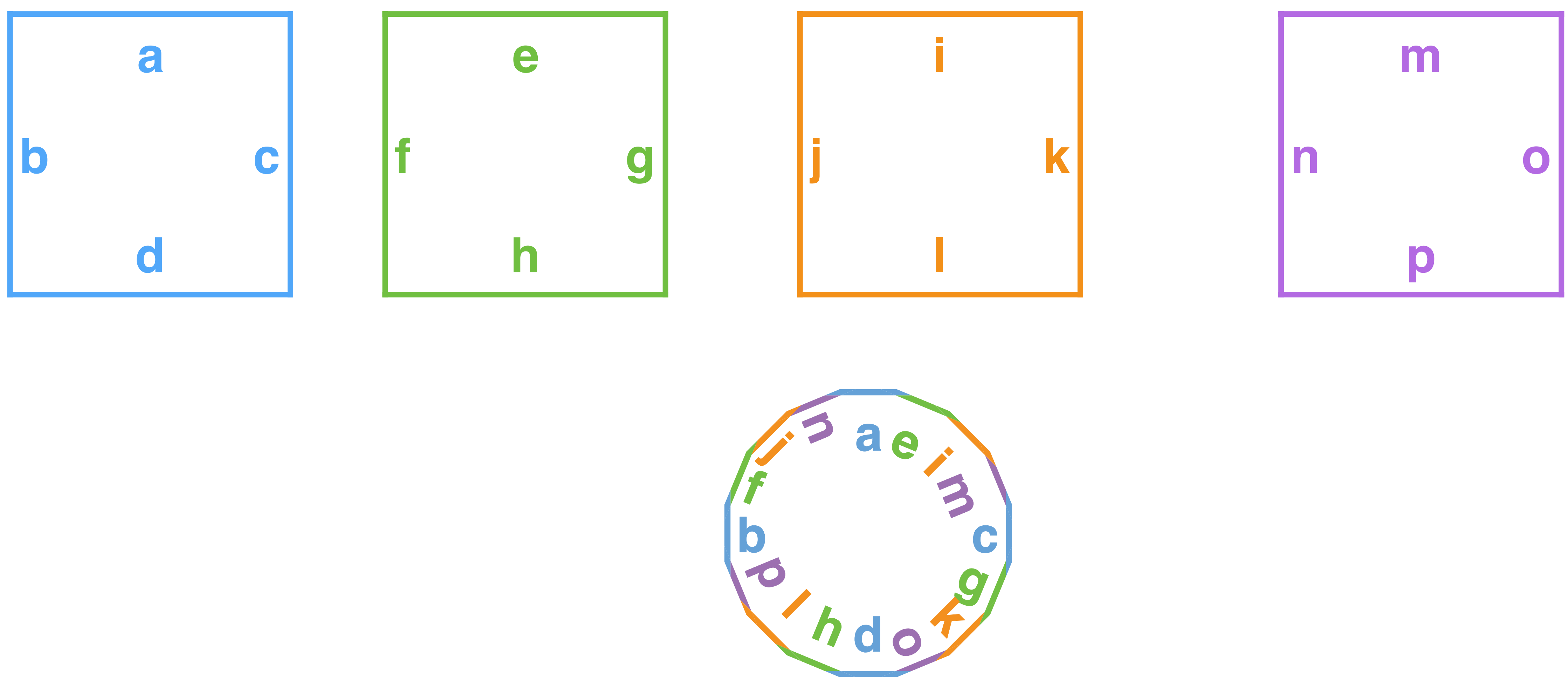

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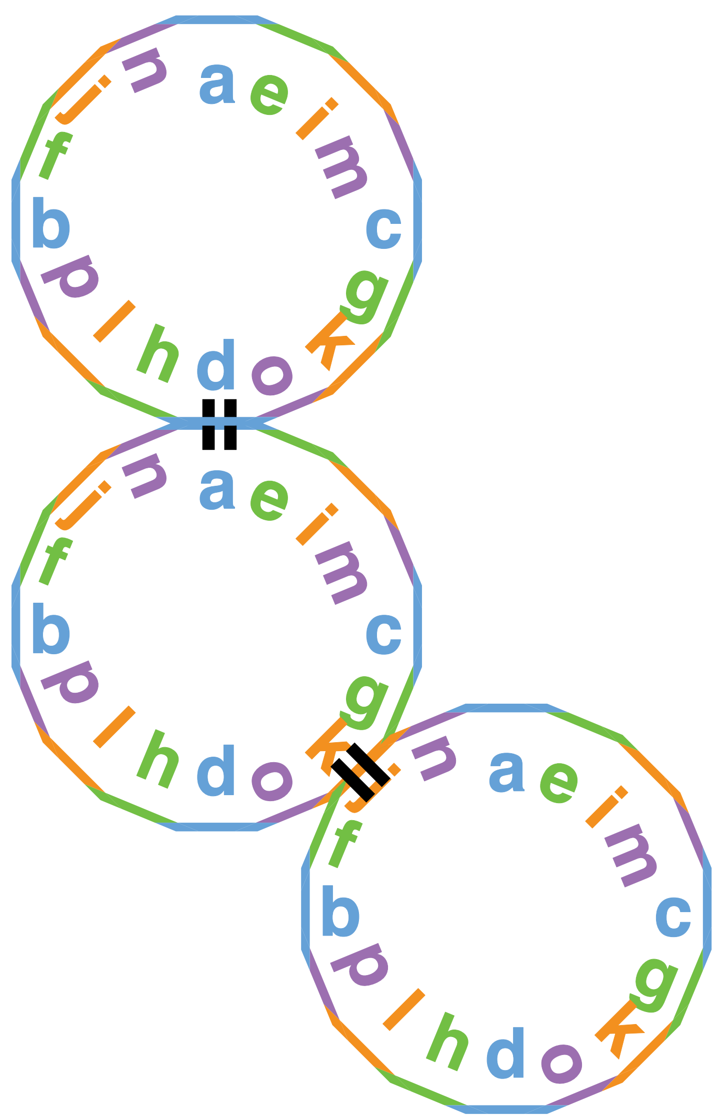
Problem with glue of strength 2 !!!



