A COMPLETE ENSEMBLE EMPIRICAL MODE DECOMPOSITION WITH ADAPTIVE NOISE

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ABSTRACT

In this paper an algorithm based on the ensemble empirical mode decomposition (EEMD) is presented. The key idea on the EEMD relies on averaging the modes obtained by EMD applied to several realizations of Gaussian white noise added to the original signal. The resulting decomposition solves the EMD mode mixing problem, however it introduces new ones. In the method here proposed, a particular noise is added at each stage of the decomposition and a unique residue is computed to obtain each mode. The resulting decomposition is complete, with a numerically negligible error. Two examples are presented: a discrete Dirac delta function and an electrocardiogram signal. The results show that, compared with EEMD, the new method here presented also provides a better spectral separation of the modes and a lesser number of sifting iterations is needed, reducing the computational cost.

***Index Terms*—** Empirical Mode Decomposition, Biomedical Signal Processing, Heart Rate Variability

1. INTRODUCTION

Empirical Mode Decomposition (EMD) [1] is an adaptive method introduced to analyze non-linear and non-stationary signals. It consists in a local and fully data-driven separation of a signal in fast and slow oscillations. However, EMD experiences some problems, such as the presence of oscillations of very disparate amplitude in a mode, or the presence of very similar oscillations in different modes, named as "mode mixing". To overcome these problems, a new method was proposed: the Ensemble Empirical Mode Decomposition (EEMD) [2]. It performs the EMD over an ensemble of the signal plus Gaussian white noise. The addition of white Gaussian noise solves the mode mixing problem by populating the whole time-frequency space to take advantage of the dyadic filter bank behavior of the EMD [3]; however it creates some new ones. Indeed, the reconstructed signal includes residual noise and different realizations of signal plus noise may produce different number of modes. In order to overcome these situations, in this paper we propose a variation of the EEMD algorithm that provides an exact reconstruction of the original signal and a better spectral separation of the modes, with a lower computational cost.

The paper is organized as follows. In Sec. 2 the main EEMD concepts are recalled, the new method is introduced and the data used for the experiments is described. In Sec. 3 the results obtained via the method here proposed are presented and compared with EEMD. Finally, the conclusions are discussed in Sec. 4.

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2. MATERIALS AND METHODS

2.1. Ensemble Empirical Mode Decomposition

EMD [1] decomposes a signal $x(t)$ into a (usually) small number of Intrinsic Mode Functions (IMFs) or modes. To be considered as an IMF, a signal must satisfy two conditions: (i) the number of extrema and the number of zero crossing must be equal or differ at most by one; and (ii) the mean value of the upper and lower envelope is zero everywhere.

EEMD defines the "*true*" IMF components (here notated as $\overline{IMF}$ in what follows) as the mean of the corresponding IMFs obtained via EMD over an ensemble of trials, generated by adding different realizations of white noise of finite variance to the original signal $x[n]$. EEMD algorithm can be described as:

1. generate $x^i[n] = x[n] + w^i[n]$, where $w^i[n]$ ($i = 1, \ldots, I$) are different realizations of white Gaussian noise,
2. each $x^i[n]$ ($i = 1, \ldots, I$) is fully decomposed by EMD getting their modes $IMF^i_k[n]$, where $k = 1, \ldots, K$ indicates the modes,
3. assign $\overline{IMF}_k$ as the k-th mode of $x[n]$, obtained as the average of the corresponding IMF^i_k: $\overline{IMF}_k[n] = \frac{1}{I}\sum_{i=1}^{I} IMF^i_k[n]$.

2.2. Our Method

Observe that in EEMD, each $x^i[n]$ is decomposed independently from the other realizations and so for each one a residue $r^i_k[n] = r^i_{k-1}[n] - IMF^i_k[n]$ is obtained.

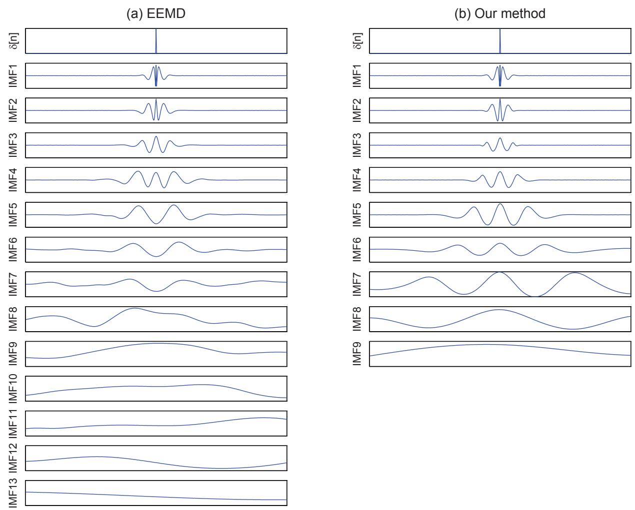

Fig. 1. Decomposition of a 512-sample long delta function by: (a) EEMD ($\overline{IMF}$); (b) Our method ($\widetilde{IMF}$). Noise standard deviation $\varepsilon = 0.02$. Ensemble size $I = 500$.

