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Three ways to obtain flat rotation curves: A problem in undergraduate computational physics

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We have addressed the need to introduce undergraduates to computational methods in physics by making computation an integral part of our intermediate laboratory experience. We discuss an example of a senior project that follows an introduction to a suite of computational tools in our intermediate laboratory and focuses on a topic in astrophysics. We report on computations of flat rotation curves for spiral galaxies using three models and compare the results to the available data for the spiral galaxy NGC 3198. The models are the "standard" exponential disk+dark matter halo model, the modified Newtonian dynamic model, and a general computational procedure that allows for testing arbitrary nonstandard surface mass density distributions. The models can also be applied to other spiral galaxies for which data are readily available.

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I. INTRODUCTION

In the 1930s, Zwicky[1] argued that clusters of galaxies must hold enormous amounts of nonluminous matter to explain the dynamics of the member galaxies. Many astronomers disregarded his argument, believing that he must have made a mistake (see, for example, Refs. 2 and 3). In the 1970s, following the measurements of galactic rotation curves by Rubin *et al.*,[4] it became increasingly apparent that the visible matter in many galaxies was insufficient to explain their observed flat rotation curves.

It is now generally accepted that a mysterious form of nonbaryonic matter (dark matter) pervades galaxies (see, for example, Ref. 5) and that dark matter is necessary to explain the flat rotation curves of visible matter about the center of spiral galaxies. The best evidence for the existence of dark matter, other than the flat rotation curves of many spiral galaxies, comes from observations of gravitational lensing by the two colliding galaxy clusters that make up the Bullet cluster.[6] Measurements of gravitational lensing indicate that the center of mass of the Bullet cluster occurs at a position that does not coincide with the position of the center of mass of the baryonic matter as determined from the distribution of x-ray producing matter, which has arisen due to the violent collision of the two clusters, and is believed to form the major baryonic component of the two colliding galaxy clusters.

It is believed that dark matter constitutes roughly 23% of the matter-energy content of the universe, with 73% dark energy, and only 4% of baryonic matter. However, as suggested by several researchers,[7] it is instructive to consider alternative explanations for the observed flat rotation curves of spiral galaxies.

The alternatives, except for those that propose a modification of general relativity, provide an opportunity for the upper level undergraduates who have been exposed to some of the basic tools of computational physics to gain insight into one of the currently most active and exciting areas of physics and astrophysics.

We have approached the need for undergraduates to be introduced to the powerful computer software tools that are available for computational physics by incorporating a significant amount of computation into our intermediate experimental physics course. There are a number of software packages to choose from and approaches to computational physics vary from department to department.[8] We have utilized the scientific spreadsheet program PSIPLOT (Ref. 9) to plot and fit data, the mathematics software package MATHCAD,[10] the systems modeling software package STELLA,[11] and more recently, VENSIM (Ref. 12) (because it is free for faculty and student use), which we use to solve complex ordinary differential equations. We also use the partial differential equation solver FLEX,[13] the video capture and analysis package VIDEOPOINT,[14] and the data acquisition package LOGGERPRO.[15]

In the following, we outline a computational physics project that one of us (Sharrar) chose as a senior project. The project utilized MATHCAD and PSIPLOT. The project involved looking at three ways of obtaining flat rotation curves for spiral galaxies. In particular, the project involved investigating the standard "exponential disk+dark matter halo model," the modified Newtonian dynamics model of Millgrom,[16] and a disk with a nonstandard (that is nonexponential) surface mass density distribution. In each case the model was fitted to the data for the spiral galaxy NGC 3198.[17,18]

Section II gives a summary of the relevant details of the spiral galaxy NGC 3198. Section III covers the standard exponential disk+dark matter halo model, and Sec. IV applies the modified Newtonian dynamics model to NGC 3198. Section V develops a method for numerically treating a thin disk with an arbitrary surface mass density. A discussion is given in Sec. VI.

II. THE GALAXY NGC 3198

One of the best studied spiral galaxies is NGC 3198. A black and white image of a color-composite picture of this galaxy is shown in Fig. 1.[19] For our purposes, we use the readily available data of Begeman,[17,18] who studied this galaxy using the Westerbork synthesis radio telescope by measuring the Doppler shift of the neutral hydrogen (HI) 21 cm line. If the hydrogen gas moves in circular orbits around the center of the galaxy, the data yields an odd set of velocity contours.[17] The circular orbital velocity of the hydrogen gas can be determined by knowing the inclination of NGC 3198 relative to the observer's line of sight and using these veloc-

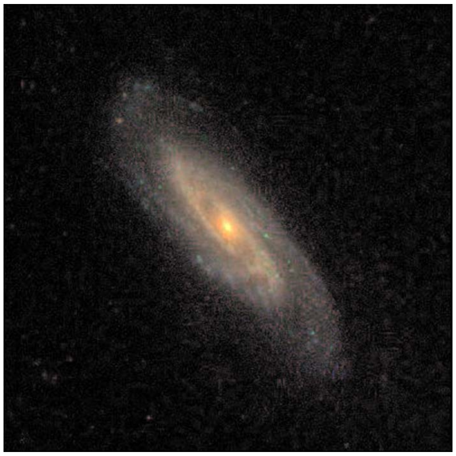

Fig. 1. A color-composite image of NGC 3198 taken at the Palomar Observatory (Ref. 19).

ity contours. A tabulation of the circular orbital velocities as a function of radius, as determined in Refs. 17 and 18, is given in Table I.

