Formalizing Linear Logic

Linearity & TLLA 2020

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The Structure of Sequents

The Curry-Howard Correspondence

Intuitionistic Case

λ-Calculus

$\texttt{Bool} = X \to X \to X \quad \texttt{true} = \lambda x.\lambda y.x \quad \texttt{false} = \lambda x.\lambda y.y$

Sequent Calculus

set based $\{A_1, \ldots, A_n\} \vdash A$

$$\dfrac{\dfrac{\dfrac{}{\{X\} \vdash X}\,ax}{\{X\} \vdash X \to X}\to R}{\{\} \vdash X \to X \to X}\to R$$

multiset based $[A_1, \ldots, A_n] \vdash A$

$$\dfrac{\dfrac{\dfrac{}{[X, X] \vdash X}\,ax}{[X] \vdash X \to X}\to R}{[\,] \vdash X \to X \to X}\to R$$

sequence (or list) based $A_1, \ldots, A_n \vdash A$

$$\dfrac{\dfrac{\dfrac{}{X, X \vdash X}\,ax}{X \vdash X \to X}\to R}{\vdash X \to X \to X}\to R \qquad \dfrac{\dfrac{\dfrac{}{X, X \vdash X}\,ax}{X \vdash X \to X}\to R}{\vdash X \to X \to X}\to R$$

The Curry-Howard Correspondence

Linear Case

Classical Multiplicative Booleans

$$\vdash X^\perp \parr X^\perp, X \otimes X$$

Sequent Calculus

multiset based $\vdash [A_1, \ldots, A_n]$

$$\dfrac{\dfrac{}{\vdash [X^\perp, X]}\,ax \qquad \dfrac{}{\vdash [X^\perp, X]}\,ax}{\dfrac{\vdash [X^\perp, X^\perp, X \otimes X]}{\vdash [X^\perp \parr X^\perp, X \otimes X]}\,\parr}\,\otimes$$

sequence (or list) based $\vdash A_1, \ldots, A_n$

$$\dfrac{\dfrac{}{\vdash X^\perp, X}\,ax \qquad \dfrac{}{\vdash X^\perp, X}\,ax}{\dfrac{\vdash X^\perp, X^\perp, X \otimes X}{\vdash X^\perp \parr X^\perp, X \otimes X}\,\parr}\,\otimes \qquad \dfrac{\dfrac{}{\vdash X^\perp, X}\,ax \qquad \dfrac{}{\vdash X^\perp, X}\,ax}{\dfrac{\vdash X^\perp, X^\perp, X \otimes X}{\vdash X^\perp \parr X^\perp, X \otimes X}\,\parr}\,\otimes$$

The Exchange Rule

Exchange

$$\frac{\vdash A_1, \ldots, A_n \qquad \sigma \in \mathfrak{S}_n}{\vdash A_{\sigma(1)}, \ldots, A_{\sigma(n)}} \; ex$$

- a required rule (or integrated in all the other ones)
- often made implicit (thus the confusion with multisets)
- unavoidable in semantics or (computation-aware) formalizations

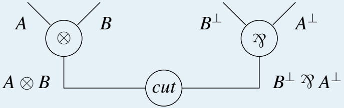
Exchange Control

- linear logic: take back control on structural rules
- control exchange (the last structural rule): restrict $\mathfrak{S}_n$

Non Commutative Logics

The Geometric Intuition

no exchange $\Longrightarrow$ no crossing $\Longrightarrow$ planarity



in particular: $(A \otimes B)^\perp = B^\perp ⅋ A^\perp \qquad (A ⅋ B)^\perp = B^\perp \otimes A^\perp$

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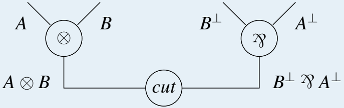

in particular: $(A \otimes B)^{\perp} = B^{\perp} \parr A^{\perp}$ $\quad (A \parr B)^{\perp} = B^{\perp} \otimes A^{\perp}$

