Internship, Master 2
Strucutre of classes of graphs defined by constraints on chords
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Description

The goal of this intership is to get familiar with structural graph theory through the study of a particular class of graphs. Here graphs are non oriented and simple. A cycle in a graph is a *cluc* if it has length at least 5 and a unique chord (cluc stands for "Cycle, Long, with a Unique Chord"). We call $\mathcal{C}$ the class of graphs that contains no cluc as an induced subgraph. We call $\mathcal{B}$ the class of graphs such that no cycle has a unique chord. Note that $\mathcal{B}$ is a subclass of $\mathcal{C}$. Note that if a graph is in $\mathcal{C}$ and not in $\mathcal{B}$, then it must contain a cycle of length 4 with a unique chord. We denote by $\chi(G)$ the chromatic number of G and by $\omega(G)$ the maximum number of pairwise adjacent vertices in G.

The following two questions are open.

- Is there a polynomial time algorithm that decides whether an input graph is in $\mathcal{G}$?
- Is there a polynomial f such that every graph G in $\mathcal{C}$ satisfies $\chi(G) \leq f(\omega(G))$?

The class under consideration is a generalisation of two known classes: the class $\mathcal{B}$ defined above, and the class of chordal graphs (these received a lot of attention, a wikipedia page is devoted to them). The papers cited below all give hints toward the solutions of these two questions. First both questions are solved for $\mathcal{B}$ in [5], and for chordal graphs (classical result). A function f such that $\chi(G) \leq f(\omega(G))$ for all graphs in $\mathcal{C}$ is known [4], but it is not a polynomial. The class $\mathcal{B}$ and the class of chordal graphs share common structural properties [3, 1], and the so-called class of HHD-free graphs introduced in [2] might be important to undestand the structure of the graphs that are in $\mathcal{C} \setminus \mathcal{B}$.

Skills

Knowledge of graph theory is the main required skill. Basic knowledge of complexity theory and programming is appreciated.

References

[1] A. Brandstädt, F.F. Dragan, V.B. Le, and T. Szymczak. On stable cutsets in graphs. *Discrete Applied Mathematics*, 105(1-3):39–50, 2000.

[2] C.T. Hoàng and N. Khouzam. On brittle graphs. *Journal of Graph Theory*, 12(3):391–404, 1988.

[3] T. McKee. Independent separator graphs. *Utilitas Mathematica*, 73:217–224, 2007.

[4] I. Penev. Amalgams and χ-boundedness. Manuscript available at http://perso.ens-lyon.fr/irena.penev/, 2014.

[5] N. Trotignon and K. Vušković. A structure theorem for graphs with no cycle with a unique chord and its consequences. *Journal of Graph Theory*, 63(1):31–67, 2010.