Table I shows the experimental points to which each of the models described in the following is fitted. R is in kpc (kiloparsecs) instead of arcminutes as in Refs. 17 and 18. The conversion assumes that NGC is a distance of 9.4 Mpc (megaparsecs) from Earth (1 arc min is equal to 2.73 kpc). References 17 and 18 assumed a Hubble constant of 75 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ to determine the NGC distance of 9.4 Mpc. This value is consistent with the latest value ($74.2 \pm 3.6\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$) for the Hubble constant.[20] Another useful conversion is 1 kpc$=3.26\times 10^3$ Ly (light-years) or 3.084×10^{16} km. The radius of the visible extent of NGC 3198 is ≈ 11.4 kpc or 3.72×10^4 Ly. A plot of Begeman's data is shown in Fig. 2.

Table I. Rotation curve data for NGC 3198 (Ref. 18).

R (kpc)	v_c (km/s)
0.68	55
1.36	92
2.04	110
2.72	123
3.4	134
4.08	142
4.76	145
5.44	147
6.12	148
6.8	152
7.48	155
8.16	156
9.52	157
10.88	153
12.24	153
13.6	154
14.96	153
16.32	150
17.68	149
19.04	148
20.4	146
21.76	147
23.12	148
24.48	148
25.84	149
27.2	150
28.56	150
29.92	149

Note that the data in Table I and Fig. 2 go out to almost 30 kpc. In contrast, the luminous matter of the galaxy extends to only 10–11 kpc, or to a little past the peak in the circular velocity curve. If all the matters were luminous, we would expect the rotation curve to start decreasing beyond the luminous extent of the galaxy (and by assumption, beyond the baryonic mass distribution also). This behavior is commonly referred to as a Keplerian decrease, and for NGC 3198, should start around 10–11 kpc. Such is not the case as can be seen in Fig. 2.

III. STANDARD EXPONENTIAL DISK+DARK MATTER HALO MODEL

Spiral galaxies consist of a central spheroid or bulge and a disk of baryonic matter in the form of stars that emit light in the visible, and baryonic neutral hydrogen emitting radiation with a wavelength of 21 cm. These are features that can be seen by the emission of various types of electromagnetic radiation. There likely is also a contribution to the baryonic mass from massive compact halo objects, which do not emit visible radiation. All of these components can be modeled and fitted to circular orbital velocity data where available. NGC 3198 was chosen because it does not have a well-defined bulge. The lack of a well-defined bulge is true for all of the galaxies studied by Begeman,[18] who chose these galaxies because of their apparent simplicity. We have not included the hydrogen gas in our analysis in the following because its mass is small,[17] and we have neglected massive compact halo objects because their contribution is unknown. We note that including the gravitational effects of the hydrogen gas would be a useful extension to the models discussed in this paper (see Figs. 13 and 15 in Ref. 17).

In its simplest form, it is customary to explain the flat rotation curve of spiral galaxies without spheroids or bulges by combining a thin disk of constant mass to light ratio and an isothermal spherically symmetric halo consisting of non-radiating dark matter. The disk is assumed to have a surface mass density of conventional baryonic matter, which is proportional to the surface brightness and given by

$$\Sigma(r) = \Sigma_0 e^{-r/R_D}, \tag{1}$$

where r is the distance from the galactic center to the point of interest and R_D is the radial scale length of the disk. The radial scale length is determined from the surface brightness. The assumption is that the mass to light ratio is constant for a particular spiral galaxy. This assumption is based on measurements made in the local neighborhood of our own galaxy, the Milky Way. The scale length is the distance from the galactic center to the point where the surface brightness drops to $1/e$ of its value at the galactic center. For NGC 3198, R_D is 2.63 kpc.[17,18] Σ_0 is the surface mass density at the center of the galaxy.

