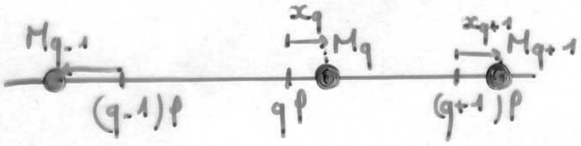
Modèle de Debye

- Cristal 1D



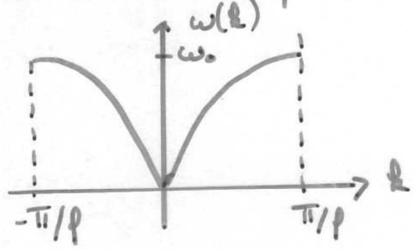
PFD : $\dfrac{d^2 x_q}{dt^2} = -\omega_0^2 \left(2x_q - x_{q+1} - x_{q-1} \right)$

$\omega_0 = \sqrt{\dfrac{K}{m}}$ Solutions harmoniques

$\hookrightarrow$ $\omega(k) = 2\omega_0 \left| \sin\left(\dfrac{k\ell}{2}\right) \right|$

$\longrightarrow$ Conditions aux limites périodiques : $k = \dfrac{2\pi m}{L}$

$\longrightarrow$ Relation de dispersion : $k \in \left[-\dfrac{\pi}{\ell} ; \dfrac{\pi}{\ell} \right]$



. $\omega \simeq 2\omega_0 \times \left| \dfrac{k\ell}{2} \right|$

$\omega \simeq \underbrace{(\ell\omega_0)}_{c} |k|$

N valeurs de $\vec{k}$ $\Rightarrow$ N CL des x_q à k fixé

$\Rightarrow$ N modes normaux $\xi(k,t) = \sum_{q=1}^{N} e^{ikq\ell} x_q(t)$

vérifiant chacun éq osc. harm.

- Cristal 3D

$\hookrightarrow$ 3N éq découplées $E_{\{m_{k\lambda}\}} = \sum_{k\lambda \to 3N \text{ valeurs}} \left(m_{k\lambda} + \dfrac{1}{2} \right) \hbar\omega_\lambda(k_\lambda)$

$\hookrightarrow Z \longrightarrow C_V$

Approximation de Debye : $\omega_\lambda(\vec{k}) = c_\lambda |\vec{k}|$

$\vec{k}$ proches $\Rightarrow$ approximation continue

Intégrale bornée : $\int_0^{\omega_M} \rho(\omega)\, d\omega = 3N \longrightarrow \omega_D = c \times \left(\dfrac{6\pi^2 N}{V} \right)^{1/3}$

$\hookrightarrow E \longrightarrow C_V \overset{T \ll T_D}{\Longrightarrow} \boxed{C_V \propto T^3}$