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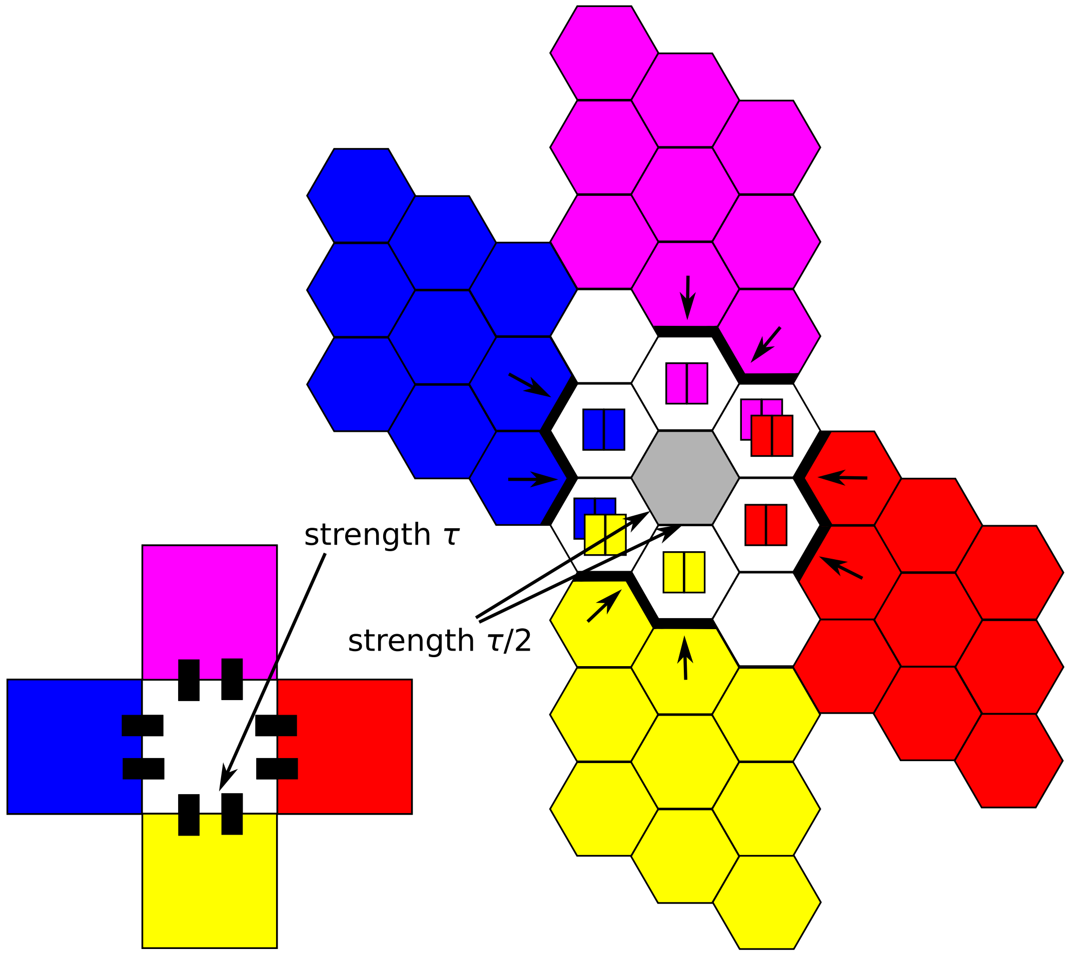
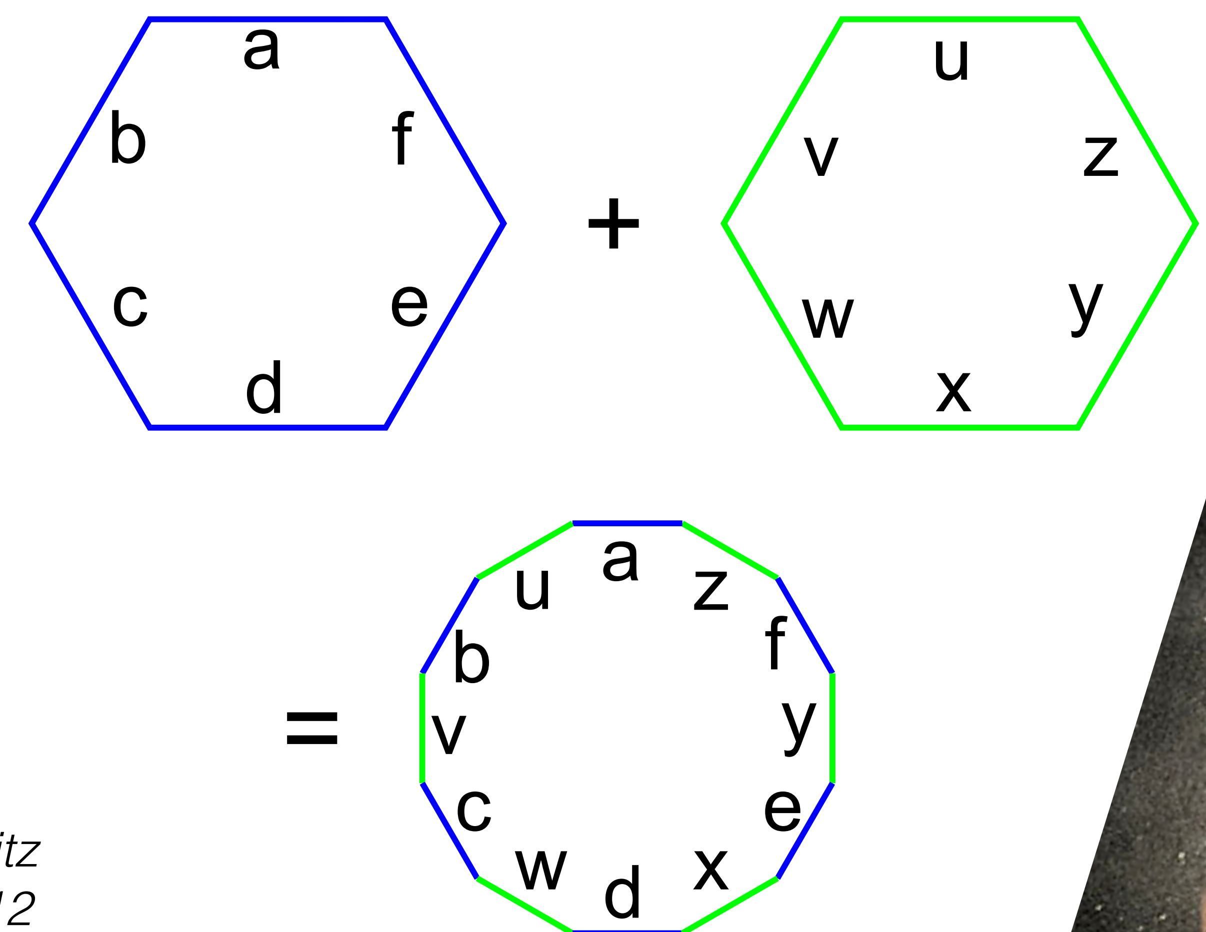
Encoding strength 2 glues into strength 1 glue in hexagonal tiles



A **single** (polygonal) tile is enough !



Demaine Demaine Fekete Patitz Schweller Winslow Woods 2012

The magic powder can assemble anything!

One molecule is enough !

Co-transcriptional folding

Joint work with Cody Geary, Pierre-Étienne Meunier, Daria Pchelina, Shinnosuke Seki, Guillaume Theyssier and Yuki Ubukata