A Cut Rule for Cut Elimination

$$\dfrac{\dfrac{\vdash \Gamma_1, A \qquad \vdash B, \Gamma_2}{\vdash \Gamma_1, A \otimes B, \Gamma_2}\otimes \qquad \dfrac{\vdash \Delta_1, B^{\perp}, A^{\perp}, \Delta_2}{\vdash \Delta_1, B^{\perp} \parr A^{\perp}, \Delta_2}\parr}{\vdash ???}\,cut$$

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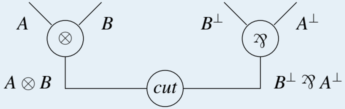

in particular: $(A \otimes B)^\perp = B^\perp \parr A^\perp \qquad (A \parr B)^\perp = B^\perp \otimes A^\perp$

A Cut Rule for Cut Elimination

$$\dfrac{\dfrac{\vdash \Gamma_1, A \qquad \vdash B, \Gamma_2}{\vdash \Gamma_1, A \otimes B, \Gamma_2}\otimes \qquad \dfrac{\vdash \Delta_1, B^\perp, A^\perp, \Delta_2}{\vdash \Delta_1, B^\perp \parr A^\perp, \Delta_2}\parr}{\vdash \color{red}{???}}\mathit{cut}$$

$$\dfrac{\vdash \Gamma_1, A \qquad \dfrac{\vdash B, \Gamma_2 \qquad \vdash \Delta_1, B^\perp, A^\perp, \Delta_2}{\vdash \Delta_1, \Gamma_2, A^\perp, \Delta_2}\mathit{cut}}{\vdash \Delta_1, \Gamma_2, \Gamma_1, \Delta_2}\mathit{cut}$$

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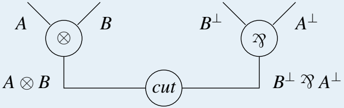


in particular: $(A \otimes B)^{\perp} = B^{\perp} \parr A^{\perp}$ $\quad$ $(A \parr B)^{\perp} = B^{\perp} \otimes A^{\perp}$

A Cut Rule for Cut Elimination

$$\dfrac{\dfrac{\vdash \Gamma_1, A \qquad \vdash B, \Gamma_2}{\vdash \Gamma_1, A \otimes B, \Gamma_2}\,\otimes \qquad \dfrac{\vdash \Delta_1, B^{\perp}, A^{\perp}, \Delta_2}{\vdash \Delta_1, B^{\perp} \parr A^{\perp}, \Delta_2}\,\parr}{\vdash \Delta_1, \Gamma_2, \Gamma_1, \Delta_2}\,cut$$

$$\dfrac{\vdash \Gamma_1, A \qquad \dfrac{\vdash B, \Gamma_2 \qquad \vdash \Delta_1, B^{\perp}, A^{\perp}, \Delta_2}{\vdash \Delta_1, \Gamma_2, A^{\perp}, \Delta_2}\,cut}{\vdash \Delta_1, \Gamma_2, \Gamma_1, \Delta_2}\,cut$$

Cyclic Linear Logic

Cut Elimination Cycles

$$\frac{\vdash \Gamma_1, C, \Gamma_2 \qquad \vdash \Delta_1, C^\perp, \Delta_2}{\vdash \Delta_1, \Gamma_2, \Gamma_1, \Delta_2}\ cut$$

$$\frac{\dfrac{\vdash \Gamma, \Delta}{\vdash \Gamma, \perp, \Delta}\ \perp \qquad \dfrac{}{\vdash 1}\ 1}{\vdash \Delta, \Gamma}\ cut$$

Cyclic Exchange

$$\frac{\vdash \Gamma, \Delta}{\vdash \Delta, \Gamma} \qquad \leftrightsquigarrow \qquad \frac{\vdash A_1, \ldots, A_n \qquad \sigma \in \mathfrak{C}_n}{\vdash A_{\sigma(1)}, \ldots, A_{\sigma(n)}}\ ex$$