The dark matter halo (spherical in shape) is assumed to have a volume density,

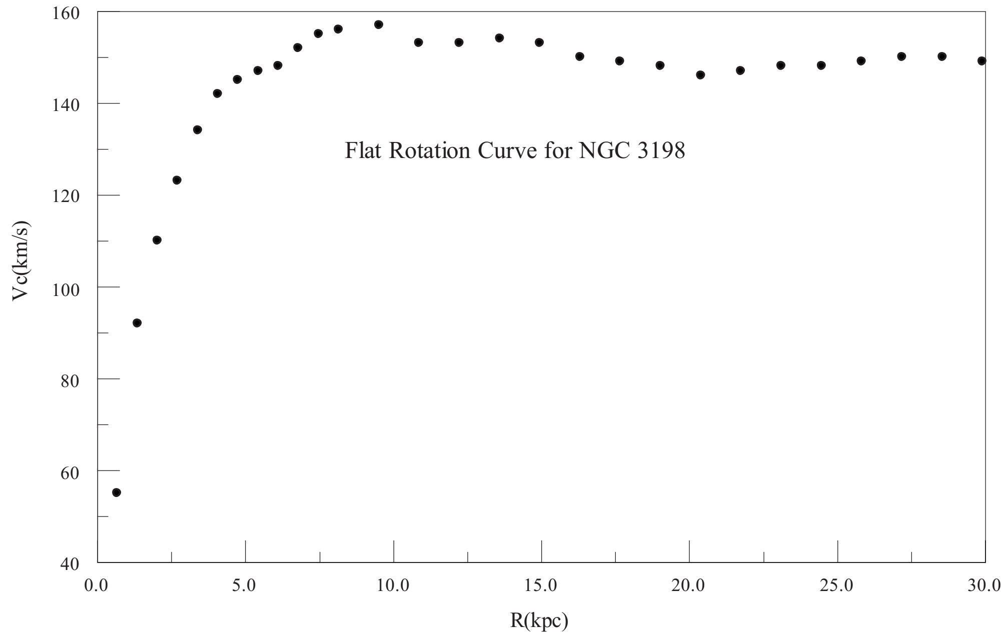

Fig. 2. A plot of rotational speed (km/s) versus radial distance (kpc) for NGC 3198 using the data in Refs. 17 and 18.

$$\rho(r) = \frac{\rho_0}{(1 + (r/r_c)^2)}, \tag{2}$$

where r_c is a halo scale length. The peak density occurs when r is equal to zero and the density at $r = r_c$ is $\rho_0/2$. A plot of $\rho(r)$ versus r for $r_c = 6.414$ kpc (see the fit of the model to "maximum disk" given in the following) is shown in Fig. 3. Note that the total halo mass obtained by integrating the density over a spherical volume from zero to infinity leads to an infinite mass of the dark matter. Hence, if Eq. (2) is correct, the halo must end at some radius to be physically acceptable. After that radius, there should be a Keplerian decrease. Such a decrease has not been observed. Students could try halos with different halo densities and, in particular, ones that don't lead to infinite masses as the radius tends to infinity. The method would be essentially the same as that outlined in the following.

The gravitational field due to the disk and halo is given by

$$g(r) = g_{\text{disk}}(r) + g_{\text{halo}}(r), \tag{3}$$

where $g_{\text{disk}}(r)$ is given by (see Ref. 3, p. 77)

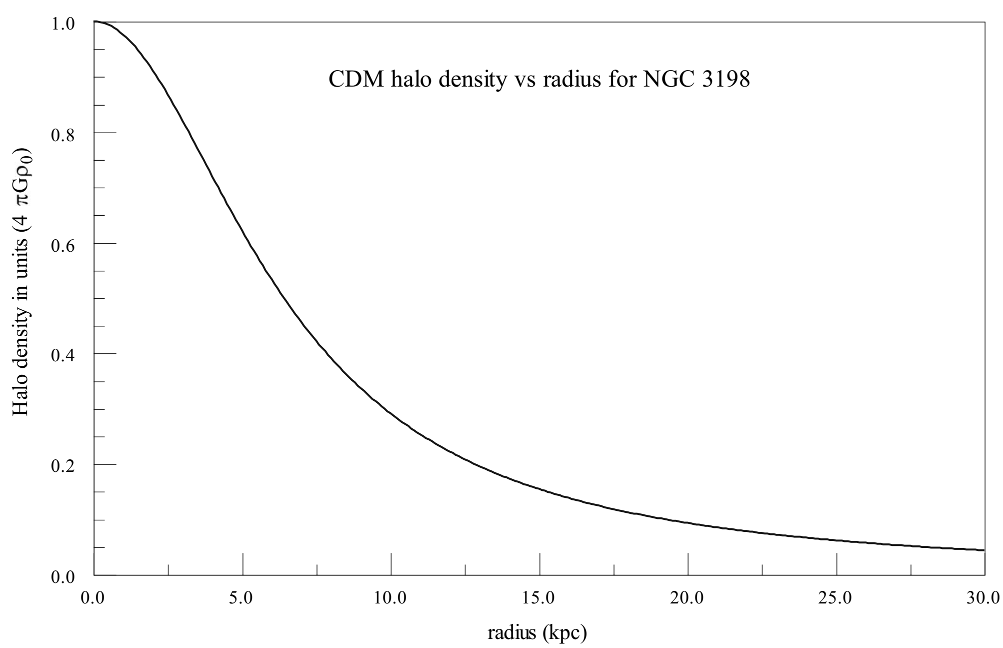

Fig. 3. Normalized dark matter halo density [Eq. (2)] with $r_c = 6.414$ kpc, corresponding to a "maximum disk" fitting to the circular orbital velocity for NGC 3198, plotted as a function of radius.

