Réécriture de diagrammes convergente pour l'algèbre linéaire

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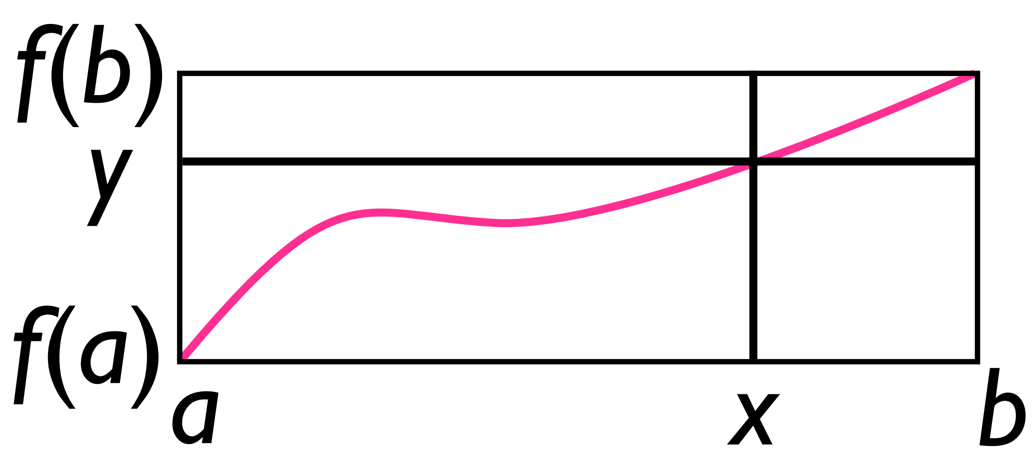
Prologue : Le langage mathématique

Si la fonction *f* est *continue*, alors on a :

langage naturel (français)

$$\forall\, y,\ f(a) \leq y \leq f(b) \ \Rightarrow\ \exists\, x \in [a,b],\ f(x) = y.$$

langage formel (ou symbolique)



figure

Diagramme sagittal

$$f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$$

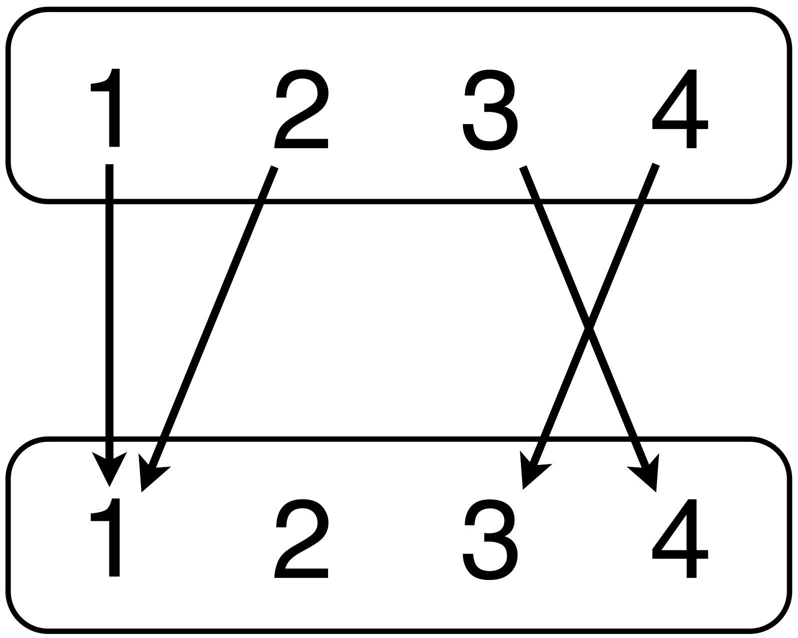
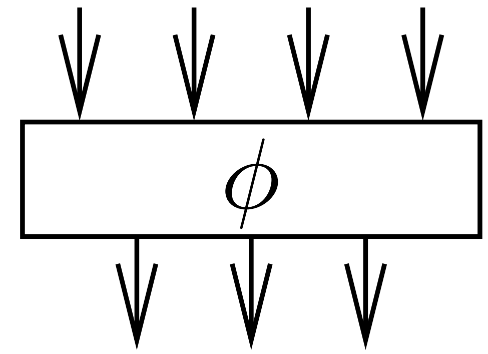


figure ?

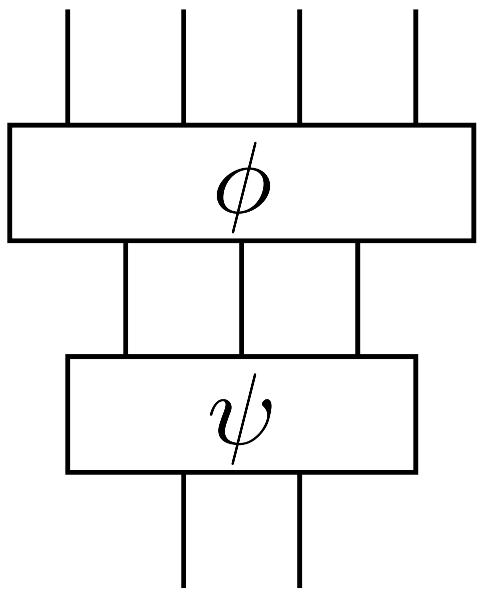
ou langage formel ?