Parameterized Exchange

One Rule

$$\frac{\vdash A_1, \ldots, A_n \qquad \sigma \in ???}{\vdash A_{\sigma(1)}, \ldots, A_{\sigma(n)}}\ ex$$

Various Instances

- $??? = \mathfrak{S}_n$ ✓ Linear Logic
- $??? = \emptyset$ ✗
- $??? = \mathfrak{C}_n$ ✓ Cyclic Linear Logic

A Common Feature

Control of principal formulas:

$$\frac{\vdash A, \Gamma \qquad \vdash B, \Delta}{\vdash A \otimes B, \Delta, \Gamma}\ \otimes$$

The Exponentials Strike Back

Contraction

$$\dfrac{\dfrac{\vdash ?A, ?A}{\vdash ?A}\,?c \qquad \vdash !A^{\perp}, ?A_1, \ldots, ?A_n}{\vdash ?A_1, \ldots, ?A_n}\,\mathit{cut} \qquad \rightsquigarrow$$

$$\dfrac{\dfrac{\dfrac{\vdash ?A, ?A \qquad \vdash !A^{\perp}, ?A_1, \ldots, ?A_n}{\vdash ?A, ?A_1, \ldots, ?A_n}\,\mathit{cut} \qquad \vdash !A^{\perp}, ?A_1, \ldots, ?A_n}{\vdash ?A_1, \ldots, ?A_n, ?A_1, \ldots, ?A_n}\,\mathit{cut}}{\vdash ?A_1, \ldots, ?A_n}\,???$$

Exponential Exchange

$$\dfrac{\vdash ?\Gamma, ?\Gamma, \Delta}{\vdash ?\Gamma, \Delta} \qquad \leftrightsquigarrow \qquad \dfrac{\vdash ?A, ?\Sigma, ?A, \Delta}{\vdash ?A, ?\Sigma, \Delta} \qquad \leftrightsquigarrow \qquad \dfrac{\vdash ?A, ?B, \Delta}{\vdash ?B, ?A, \Delta}$$

intermezzo

Axioms

Axiom Rules

Axiom Expansion

$$\frac{}{\vdash X^\perp, X}\ ax \qquad \Longrightarrow \qquad \frac{}{\vdash A^\perp, A}$$

definition with restricted rules — use with admissible rules

Generalized Axioms

$$\frac{}{\vdash A_1, \ldots, A_n}\ ax$$

- open proofs

$$\begin{array}{c} \vdash \Gamma \\ \vdots \\ \vdash \Delta \end{array} \quad \mapsto \quad \begin{array}{c} \frac{}{\vdash \Gamma}\ ax \\ \vdots \\ \vdash \Delta \end{array}$$

  warning: cut is not admissible

- linear atomic theories

$$\vdash X, X_1, \ldots, X_n \quad \wedge \quad \vdash X^\perp, Y_1, \ldots, Y_m$$

$$\Rightarrow \quad \vdash X_1, \ldots, X_n, Y_1, \ldots, Y_m$$

Intuitionistic Linear Logic

Intuitionistic Exchange Rule

$$\frac{A_1, \ldots, A_n \vdash A \qquad \sigma \in \mathfrak{S}_n}{A_{\sigma(1)}, \ldots, A_{\sigma(n)} \vdash A}\, ex$$

Non Commutative Variants

- $\sigma \in \emptyset$ ✓ Lambek Calculus
- $\sigma \in \mathfrak{C}_n$ ✗

$$\dfrac{\dfrac{\dfrac{\vdots}{C, B \vdash C \otimes B} \qquad \dfrac{}{(0\,1) \in \mathfrak{C}_2}}{B, C \vdash C \otimes B}\, ex \qquad \dfrac{\dfrac{\dfrac{\vdots}{A, C \vdash A \otimes C} \qquad \dfrac{\vdots}{B, D \vdash B \otimes D}}{A, C, B, D \vdash (A \otimes C) \otimes (B \otimes D)}\, \otimes R}{A, C \otimes B, D \vdash (A \otimes C) \otimes (B \otimes D)}\, \otimes L}{A, B, C, D \vdash (A \otimes C) \otimes (B \otimes D)}\, cut$$

Strategies for Cut Elimination

1. 1 proof for LL + 1 proof for ILL
   but would like to avoid redundancy...