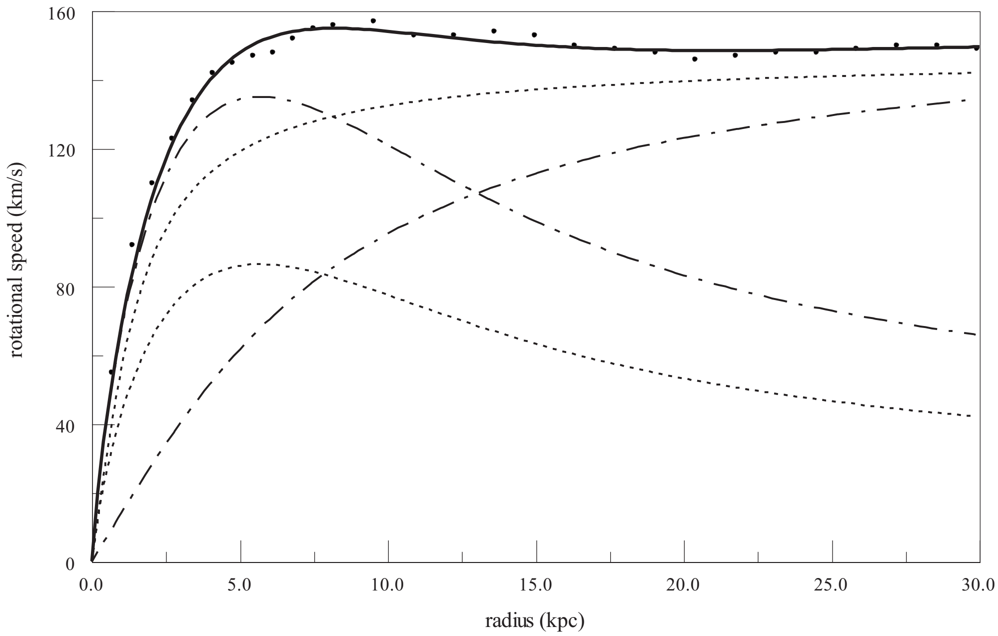

Fig. 4. Circular orbital velocities versus radius for NGC 3198. Solid dots are data points Refs. 17 and 18. Dot-dash lines are the best fit to the data with maximum disk surface mass density and minimum dark matter halo volume density. Dashed lines are for minimum disk surface mass density and maximum dark matter halo volume density. The solid curve is the sum (in quadrature) of disk+halo for both maximum and minimum disk situations.

$$g_{\rm disk}(r) = 4\pi G\Sigma_0 (R_D/r)(y)^2[I_0(y)K_0(y) - I_1(y)K_1(y)]. \tag{4}$$

In Eq. (4) G is the gravitational constant. $I_0(y)$ and $I_1(y)$ are the modified Bessel functions of the first kind, and $K_0(y)$ and $K_1(y)$ are the modified Bessel functions of the second kind with $y=r/(2R_D)$.

The gravitational field due to the halo can be obtained by recalling that for a spherically symmetric matter distribution, $g(r)=GM_{\rm inside}/r^2$. We evaluated the integral $(4\pi G\rho_0/r^2)\int_0^r [s^2/(1+(s/r_c)^2)]ds$. The well known result is

$$g_{\rm halo}(r) = (4\pi G\rho_0/r)(r_c^2 - (r_c^3/r)\arctan(r/r_c)). \tag{5}$$

For circular orbits with circular orbital velocity v_c, the acceleration of an object of mass m is v_c^2/r. Newton's second law then yields

$$g(r)m = mv_c^2/r. \tag{6}$$

Solving for v_c^2 and substituting Eqs. (3)–(5) into Eq. (6) yields

$$v_c^2 = u_0 R_D (y)^2[I_0(y)K_0(y) - I_1(y)K_1(y)] + u_1[u_2^2 - (u_2^3/r)\arctan(r/u_2)], \tag{7}$$

where $u_0=4\pi G\Sigma_0$ $[(\mathrm{km/s})^2(1/\mathrm{kpc})]$, $u_1=4\pi G\rho_0$ $[(\mathrm{km/s})^2(1/\mathrm{kpc})^2]$, and $u_2=r_c$ (kpc). These quantities are used as adjustable parameters in a nonlinear fit of Eq. (7) to the data for NGC 3198. We used the *genfit* function in MATHCAD to do the fitting. Two possible best fits are shown in Fig. 4.

