# HW 2: circuits, randomness.

(due Thursday, April 11th, before tutorial)

**Problem 1** (Circuits)**.** Show that any boolean function $f : \{0,1\}^n \to \{0,1\}$ can be computed by a circuit of size $O(2^n)$.

**Bonus:** Improve this bound to $O(\frac{2^n}{n})$.

**Problem 2.** The class $\mathbf{PP}$ is defined as the set of languages $L$ such that there exists a polynomial $q(n)$ and a polynomial time machine $M$ such that the following holds.

$$x \in L \implies \underset{r\in\{0,1\}^{q(n)}}{\mathbf{P}} \{M(x,r) = 1\} > \frac{1}{2}$$
$$x \notin L \implies \underset{r\in\{0,1\}^{q(n)}}{\mathbf{P}} \{M(x,r) = 1\} \leq \frac{1}{2} ,$$

where $n = |x|$.

1. Show that $\mathbf{NP} \subseteq \mathbf{PP}$.
2. Show that the problem $\#\text{SAT} = \{(\varphi, k) : \text{ the formula } \varphi \text{ has } > k \text{ satisfying assignments}\}$ is $\mathbf{PP}$-complete for Karp reductions.