DETRENDING AND DENOISING WITH EMPIRICAL MODE DECOMPOSITIONS

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ABSTRACT

Empirical Mode Decomposition (EMD) has recently been introduced as a local and fully data-driven technique aimed at decomposing nonstationary multicomponent signals in "intrinsic" AM-FM contributions. Although the EMD principle is appealing and its implementation easy, performance analysis is difficult since no analytical description of the method is available. We will here report on numerical simulations illustrating the potentialities and limitations of EMD in two signal processing tasks, namely detrending and denoising. In both cases, the idea is to make use of partial reconstructions, the relevant modes being selected on the basis of the statistical properties of modes that have been empirically established.

1. INTRODUCTION AND MOTIVATION

Empirical Mode Decomposition (EMD) has been recently pioneered by Huang *et al.* [2] for adaptively decomposing signals in a sum of "well-behaved" AM-FM components. The technique has already received some attention in terms of both applications [1, 2, 7, 9, 11] and interpretation [3, 4, 5, 8]. The method consisting in a local and fully data-driven splitting of a (possibly nonstationary) signal in fast and slow oscillations, the purpose of this paper is to investigate further its capabilities in terms of detrending and denoising.

2. BASICS OF EMPIRICAL MODE DECOMPOSITION

2.1 Principle

Empirical Mode Decomposition (EMD) [2] is a technique which has been designed primarily for obtaining AM-FM type representations in the case of signals which are oscillatory (possibly nonstationary and/or generated by a nonlinear system), in some automatic, fully data-driven, way. In a nutshell, the starting point of EMD is to consider oscillatory signals at the level of their local oscillations and to formalize the idea that:

"signal = fast oscillations superimposed to slow oscillations,"

and to iterate on the slow oscillations component considered as a new signal.

2.2 Algorithm

More precisely, if we look at the evolution of a signal $x(t)$ between two consecutive local extrema (say, two minima occurring at times t_- and t_+), we can heuristically define a (local) "high-frequency" part $\{d(t), t_- \leq t \leq t_+\}$. This *detail* $d(t)$ corresponds to the oscillation terminating at the two minima and passing through the maximum which necessarily exists in between them. For the picture to be complete, we also identify the corresponding (local) "low-frequency" part $m(t)$, or local *trend*, so that we have $x(t) = m(t) + d(t)$ for $t_- \leq t \leq t_+$. Assuming that this is done in some proper way for all the oscillations composing the entire signal, we get what is referred to as an *Intrinsic Mode Function* (IMF) as well as a *residual* consisting of all local trends. The procedure can then be applied to this residual, considered as a new signal to decompose, and successive constitutive components of a signal can therefore be iteratively extracted.

Given a signal $x(t)$, the effective algorithm of EMD can therefore be summarized as the following main loop [2]:

1. identify all extrema of $x(t)$;
2. interpolate between minima (resp. maxima), ending up with some "envelope" $e_{\min}(t)$ (resp. $e_{\max}(t)$);
3. compute the average $m(t) = (e_{\min}(t) + e_{\max}(t))/2$;
4. extract the detail $d(t) = x(t) - m(t)$;
5. iterate on the residual $m(t)$.

In practice, the above procedure has to be refined by a *sifting* process, an inner loop that iterates steps (1) to (4) upon the detail signal $d(t)$, until this latter can be considered as zero-mean according to some stopping criterion[1]. Once this is achieved, the detail is considered as the effective IMF, the corresponding residual is computed and only then, step (5) applies. Eventually, the original signal $x(t)$ is first decomposed through the main loop as

$$x(t) = d_1(t) + m_1(t), \tag{1}$$

and the first residual $m_1(t)$ is itself decomposed as

$$m_1(t) = d_2(t) + m_2(t), \tag{2}$$

so that

$$\begin{aligned} x(t) &= d_1(t) + m_1(t) \\ &= d_1(t) + d_2(t) + m_2(t) \\ &= \dots \\ &= \sum_{k=1}^{K} d_k(t) + m_K(t). \end{aligned} \tag{3}$$

[1]It is not the purpose of this paper to address algorithmic issues which have been considered in some detail elsewhere [2, 8]. Let us just mention that the main reason for which a proper IMF has to be zero-mean is that this is a pre-requisite for its AM-FM demodulation with Hilbert transform techniques [2], a post-processing aspect of EMD that will not be considered here.

2.3 Interpretations

Modes and residuals have been heuristically introduced on "spectral" arguments, but this must not be considered from a too narrow perspective. First, the decomposition makes no assumption about the *harmonic* nature of oscillations, and it can thus guarantee a compact representation (i.e., with fewer modes than a Fourier or wavelet decomposition) in situations involving *nonlinear* oscillations. Second, it is worth stressing the fact that, even in the case of harmonic oscillations, the high vs. low frequency discrimination mentioned above applies only *locally* and corresponds by no way to a predetermined sub-band filtering. Indeed, selection of modes rather corresponds to an automatic and adaptive (data-driven) time-variant filtering.