The dot-dash curves are the best fit for a maximum disk contribution with $u_0=35680$ $(\mathrm{km/s})^2(1/\mathrm{kpc})$, $u_1=619$ $(\mathrm{km/s})^2(1/\mathrm{kpc})^2$, and $u_2=6.414$ kpc. Here, u_0 is a measure of the disk surface mass density Σ_0 at the center of the galaxy and u_1 is a measure of the volume mass density ρ_0 of the dark matter halo at the center of the galaxy. If masses are measured in solar masses $M_\oplus$, distances are measured in kpc, and speed is measured in km/s, $G=4.306\times 10^{-6}$ $\mathrm{km}^2\,\mathrm{kpc}/(\mathrm{s}^2\,M_\oplus)$, $\Sigma_0=6.594\times 10^8$ $M_\oplus/\mathrm{kpc}^2$, and $\rho_0=1.144\times 10^7$ $M_\oplus/\mathrm{kpc}^3$. The dotted curves are for a minimum disk contribution and yield $u_0=14690$ $(\mathrm{km/s})^2(1/\mathrm{kpc})$, $u_1=12980$ $(\mathrm{km/s})^2(1/\mathrm{kpc})^2$, and $u_2=1.291$ kpc. The corresponding values for Σ_0 and ρ_0 are 2.715×10^8 $M_\oplus/\mathrm{kpc}^2$ and 2.399×10^7 $M_\oplus/\mathrm{kpc}^3$, respectively. Note that both sets of fitted parameters gave the same fit to the data (solid curve). Presumably, a variety of fitted curves could be obtained by fixing the disk contribution somewhere between the maximum and minimum and then adjusting the halo parameters. A variety of dark matter halo densities could also be tried. It should be clear that the exact form of the dark matter contribution is far from established. Note that the individual contributions from the disk and the halo must be added in quadrature and the square root taken to obtain the fitted curve.

The data for seven other spiral galaxies can be obtained from Ref. 18 and the same fitting procedure could be done. Such a fit would be a useful and interesting extension to the calculations we have reported.

IV. MODIFIED NEWTONIAN DYNAMIC MODEL

An often studied alternative to a dark matter halo is the modified Newtonian dynamic model.[16] In its initial form, Milgrom proposed a modification of Newton's second law for small accelerations. However, in the context of galactic rotations with circular orbits, this modification can also be interpreted as a modification of gravity, which leads to many complications as far as Einstein's general theory of relativity is concerned. In this paper, we interpret the model as a modification of Newton's second law for very small accelerations.

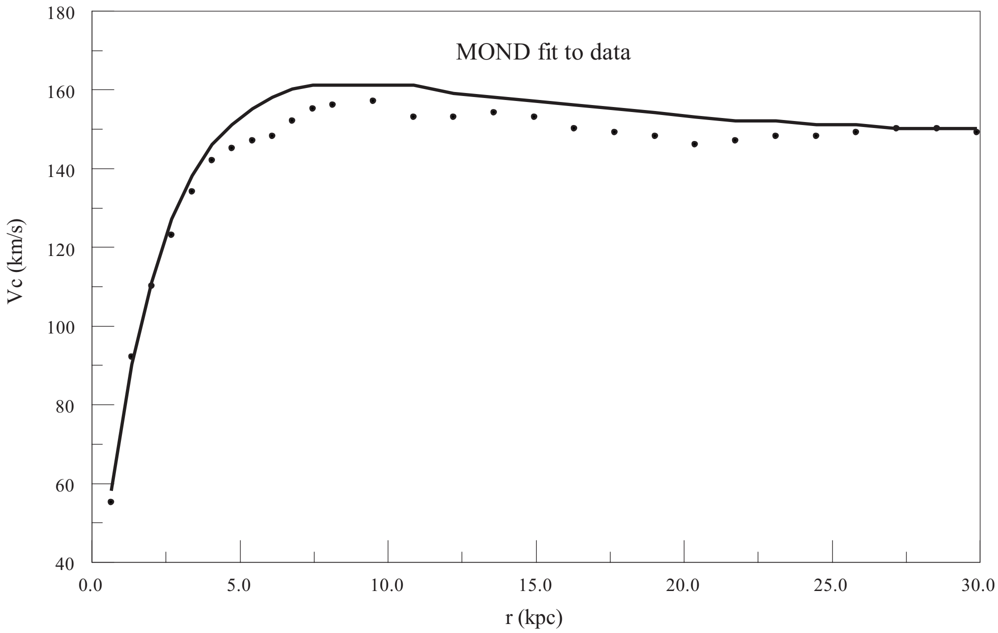

Fig. 5. Modified Newtonian dynamics fit to the data (Refs. 17 and 18) for NGC 3198.