Strategies for Cut Elimination

1. 1 proof for LL + 1 proof for ILL
   but would like to avoid redundancy...

2. embedding ILL into LL

Conservativity of LL over ILL

Cut Elimination through Conservativity

$$\Gamma \vdash_{\mathsf{ILL}} A \implies \vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp} \implies \underset{\text{cut free}}{\vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp}} \implies \underset{\text{cut free}}{\Gamma \vdash_{\mathsf{ILL}} A}$$

Embedding ILL into LL (twice)

$$\Gamma \vdash_{\mathsf{ILL}} A = \Gamma \vdash^{\mathfrak{S}}_{\mathsf{ILL}} A \mapsto \vdash^{\mathfrak{S}}_{\mathsf{LL}} A^\star, \Gamma^{\star\perp} = \vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp}$$

$$\Gamma \vdash_{\mathcal{L}} A = \Gamma \vdash^{\emptyset}_{\mathsf{ILL}} A \mapsto \vdash^{\mathfrak{C}}_{\mathsf{LL}} A^\star, \overline{\Gamma}^{\star\perp} = \vdash_{\mathsf{cyLL}} A^\star, \overline{\Gamma}^{\star\perp}$$

Conservativity of LL over ILL

Cut Elimination through Conservativity

$$\Gamma \vdash_{\mathsf{ILL}} A \quad \Longrightarrow \quad \vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp} \quad \Longrightarrow \quad \underset{\text{cut free}}{\vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp}} \quad \Longrightarrow \quad \underset{\text{cut free}}{\Gamma \vdash_{\mathsf{ILL}} A}$$

Embedding ILL into LL (twice)

$$\begin{array}{ccccccc}
\Gamma \vdash_{\mathsf{ILL}} A & & & & & & \vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp} \\
& \searrow & & & & \nearrow & \\
& & \Gamma \vdash^{P}_{\mathsf{ILL}} A & \mapsto & \vdash^{P^\star}_{\mathsf{LL}} A^\star, \overline{\Gamma}^{\star\perp} & & \\
& \nearrow & & & & \searrow & \\
\Gamma \vdash_{\mathcal{L}} A & & & & & & \vdash_{\mathsf{cyLL}} A^\star, \overline{\Gamma}^{\star\perp}
\end{array}$$

Conservativity of LL over ILL

Cut Elimination through Conservativity

$$\Gamma \vdash_{\mathsf{ILL}} A \quad\Longrightarrow\quad \vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp} \quad\Longrightarrow\quad \underset{\text{cut free}}{\vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp}} \quad\Longrightarrow\quad \underset{\text{cut free}}{\Gamma \vdash_{\mathsf{ILL}} A}$$

Embedding ILL into LL (twice)

$$\Gamma \vdash_{\mathsf{ILL}} A \quad\searrow \qquad\qquad\qquad\qquad \nearrow \quad \vdash_{\mathsf{LL}} A^\star, \Gamma^{\star\perp}$$

$$\Gamma \vdash^{P}_{\mathsf{ILL}} A \quad\mapsto\quad \vdash^{P^\star}_{\mathsf{LL}} A^\star, \overline{\Gamma}^{\star\perp}$$

$$\Gamma \vdash_{\mathcal{L}} A \quad\nearrow \qquad\qquad\qquad\qquad \searrow \quad \vdash_{\mathsf{cyLL}} A^\star, \overline{\Gamma}^{\star\perp}$$

