Homework I (due March 7st, before tutorial)

Problem 1 (NP-hardness)**.** Prove that the following problems are $\mathbf{NP}$-hard

- The halting problem HALT $= \{(\alpha, x) : M_\alpha$ halts on input $x\}$.
- The problem Integer Linear Programming

$$\text{ILP} = \{(A, b) : A \text{ integer } n \times m \text{ matrix and } b \text{ column vector of } m \text{ integers s.t. there exists a vector } x \text{ of } n \text{ integers such that } Ax \geq b\}$$

We use the notation $v \geq w$ for vectors v and w when $v_i \geq w_i$ for all coordinates i.

Are they $\mathbf{NP}$-complete? For this, you may choose one of three options with justification: not in $\mathbf{NP}$, in $\mathbf{NP}$, seems to be in $\mathbf{NP}$ but there is a difficulty and say what the difficulty is.

Problem 2 (Reductions)**.** We saw that a language L is polynomial-time Karp reducible to L' if there is a polynomial-time computable functions f such that $x \in L$ iff $f(x) \in L'$. Give an example of a class that is closed under polynomial-time Karp reductions (i.e., if L is reducible to L' and $L' \in \mathcal{C}$, then $L \in \mathcal{C}$) and one that is not.

Problem 3 (Difference of $\mathbf{NP}$ problems)**.** Let $\mathbf{DP} = \{L_1 \setminus L_2 : L_1 \in \mathbf{NP}$ and $L_2 \in \mathbf{NP}\}$. Show that the problem EXACTINDSET $= \{(G, k) :$ the largest independent set of G has size exactly $k\}$ is $\mathbf{DP}$-complete (for the usual polynomial-time Karp reductions).