In the method here presented, the decomposition modes will be noted as $\widetilde{IMF}_k$ and we propose to calculate a unique first residue as:

$$r_1[n] = x[n] - \widetilde{IMF}_1[n], \tag{1}$$

where $\widetilde{IMF}_1[n]$ is obtained in the same way as in EEMD. Then, compute the first EMD mode over an ensemble of $r_1[n]$ plus different realizations of a given noise obtaining $\widetilde{IMF}_2$ by averaging. The next residue is defined as: $r_2[n] = r_1[n] - \widetilde{IMF}_2[n]$. This procedure continues with the rest of the modes until the stopping criterion is reached.

Let us define the operator $E_j(\cdot)$ which, given a signal, produces the $j-$th mode obtained by EMD. Let w^i be white noise with $\mathcal{N}(0,1)$. If $x[n]$ is the targeted data, we can describe our method by the following algorithm:

1. decompose by EMD I realizations $x[n] + \varepsilon_0 \, w^i[n]$ to obtain their first modes and compute

$$\widetilde{IMF}_1[n] = \frac{1}{I}\sum_{i=1}^{I} IMF_1^i[n] = \overline{IMF}_1[n].$$

2. at the first stage ($k = 1$) calculate the first residue as in Eq. (1): $r_1[n] = x[n] - \widetilde{IMF}_1[n]$.
3. decompose realizations $r_1[n] + \varepsilon_1 \, E_1(w^i[n])$, $i = 1, \ldots, I$, until their first EMD mode and define the second mode:

$$\widetilde{IMF}_2[n] = \frac{1}{I}\sum_{i=1}^{I} E_1\left(r_1[n] + \varepsilon_1 \, E_1(w^i[n])\right).$$

4. for $k = 2, \ldots, K$ calculate the k-th residue:

$$r_k[n] = r_{(k-1)}[n] - \widetilde{IMF}_k[n]. \tag{2}$$

5. decompose realizations $r_k[n] + \varepsilon_k \, E_k(w^i[n])$, $i = 1, \ldots, I$, until their first EMD mode and define the $(k+1)$-th mode as

$$\widetilde{IMF}_{(k+1)}[n] = \frac{1}{I}\sum_{i=1}^{I} E_1(r_k[n] + \varepsilon_k \, E_k(w^i[n])). \tag{3}$$

6. go to step 4 for next k.

Steps 4 to 6 are performed until the obtained residue is no longer feasible to be decomposed (the residue does not have at least two extrema). The final residue satisfies:

$$R[n] = x[n] - \sum_{k=1}^{K} \widetilde{IMF}_k, \tag{4}$$

with K the total number of modes. Therefore, the given signal $x[n]$ can be expressed as:

$$x[n] = \sum_{k=1}^{K} \widetilde{IMF}_k + R[n]. \tag{5}$$

Eq. (5) makes the proposed decomposition complete and provides an exact reconstruction of the original data.

Observe that the ε_i coefficients allow to select the SNR at each stage. Concerning the amplitude of the added noise, Wu and Huang suggested [2] to use small amplitude values for data dominated by high-frequency signals, and vice versa. Following then, in this work, we used a few hundred of realizations and fixed the same SNR for all the stages. This value might depend on the application. In all the implementations we use the EMD toolbox available on: `http://perso.ens-lyon.fr/patrick.flandrin/emd.html`

2.3. Data

Synthetic and real signals will be analyzed in the present paper. We consider a synthesized single $\delta[n]$ Dirac signal of 512 samples. This function was used in [4] to suggest that noise could help data analysis in cases where EMD cannot be performed, giving birth to EEMD in [2]. A second example will be considered, using real data: Electrocardiogram (ECG) signals from the MIT-BIH Normal Sinus Rhythm Database[1].

3. RESULTS AND DISCUSSIONS

As a first example, we apply the proposed method to a single delta signal $\delta[n]$. In Fig. 1 are depicted the decompositions obtained by EEMD and the one obtained by the new method here proposed. In both cases, an ensemble size of $I = 500$ were used, with $\varepsilon_0 = 0.02$, corresponding to a SNR of 34 dB. In the left panel, it can be seen that EEMD produces thirteen modes, while in the right one, only nine modes are obtained by the method here proposed.