Conservativity Failures

$((X \multimap Y) \multimap 0) \multimap (X \otimes \top)$

$((((0 \multimap 1) \multimap 1) \multimap 0) \multimap 0) \multimap 1$

$(((X \otimes \top) \,\&\, (Y \otimes \top)) \multimap 0) \multimap ((X \multimap X') \oplus (Y \multimap Y'))$

Sufficient Conditions for Conservativity

Control Additive Units

no $\_ \multimap \mathbb{Z}$ in negative position

$$\mathbb{Z} ::= 0 \mid \mathbb{Z} \otimes A \mid A \otimes \mathbb{Z} \mid \mathbb{Z} \,\&\, A \mid A \,\&\, \mathbb{Z} \mid \mathbb{Z} \oplus \mathbb{Z} \mid {!\mathbb{Z}}$$

Use Exponentials

only $\mathbb{P} \multimap \_$ in negative position

$$\mathbb{P} ::= X \mid 1 \mid 0 \mid \mathbb{P} \otimes \mathbb{P} \mid \mathbb{P} \oplus \mathbb{P} \mid \mathbb{P} \,\&\, A \mid A \,\&\, \mathbb{P} \mid {!A}$$

Corollaries

- LL is conservative over TL
- LL is conservative over targets of (all?) translations of LK and LJ

Strategies for Cut Elimination

1. 1 proof for LL + 1 proof for ILL

   but would like to avoid redundancy...

2. embedding ILL into LL

   but not always conservative...

3. double-negation translations of LL into ILL

(Double) Negation Translation

From LL to ILL

$$
\begin{array}{rclcrcl}
(X^{\perp})^{\bullet} & = & X & & & & \\
(A \parr B)^{\bullet} & = & A^{\bullet} \otimes B^{\bullet} & & \perp^{\bullet} & = & 1 \\
(A \& B)^{\bullet} & = & A^{\bullet} \oplus B^{\bullet} & & \top^{\bullet} & = & 0 \\
(?A)^{\bullet} & = & !\neg(A^{\perp})^{\bullet} & & & & \\
P^{\bullet} & = & \neg(P^{\perp})^{\bullet} & & \neg A & := & A \multimap \mathrm{R}
\end{array}
$$

$$\vdash_{\mathsf{LL}} \Gamma \quad \mapsto \quad \Gamma^{\bullet} \vdash_{\mathsf{ILL}} \mathrm{R}$$

- sober on formulas
- adds a lot of cuts

Analysis of the Target: $\multimap$-free subsystems of ILL

- TL: $A ::= X \mid 1 \mid 0 \mid A \otimes A \mid A \oplus A \mid !A \mid \neg A$
- $\mathsf{TL}^{\neg}$: $A ::= X \mid 1 \mid 0 \mid A \otimes A \mid A \oplus A \mid !\neg A \mid \neg A$

Applications

Cut Elimination

$$\vdash_{\mathsf{LL}} \Gamma \;\Rightarrow\; \Gamma^{\bullet} \vdash_{\mathsf{TL}^{\neg}} \mathtt{R} \;\overset{\mathsf{ILL}}{\Rightarrow}\; \underset{\text{cut free}}{\Gamma^{\bullet} \vdash_{\mathsf{TL}^{\neg}} \mathtt{R}} \;\Rightarrow\; \underset{\text{cut free}}{\vdash_{\mathsf{LL}} \Gamma^{\bullet\perp}, \mathtt{R}} \;\overset{[\perp/\mathtt{R}]}{\Rightarrow}\; \underset{\text{cut free}}{\vdash_{\mathsf{LL}} \Gamma}$$

Focusing

$$\vdash_{\mathsf{LL}} \Gamma \;\Rightarrow\; \Gamma^{\bullet} \vdash_{\mathsf{TL}^{\neg}} \mathtt{R} \;\overset{\mathsf{ILL}}{\Rightarrow}\; \underset{\text{cut free}}{\Gamma^{\bullet} \vdash_{\mathsf{TL}^{\neg}} \mathtt{R}} \;\Rightarrow\; \underset{\text{cut free}}{\Gamma^{\bullet} \vdash^{!\neg}_{\mathsf{TL}^{\neg}} \mathtt{R}} \;\Rightarrow\; \vdash_{\mathtt{foc}} \Gamma$$

Weakness

- not compatible with atomic generalized axioms
- non-commutative cases still to be studied

Strategies for Cut Elimination

1. 1 proof for LL + 1 proof for ILL

   but would like to avoid redundancy...