Syntaxe des diagrammes

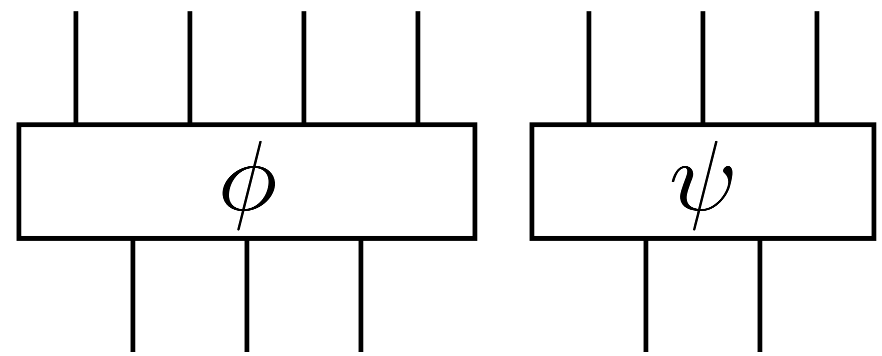
entrées/sorties :



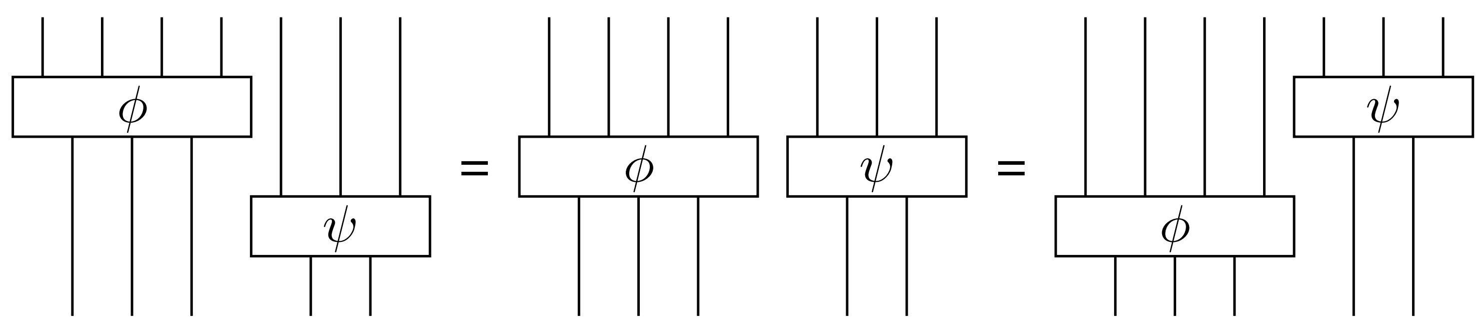
composition séquentielle



composition parallèle



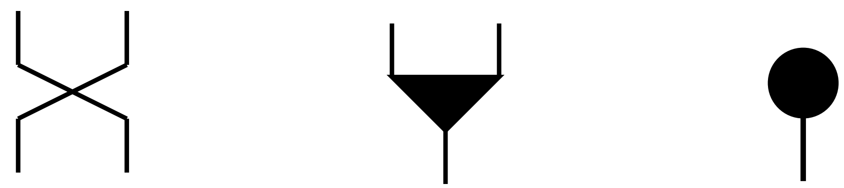
règle d’échange :