In both decompositions the amplitudes of the modes one to five are similar ($0.01 \leq |\overline{IMF}_k| \leq 0.5$ for $k = 1, \ldots, 5$)

[1] `http://www.physionet.org/cgi-bin/ATM`

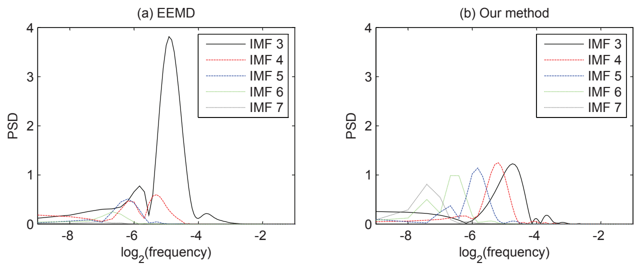

Fig. 2. Spectra of modes 3 to 7 obtained by: (a) EEMD ($\overline{IMF}$); (b) Our method ($\widetilde{IMF}$).

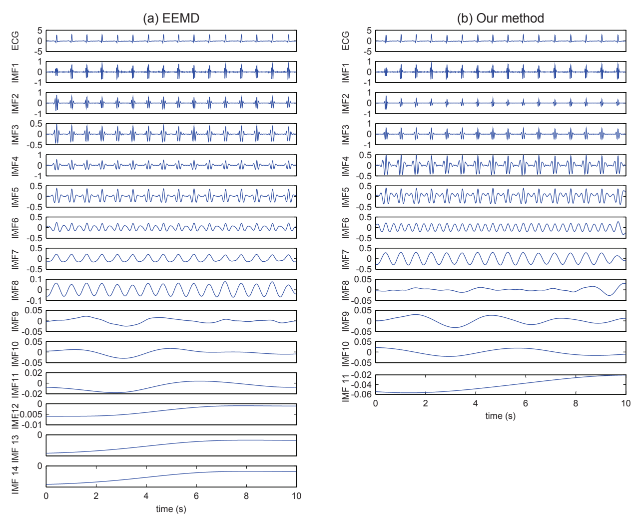

Fig. 3. Decomposition of a 10-second length ECG signal by: (a) EEMD ($\overline{IMF}$); (b) Our method ($\widetilde{IMF}$). Noise standard deviation $\varepsilon = 0.2$. Ensemble size $I = 500$.

and less than 10^{-3} for $k = 6, \ldots, 8$, with lower energy in the EEMD cases. EEMD modes nine to thirteen have very low amplitude ($\max(|\overline{IMF}_k|) \leq 2\times10^{-4}$.) Additionally $\overline{IMF}_k$ ($k \geq 8$) are not symmetric as expected. This low energy issue in the case of EEMD is due to the effect of averaging over all realizations, while a large variation in the number of modes could be observed. Table 1 shows how many realizations x^i produced a given number of modes in our examples. For $\delta[n]$ only one realization produced seven modes, 17 realizations produced eight modes, and so on. In order to perform the averaging it is necessary to pad with zeros the missing modes, getting low amplitudes when averaging. A different solution could be to set the number of modes (usually at $1+[log_2(N)]$, with N the signal length); in this case the EEMD method would no longer be fully adaptive. The method here proposed does not suffer from this difficulty because: (i) each realization of residue plus noise is decomposed until the first mode is reached, and (ii) for the final mode K we use as stopping criterion the one used in EMD [1].

Also, observe that in $\overline{IMF}_4$ the central maximum is smaller than the lateral maxima and that $\overline{IMF}_k$ $(5 \leq k \leq 7)$ present no central maximum. In contrast, in the right side of Fig. 1 we can appreciate a similar behavior for all modes, consistent to what would have been obtained performing a wavelet analysis. Flandrin *et al.* [4] obtained similar results when they calculated an equivalent impulse response for EMD by adding noise to a 256-sample long delta function and averaged the corresponding IMFs using an ensemble size of 5000. In our case we need just a tenth of the realizations.

Compared with EEMD, another advantage of the method here proposed concerns the number of sifting iterations needed for such δ signals. While EEMD required 278931 sifting iterations totally, the method here proposed required only 140939, which is almost a half.

In order to compare the spectral separation properties, the PSD of modes three to seven for both methods are shown in Fig. 2. On the right it can be observed that the mode's spectra obtained by the new decomposition are less overlapped than those obtained from EEMD, showing a clearer separation of the frequency content between the modes.

In summary, it can be appreciated that in the case of delta like signals, the method here proposed provides a clearer net decomposition than the EEMD.