2. embedding ILL into LL

   but not always conservative...

3. double-negation translations of LL into ILL

   but not compatible with atomic generalized axioms...

4. back to 2 proofs...

Yalla

a Coq library

The Inductive Type

```
Inductive ll P : list formula → Type :=
| ax_r : ∀ X, ll P (covar X :: var X :: nil)
| ex_r : ∀ l1 l2, ll P l1 → PCperm_Type (pperm P) l1 l2 → ll P l2
| ex_wn_r : ∀ l1 lw lw' l2, ll P (l1 ++ map wn lw ++ l2) →
                         Permutation_Type lw lw' → ll P (l1 ++ map wn lw' ++ l2)
| one_r : ll P (one :: nil)
| bot_r : ∀ l, ll P l → ll P (bot :: l)
| tens_r : ∀ A B l1 l2, ll P (A :: l1) → ll P (B :: l2) → ll P (tens A B :: l2 ++ l1)
| parr_r : ∀ A B l, ll P (A :: B :: l) → ll P (parr A B :: l)
| top_r : ∀ l, ll P (top :: l)
| plus_r1 : ∀ A B l, ll P (A :: l) → ll P (aplus A B :: l)
| plus_r2 : ∀ A B l, ll P (A :: l) → ll P (aplus B A :: l)
| with_r : ∀ A B l, ll P (A :: l) → ll P (B :: l) → ll P (awith A B :: l)
| oc_r : ∀ A l, ll P (A :: map wn l) → ll P (oc A :: map wn l)
| de_r : ∀ A l, ll P (A :: l) → ll P (wn A :: l)
| wk_r : ∀ A l, ll P l → ll P (wn A :: l)
| co_r : ∀ A l, ll P (wn A :: wn A :: l) → ll P (wn A :: l)
| cut_r {f : pcut P = true} : ∀ A l1 l2, ll P (dual A :: l1) → ll P (A :: l2) → ll P (l2 ++ l1)
| gax_r : ∀ a, ll P (projT2 (pgax P) a).
```

Hiding Parameters

Recommendations

- define your own inductive type
- inject it in an instance of **ll**
- import / use results from the library

A More Natural Definition of MELL

```
Inductive mell : list formula → Type :=
| ax_r : ∀ X, mell (covar X :: var X :: nil)
| ex_r : ∀ l1 l2, mell l1 → Permutation_Type l1 l2 → mell l2
| tens_r : ∀ A B l1 l2, mell (A :: l1) → mell (B :: l2) → mell (tens A B :: l1 ++ l2)
| parr_r : ∀ A B l, mell (A :: B :: l) → mell (parr A B :: l)
| oc_r : ∀ A l, mell (A :: map wn l) → mell (oc A :: map wn l)
| de_r : ∀ A l, mell (A :: l) → mell (wn A :: l)
| wk_r : ∀ A l, mell l → mell (wn A :: l)
| co_r : ∀ A l, mell (wn A :: wn A :: l) → mell (wn A :: l).

Lemma mell2mellfrag : ∀ l, mell l ↔ ll pfrag_mell (map mell2ll l).
```

Extensions

Mix Rules

(already available)

Girard's Mix Rules

$$\frac{}{\vdash}\ mix_0 \qquad\qquad \frac{\vdash \Gamma \qquad \vdash \Delta}{\vdash \Gamma, \Delta}\ mix_2$$

$$\searrow \qquad\qquad \swarrow$$

$$\frac{\vdash \Gamma_1 \qquad \cdots \qquad \vdash \Gamma_i}{\vdash \Gamma_1, \ldots, \Gamma_i}\ mix_i$$

Some Associated Properties

- $\mathsf{LL} + \bot \multimap 1 \quad \Longleftrightarrow \quad \mathsf{LL} + mix_0 + mix_2$
- $mix_i + mix_j \quad \Longrightarrow \quad mix_{i+j-1}$
- ...