Given that the true gravitational acceleration is g, the model assumes that there is a deviation from Newton's second law for $g \ll a_0$, where a_0 is small. For $g \gg a_0$, g is reduced to the Newtonian value g_N. A relation that does this is

$$g = \frac{g_N}{\mu(x)}, \tag{8}$$

where

$$\mu(x) = \frac{x}{(1+x^2)^{1/2}} \tag{9}$$

and

$$x = \frac{g}{a_0}. \tag{10}$$

We substitute this form of $\mu(x)$ into Eq. (8) and obtain an equation for g given by

$$g^4 - g_N^2 g^2 - g_N^2 a_0^2 = 0. \tag{11}$$

For $g \gg a_0$, we see that $g = g_N$, and in the limit $g \ll a_0$, we have

$$g = \sqrt{g_N a_0}. \tag{12}$$

For circular orbits, $g = v_c^2/r$. For large r, where g is much less than a_0, we have from Eq. (12), $a_0 = v_c^4/(r^2 g_N)$. We can obtain a value for a_0 by using this expression. From the experimental data in Table I, we see that at $r \approx 30$ kpc (9.25×10^{17} km) and $v_c \approx 150$ km s^{-1}. The Newtonian gravitational field $g_N(r)$ at $r = 30$ kpc can be found from Eq. (4) with $4\pi G \Sigma_0 = 35680$ (km/s)^2(1/kpc), corresponding to the case where the visible baryonic matter makes a maximum contribution to the gravitational field. The results are $g_N(30) = 4.97 \times 10^{-15}$ km/s^2 and $a_0 = 1.2 \times 10^{-13}$ km/s^2. Once a_0 is obtained, Eq. (11) can be solved for g as a function of r. A value for $v_c = (gr)^{1/2}$ can then be found for comparison with the data. The results of this procedure for NGC 3198 are shown in Fig. 5. A small adjustment of the value of a_0 can lead to an even better fit.

A similar procedure could be applied to the other spiral galaxies studied in Ref. 18 and the values of a_0 compared. Although we have not calculated a_0 for these other galaxies, we anticipate that the value of a_0 would be almost constant and is independent of the galaxy studied, in accordance with the more involved procedure generally used (see, for example, Ref. 7) As in the case of the computations in Sec. V, this calculation would be an informative problem for students because it would indicate that a_0 might be a universal constant, albeit fortuitously.

V. A DISK WITH ARBITRARY SURFACE MASS DENSITY

In this section we set up the formalism that will allow us to calculate the rotational velocity for circular orbits in the plane of a thin disk with arbitrary surface mass density. Using this formalism and trial and error will allow us to find surface mass densities that yield flat rotation curves. The validity of our numerical technique can be tested by using an exponential surface mass density for which an analytic solution exists [see Eq. (4)].

Consider a coordinate system with its origin fixed at the center of the galaxy and with the x and y axes in the plane of the galaxy, as shown in Fig. 6. Also consider an arbitrary point P as shown. The magnitude of the gravitational field produced at point P by an element of mass dm on an annulus of radius u at a distance R from dm is given by Newton's law of universal gravitation and can be written as

$$|d\vec{g}| = G dm/R^2, \tag{13}$$

where

$$dm = \sigma(u) u du d\theta \tag{14}$$

with $\sigma(u)$ as the surface mass density. We also have

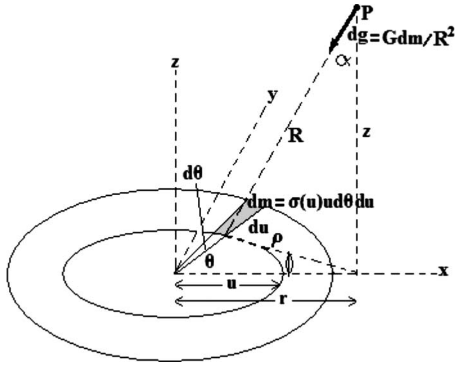

Fig. 6. Geometry used for calculating the gravitational field due to a thin disk.

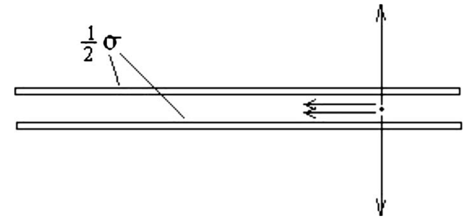

Fig. 7. Diagram illustrating the method used to deal with the singularity at the field point to facilitate the numerical calculation of the gravitational field in the plane of a thin disk.

$$R = (\rho^2 + z^2)^{1/2} \tag{15}$$

and

$$\rho^2 = u^2 + r^2 - 2ur\cos\theta. \tag{16}$$

The substitution of Eqs. (14)–(16) into Eq. (13) yields

$$|d\vec{g}| = \frac{G\sigma(u)udud\theta}{(u^2 + r^2 + z^2 - 2ur\cos\theta)}. \tag{17}$$