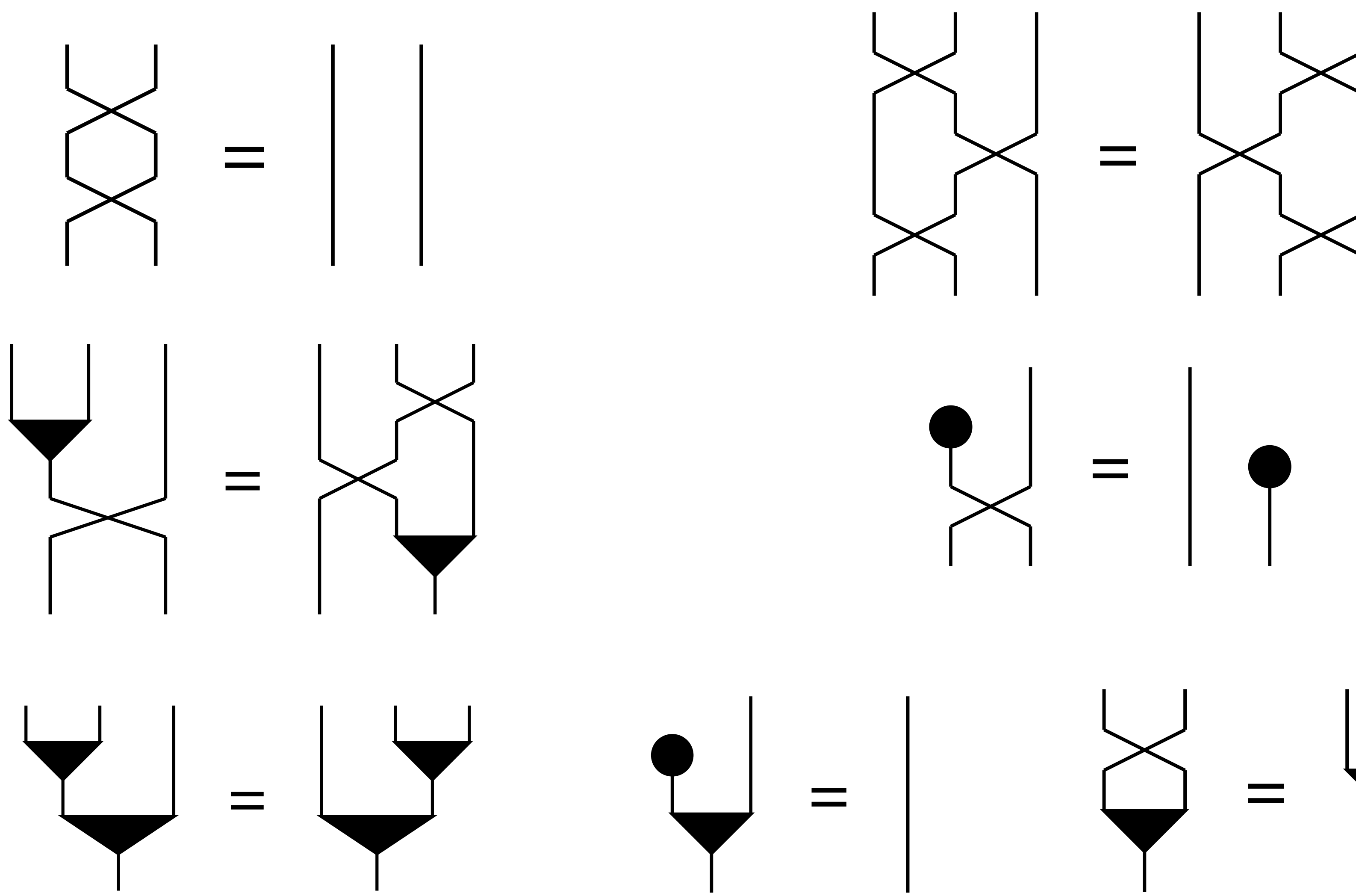
→ structure de PRO (*catégorie monoïdale stricte* dont les objets sont les *entiers naturels*)

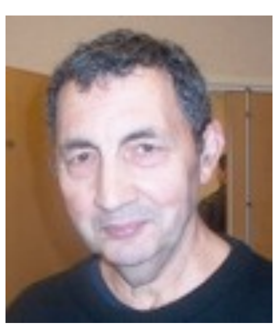
diagrammes pour les applications finies

générateurs :



relations :