Quantifiers

Some Possible Approaches

- De Bruijn indices
- (Parametric) Higher-Order Abstract Syntax
- ∇ quantifier
- Nominal logic
- First-order terms up to α-equivalence

The Current Plan

- Limit interferences with propositional logic
- Avoid renamings (and capture)

$$A ::= X\,\vec{t} \mid \cdots \mid \forall x.A \mid \exists x.A \qquad\qquad t ::= x \mid n \mid f\,\vec{t}$$

$$\forall y.A[{}^{y}/_{x}] \neq \forall x.A \text{ but } \forall x.A \dashv\vdash \forall y.A[{}^{y}/_{x}]$$

- De Bruijn constructs at proof level

$$\frac{\vdash A[{}^{t}/_{x}], \Gamma \qquad t \text{ closed}}{\vdash \exists x.A, \Gamma}\ \exists \qquad\qquad \frac{\vdash (\uparrow A)[{}^{0}/_{x}], \uparrow\Gamma}{\vdash \forall x.A, \Gamma}\ \forall$$

Alternative Exponential Rules

see next talk!

Semantics

Phase Semantics

- refactored notion of interpretation
- quotient-free list-based syntactic model
- Okada's lemma

  true in all phase models $\Longrightarrow$ cut-free provable

- yet another cut-elimination proof

Denotational Semantics

- cut-elimination steps as rewrite rules
- weak normalization proof rather than admissibility
- invariance on rewrite rules

Conclusion

Formalizing Linear Logic

- that's fun!
- opportunity to dig into subtleties of LL

Yalla

- try to be as generic as possible for multiple uses
- try to be easy to grasp for users through templates
- compatible with computational content of proofs
- formalized meta-theory in a constructive system:
  possible to extract certified transformation algorithms

Future work

- proof-nets
- circular proofs
- LL handbook

Try Yalla!!!

`https://perso.ens-lyon.fr/olivier.laurent/yalla/`

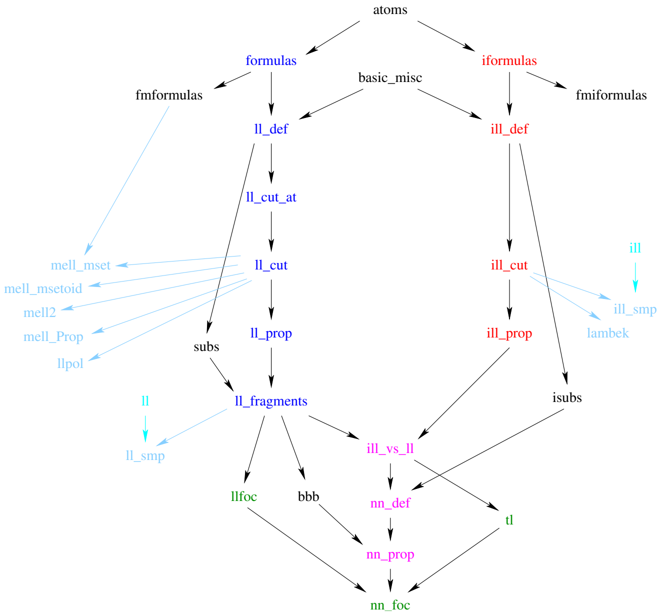
`https://github.com/olaure01/yalla/tree/working`

Users, comments and manpower are welcome!

Support guaranteed:

- `olivier.laurent@ens-lyon.fr`
- `https://github.com/olaure01/yalla/issues`

Files Dependencies



CALL: Computer-Assisted Linear Logic