From symmetry, the component of $\vec{g}$ in the y direction must be zero and we can therefore ignore dg_y. Consider the components of $d\vec{g}$ in the z and x directions. In the z direction,

$$dg_z = -|d\vec{g}|\cos\alpha. \tag{18}$$

Substituting for $|d\vec{g}|$ and $\cos\alpha$ yields

$$dg_z = -\frac{G\sigma(u)udud\theta z}{(u^2 + r^2 + z^2 - 2ur\cos\theta)^{3/2}}. \tag{19}$$

In the x direction (equivalent to the radial direction for $z=0$),

$$dg_x = -|d\vec{g}|\sin\alpha\cos\phi \tag{20}$$

$$= -\frac{G\sigma(u)udud\theta\rho\cos\phi}{(u^2 + r^2 + z^2 - 2ur\cos\theta)^{3/2}}. \tag{21}$$

From the triangle in the galactic plane, we have

$$u^2 = \rho^2 + r^2 - 2\rho r\cos\phi. \tag{22}$$

Hence,

$$\cos\phi = \frac{\rho^2 + r^2 - u^2}{2r\rho}. \tag{23}$$

Substituting Eqs. (16) and (23) into Eq. (21) and simplifying yields

$$dg_x = -\frac{G\sigma(u)udud\theta(r^2 - ur\cos\theta)}{r(u^2 + r^2 + z^2 - 2ur\cos\theta)^{3/2}}. \tag{24}$$

The integral of Eq. (24) yields the x-component of $\vec{g}$,

$$g_x(r) = -2\frac{G}{r}\int_0^{\pi}\int_0^{\infty}\frac{\sigma(u)u(r^2 - ur\cos\theta)}{(u^2 + r^2 + z^2 - 2ur\cos\theta)^{3/2}}dud\theta. \tag{25}$$

Inspection of Eq. (25) shows that there is a singularity in the galactic plane $(z=0)$ when $u=r$ and $\theta=0$ that causes the numerical integration to fail. We treat this problem by imagining what happens if we consider a field point situated between two thin disks as the spacing between them approaches zero, thus sandwiching the field point between the two disks as in Fig. 7. We assume that each disk carries half of the surface mass density of the original disk and therefore the two disks together have the same surface mass density as the original disk. That is, the two disks are equivalent to the original disk for computational purposes.

As can be seen from Fig. 7, the z components of the gravitational fields due to the two disks are equal in magnitude but opposite in direction and hence cancel. The radial (x) component due to the two disks is twice that due to a single disk with half the surface mass density. The result is that we can numerically calculate the gravitational field in the galactic plane using Eq. (25), with z small, but nonzero, and use the original surface mass density. An effective value for z was established by substituting the surface mass density for an exponential disk [Eq. (1)] into Eq. (25) and reducing z until agreement with the analytic solution in Eq. (4) was obtained to four significant figures for all r. As a result we found $z = 10^{-6}$.

As an example of the use of Eq. (25) to obtain a flat rotation curve without the need for a dark matter halo, we investigated functions of the form, $\sigma = 1/(1+(r/R)^n)$, where n is not necessarily an integer and R is a scale factor. By trial and error, we found that $\sigma = 1/(1+(r/2)^{1.4})$ leads to good agreement with the data for NGC 3198, as shown in Fig. 8.

For this form of σ, the total mass of the galaxy becomes infinite unless the distribution is cut off at some maximum value of r. Thereafter, a Keplerian drop off is expected. As mentioned in connection with the dark matter halo, such a drop off has not been observed. Note also that the use of this form for σ violates the empirically strong assumption of a constant mass to light ratio for spiral galaxies and is unacceptable, unless there is a significant amount of baryonic matter that is not visible. It is clear from this example that there are a number of possible surface mass densities that could lead to a flat rotation curve, but could possibly be excluded on the basis of other empirical evidence. Nonetheless, investigating surface mass densities that can lead to flat rotation curves provides students with an introduction to the complexity involved in trying to pin down the composition of our universe.

We note that an analytic solution, in contrast to the general numerical procedure used in this paper, for the gravitational field due to a special gravitating annulus has recently been obtained.[21] In Ref. 21, a surface mass density of $\sigma = k/r$ was assumed. This surface mass density diverges at the origin