Different authors have proposed to use EMD or EEMD for ECG denoising [5, 6]. ECG signals are characterized by spike-like events (QRS complexes), similar to discrete Dirac δ functions previously analyzed. In our experience with ECG signals, severe mode mixing was observed when decomposing them by EMD. Although EEMD alleviates the mode mixing, it is still too much time consuming because of the large number of sifting iterations required to achieve the decomposition. In Fig. 3 both decompositions of an ECG signal are presented, obtained by EEMD (left) and with the new method here proposed (right). An ensemble size of $I = 500$ was used in both cases, with standard deviation $\varepsilon = 0.2$ of the added noise (SNR = 14 dB). It can be appreciated in the right panel that in the seventh mode the fundamental frequency (F) of the signal is clearly captured, while in the case of EEMD, F appears with lower energy in modes seven and eight (left panel). Therefore, a fundamental frequency extraction algorithm could fail to identify the mode that contains it when applied to an EEMD decomposition.

The RR signal, defined as the distance between consecutive R peaks in the ECG, is widely used to study the heart rate variability (HRV) which contains information about the state of the autonomous nervous system (ANS). While this approach provides a non-uniformly and low rate sampled signal, an estimation of the instantaneous heart frequency from the proper mode obtained using our method, would allow a uniformly sampled heart rate estimation at a higher frequency.

Boxplots of the sifting iterations required by each decomposition are presented in Fig.4. Note the different ranges in the vertical axes (2500 *vs* 500). Moreover, while in the EEMD

#	7	8	9	10	11	12	13	14	I
$\delta[n]$	1	17	175	234	66	6	1	-	500
ECG	-	-	2	58	240	187	12	1	500

Table 1. Number of EEMD modes for each realization i.

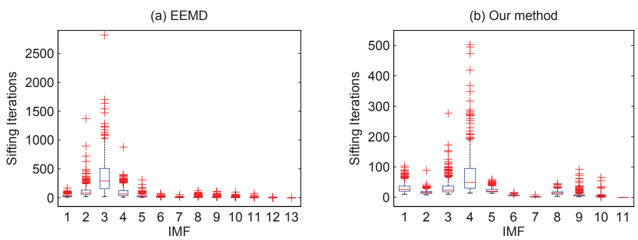

Fig. 4. Boxplots showing the sifting iterations for each mode. (a) EEMD ; (b) Our method. Note the different scales for the amplitude.

case the total number of iterations is 342199, in the case of the method here proposed it is 105314. It is clear that the new method provides an improvement in terms of computational cost, requiring only the 30.8% of the sifting iterations of the EEMD for this signal.

The completeness of the new method is guaranteed by Eq. (5). To numerically confirm this property, we present in Fig. 5 the reconstruction error, for both decompositions, computed as the difference between the ECG signal and the sum of the modes. In the case of our method, the maximum amplitude is less than 2×10^{-15} (the round off error from the precision of the computer) with a standard deviation of 2×10^{-14}. To achieve this precision with EEMD (which does not guarantee a complete decomposition) it would be necessary to increase the number of realizations to over 10^{29}, considering that in EEMD, the remaining noise has a standard deviation of $\varepsilon_r = \varepsilon/\sqrt{I}$, turning the process extremely expensive in terms of computational cost.

4. CONCLUSIONS

In this work we have presented a new algorithm for analyzing and processing non-linear and non-stationary signals. The new method was successfully tested on artificial and real signals.

The method here proposed has the advantages of requiring less than half the sifting iterations that EEMD does, and that the original signal can be exactly reconstructed by summing the modes. Decomposition completeness was theoretically demonstrated and numerically verified in the case of ECG signal. Because of that, a smaller ensemble size is needed resulting in a significant computational cost saving. In that sense, the novel method recovers some of the EMD properties lost by EEMD, such as completeness and a fully data-driven number of modes.

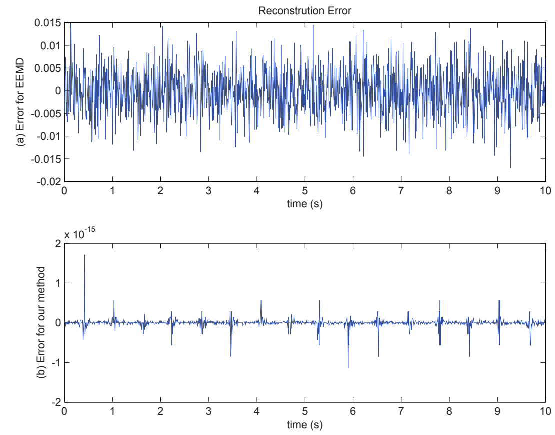

Fig. 5. Reconstruction error for: (a) EEMD ; (b) Our method.

In future works statistical studies will be carried out in order to determine the proper ensemble size and the amplitude of the added noise.

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