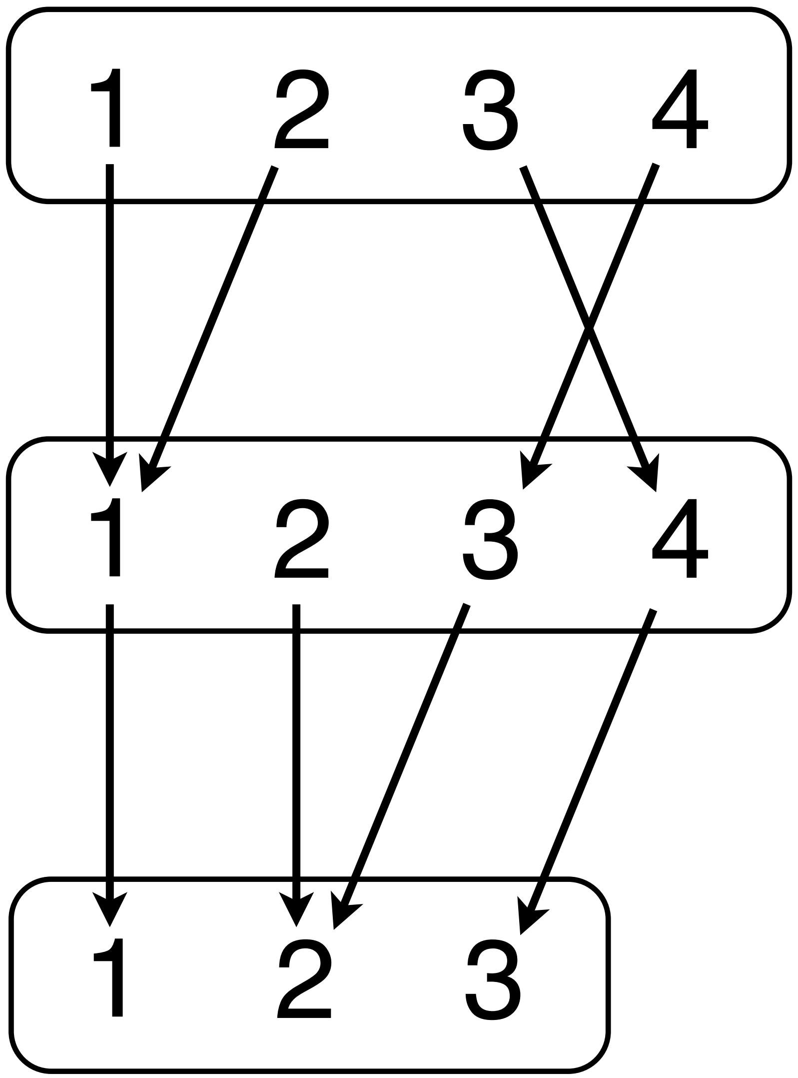


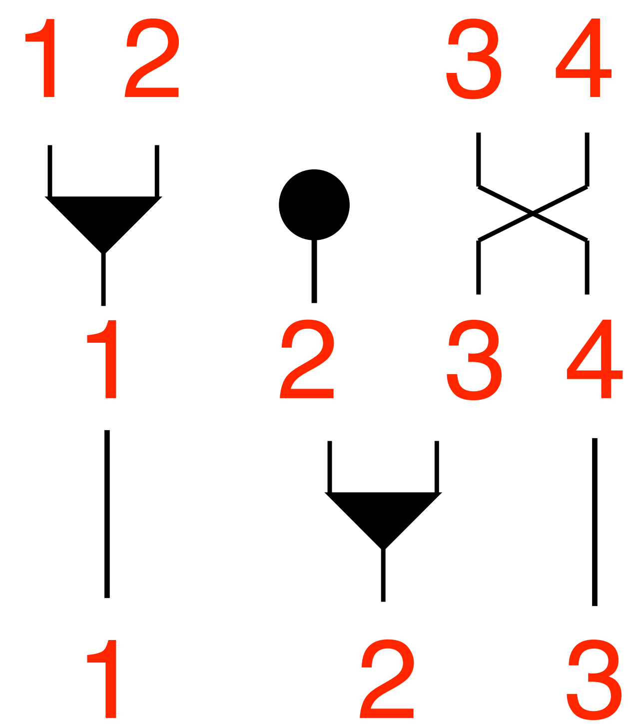


Albert Burroni
(1991)

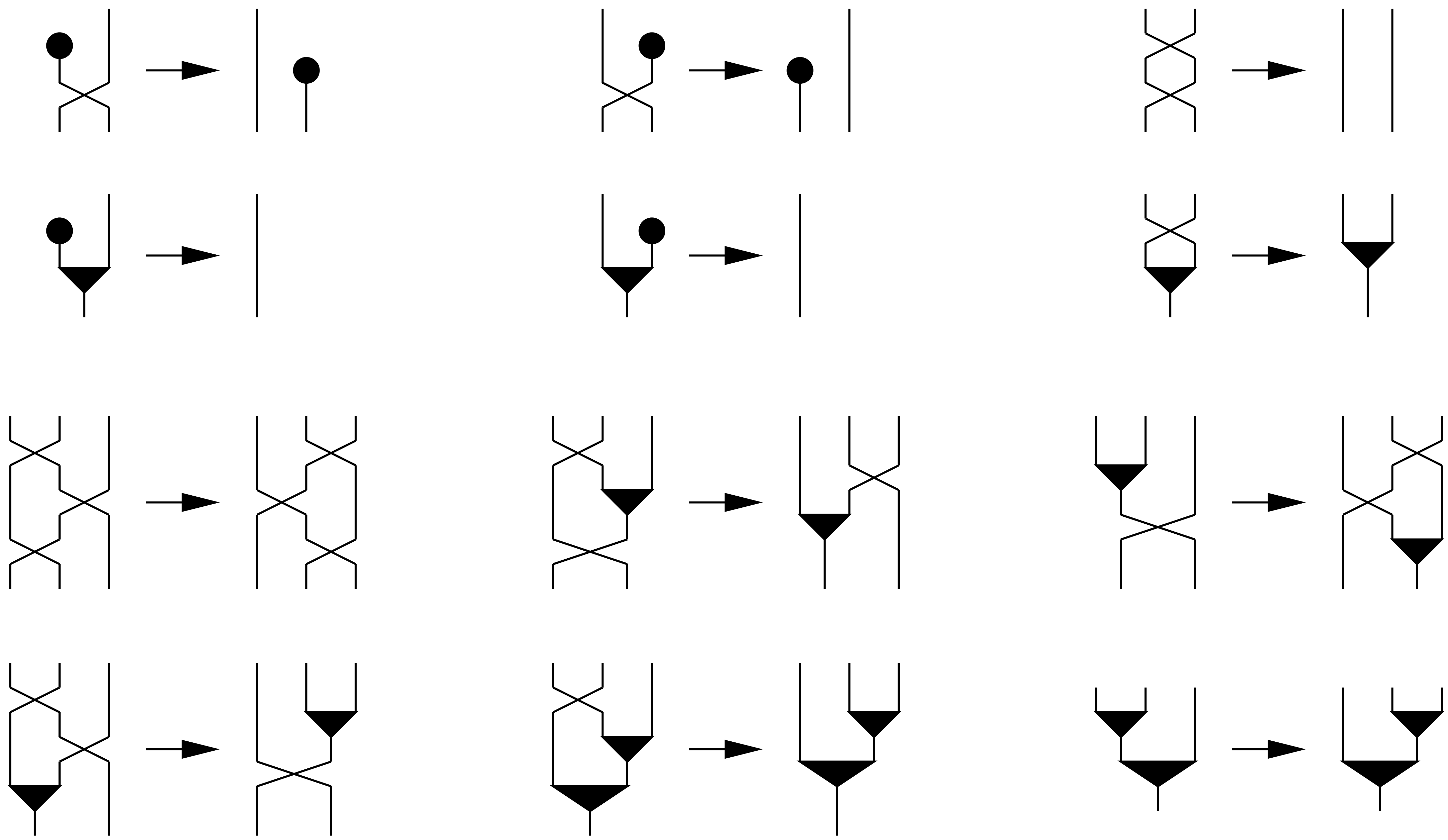
Diagramme sagittal (revu)