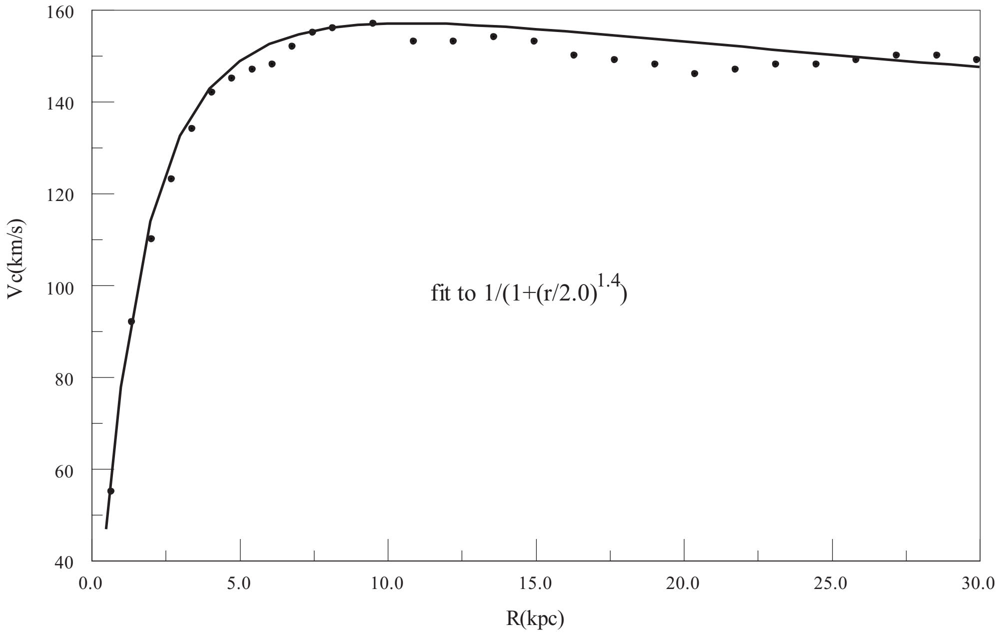

Fig. 8. Fit to the data (Refs. 17 and 18) for NGC 3198 of the circular orbital velocities versus radius for a surface mass density $\sigma = 1/[1+(r/2)^{1.4}]$ and numerical integration of Eq. (25).

(which is nonphysical) but nevertheless leads to an analytic solution and yields a constant orbital velocity for circular orbits for all r.

VI. DISCUSSION

As we have seen, it is possible to obtain a flat rotation curve using a variety of galactic models. Three of these approaches have been outlined in this paper. Are there other models? Quite possibly. For example, it may be possible to obtain a flat rotation curve by modeling the hydrogen as a fluid and using the Navier—Stokes equation, or perhaps the hydrogen is responding to the baryonic mass of the galaxy as it was in the "past" rather than as it is "now," analogous to the 81/2 min delay involving the "present" state of our Sun.

So far we have purposefully avoided coming to any nonexpert conclusions regarding the three possibilities that we considered and have focused instead on computational procedures related to three models that lead to a flat rotation curve. However, the majority of experts believe that a model that includes an exponential disk and a spherically symmetric dark matter halo is as close as one can get to a correct model at the present time. For reasons of stability, it is believed that a spherical dark matter halo is to be preferred over any suggestion that there may be a dark matter component to the disk surface mass density as is possible with the third procedure that we have discussed. (See Ref. 3, Chap. 6 for an introduction to the complex questions involving disk stability and the results of N-body simulations.) Precisely how the dark matter may be distributed in the spherical halo is an open question, and as we have suggested, different spherically symmetric dark matter distributions could be tried as an extension to the present work. One way to ultimately decide on the dark matter distribution in the halo might be through a study of orbits out of the plane of the galactic disk. Readers might be interested in a recent paper which calculated possible orbits in the galactic plane of a spiral galaxy with a flat rotation curve.[22]

The second model considered in the present paper is the MOND model proposed by Milgrom.[16] As mentioned, this model proposes a modification to Newton's second law for small accelerations and has garnered much attention since it was proposed. The successes and failures of the MOND model and a critical evaluation of its effectiveness can be found in Ref. 7. The MOND conjecture is an area of interest in its own right.[23] If and when the dark matter is detected in the ongoing laboratory experiments (as an example see Ref. 24), the MOND conjecture would presumable fall by the wayside. Therefore, the importance of the ongoing laboratory experiments to detect dark matter cannot be overemphasized. An intriguing possibility is that these experiments could prove to be the next (after Michelson–Morley) great null experiments and then what?

ACKNOWLEDGMENTS

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Superb Calculation of the Pressure in a Fluid
Bob Panoff, Shodor Foundation
Tune: Supercalifragilisticexpialidocious

Bernoulli knew he had a rule he used for wings in air
For fluid incompressible he’d never have a scare.
The density of energy’s the same at every spot
A caveat is cavitation in which case it’s not!

Oh, Superb calculation of the pressure in a fluid
Is simple so that anyone with any sense can do it.
We all deserve a force conserved among the objects paired.
Just add to pressure rho gee aitch then add half rho vee squared

A water tower tower’s o’er a town so water goes
Through every pipe, and when you turn the faucet on it flows.
The pressure head is now instead a steady stream, you see,
The pipe’s diameter determines stream velocity.

The sum at every point’s a constant, check it if you care
Each term can change within a range for water or for air.
The key’s to keep the units straight and don’t have any gap
Or else your fluid starts to leak and then you’ll just get Oh... .