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**HW VI: The probabilistic method** (due before March 21st at 8:00)

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1. Recall that the chromatic number  $\chi(G)$  is the smallest number of colors needed to color the vertices of  $G$  such that two adjacent vertices never share the same color. It might seem reasonable to believe that if the graph does not have short cycles, then  $\chi(G)$  should not be too large. This however turns out not to be true.

Prove that for any integer  $k \geq 2$ , there exists a graph  $G$  with no triangles and that has a chromatic number  $\chi(G) \geq k$ .