$$f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$$





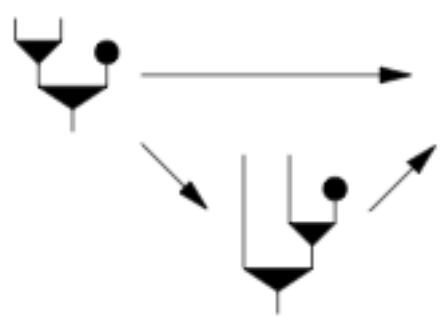
Règles de réécriture

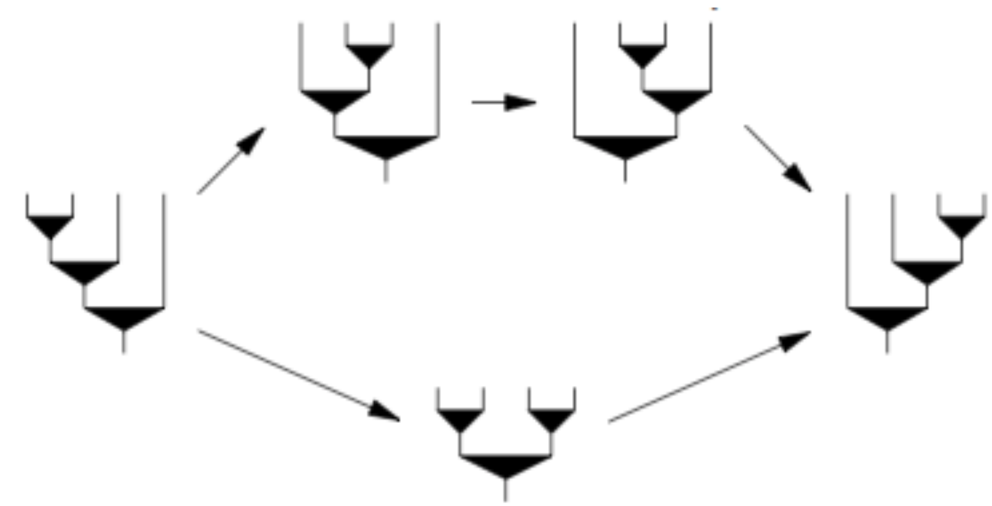


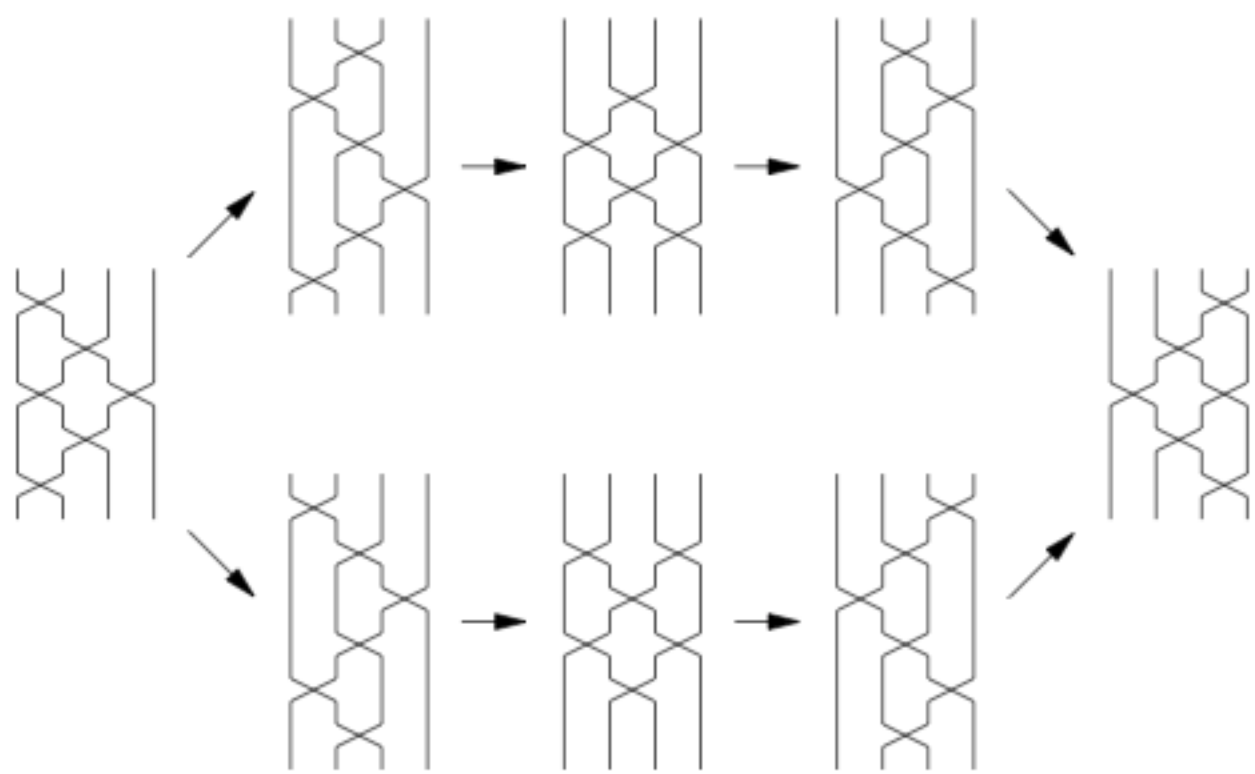
Ce système est *convergent* (Yves Lafont, 1995)

→ existence et unicité de la *forme réduite*

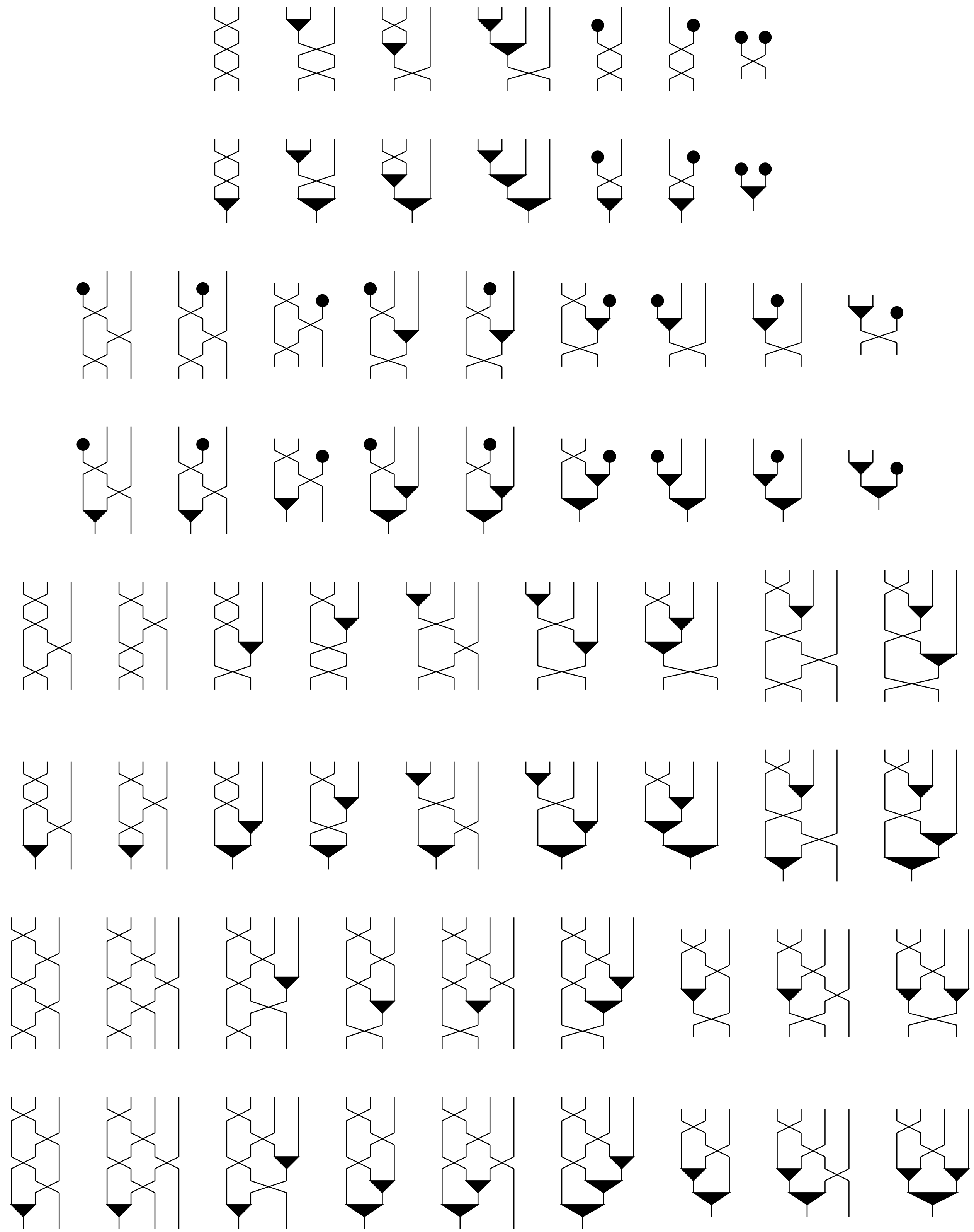
Confluence des pics critiques





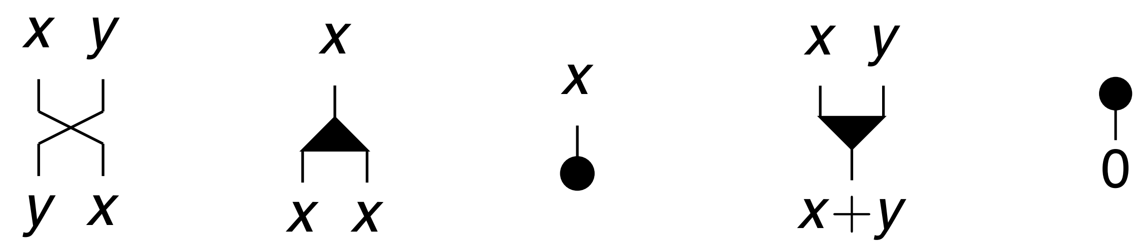


Les 68 pics critiques

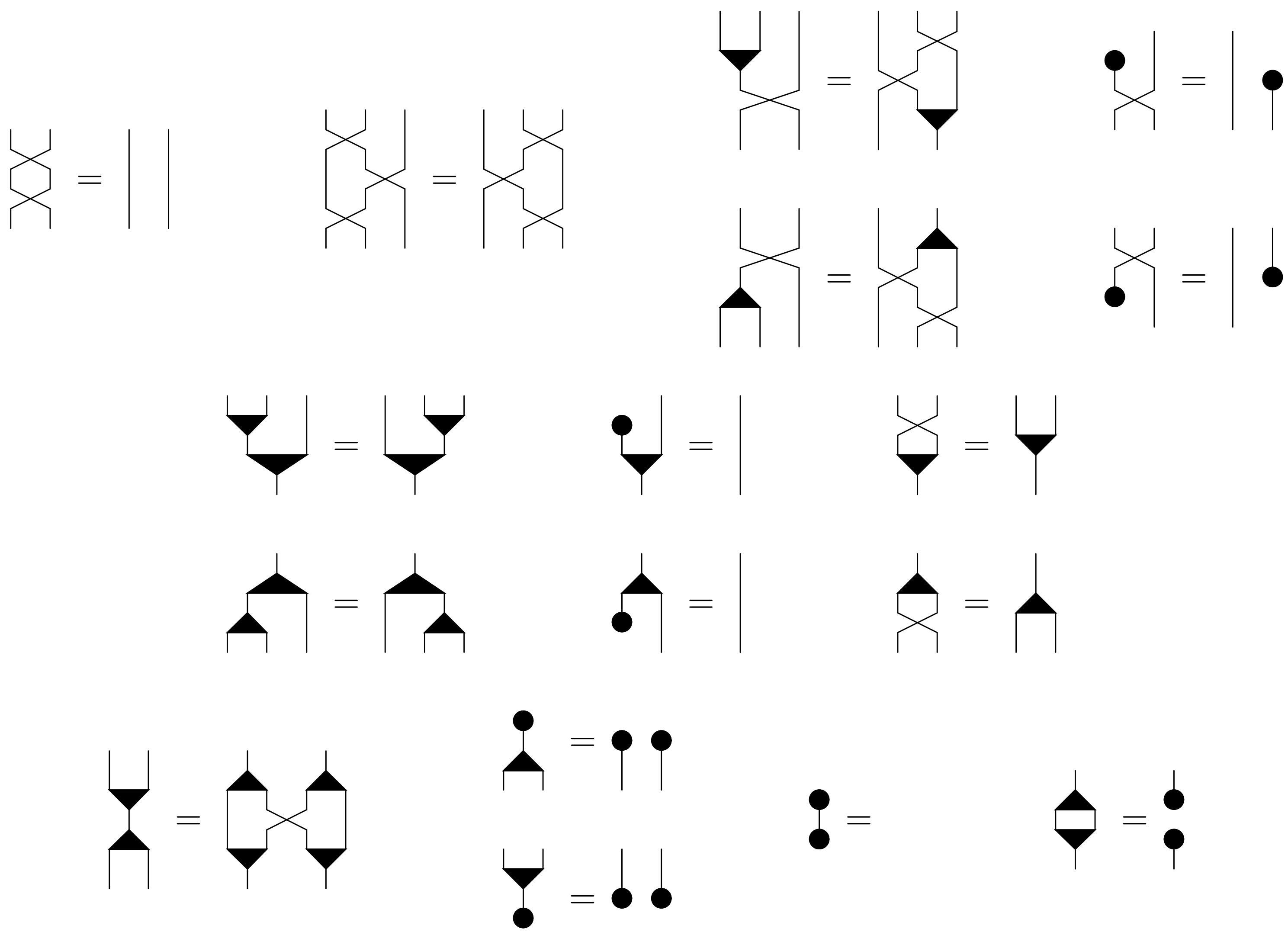


Algèbre linéaire sur $\mathbf{Z}_2$

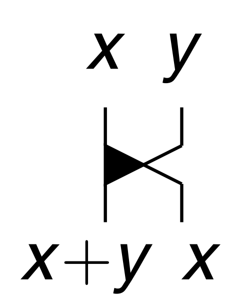
générateurs :



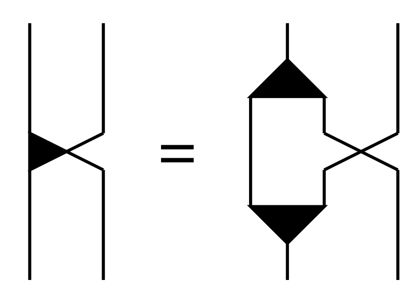
relations :



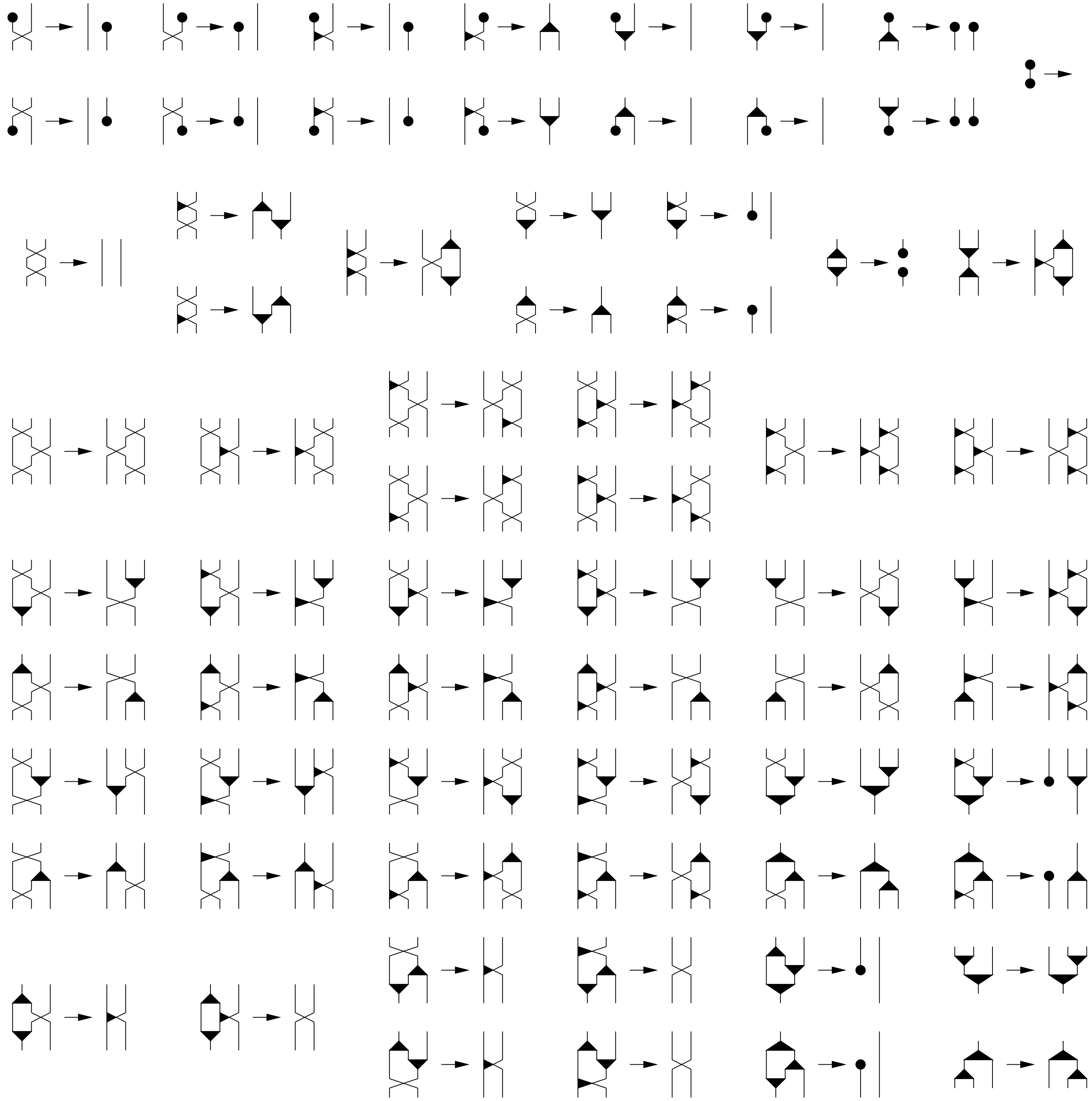
générateur supplémentaire :



$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$



Algèbre linéaire sur $\mathbf{Z}_2$



Ce système est convergent (Yves Guiraud, 2006)

→ généralisation pour un corps **K** quelconque

Références

- Y. Lafont, *Towards an algebraic theory of Boolean Circuits*, JPAA 184 (2-3) 2003
- Y. Guiraud, *Termination orders for 3-dimensional rewriting*, JPAA 207 (2) 2006
- Y. Lafont, *Diagram rewriting and operads*, Operads 2009, SMF