3. MODE MANIPULATIONS

By construction, the number of extrema decreases when going from one residual to the next, thus guaranteeing that the complete decomposition is achieved in a finite number of steps (typically, at most $O(\log_2 N)$ for N data points). Moreover, the whole decomposition being only based on elementary subtractions, it obviously allows for a perfect reconstruction of the initial signal $x(t)$, given the collection of details $\{d_k(t), k = 1, \ldots K\}$ and the residual $m_K(t)$.

This property of perfect reconstruction, together with the spectral interpretation outlined above, suggests to achieve partial reconstructions only, so as to selectively remove slow or fast oscillations (*detrending* or *denoising*, respectively).

3.1 Detrending

In the case where the analyzed signal $x(t)$ consists in a slowly varying trend superimposed to a fluctuating process $y(t)$, the trend is expected to be captured by IMFs of large indices (+ the final residual). Detrending $x(t)$, which corresponds to estimating $y(t)$, may therefore amount to computing the partial, fine-to-coarse, reconstruction

$$\hat{y}_D(t) = \sum_{k=1}^{D} d_k(t),$$

where D is the larger IMF index prior contamination by the trend. Each of the IMFs $\{d_k(t); k = 1, \ldots D\}$ being zero-mean, a rule of thumb for choosing D is to observe the evolution of the (standardized) empirical mean of $\hat{y}_d(t)$ as a function of a test order d, and to identify for which $d = D$ it departs significantly from zero. An example of this approach is given in Figure 1, where a 7000 data point segment of a Heart-Rate Variability signal is considered.

3.2 Statistics

The procedure outlined above is a rough approach that can be improved upon when a more precise model can be advocated for the signal + noise mixture. To this end, a detailed knowledge of IMFs statistics in noise only situations can help identifying the significance of a given mode. This idea, which has been pioneered by Wu and Huang [10], can be followed in two directions, namely detrending as in the previous section (by keeping only those modes which are identified as noise) and denoising (by removing them).

Previous EMD studies have considered in some detail white Gaussian noise [10] and, more generally, fractional

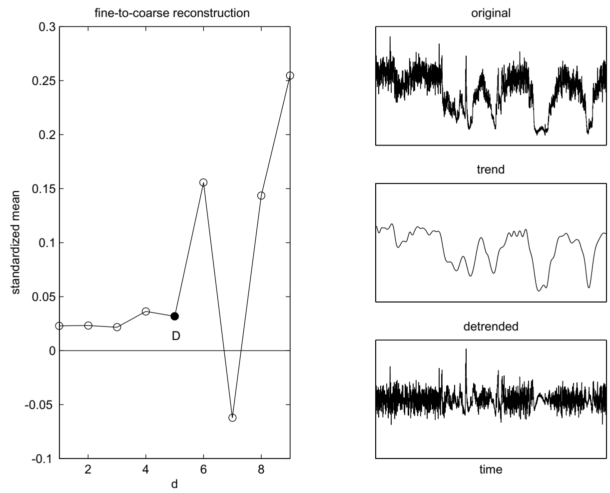

Figure 1: Detrending of a Heart-Rate Variability signal. *Left: standardized empirical mean of the fine-to-coarse EMD reconstruction, evidencing $D = 5$ as the change point. Top right: original signal. Middle right: estimated trend obtained from the partial reconstruction with IMFs 6 to 9 + the residual. Bottom right: detrended signal obtained from the partial reconstruction with IMFs 1 to 5.*

Gaussian noise (fGn) [3, 5, 6] as a versatile class for broadband noise with no dominant frequency band. What has been shown in this context is that EMD acts spontaneously as a dyadic filterbank [3, 5, 6, 10]. Furthermore, the expected IMFs log-variance has been shown to admit a simple linear model controlled by the Hurst exponent H of the considered process:

$$\log_2 V_H[k] = \log_2 V_H[2] + 2(H-1)(k-2)\log_2 \rho_H \quad (4)$$

for $k \geq 2$, with $\rho_H \approx 2$.

As far as the variability of this quantity is concerned, a quantitative yet empirical appreciation can be gained from the upper part of Figure 2 where, in 3 typical cases ($H = 0.2$, 0.5 and 0.8), the experimental mean, median and various confidence intervals have been reported, together with the model (4). This series of simulations (which has been carried out on 10000 realizations of 2048 data points in each case) evidences larger and larger fluctuations for modes of larger and larger indices, in agreement with (and generalization of) the findings reported in [10] for the only case of white noise. (Interestingly, it has to be remarked that the skewed (marginal) distribution of these "modegrams" reveals a better agreement when fitting the linear model (4) with the median rather than the mean of the realizations.)

The lower part of Figure 2 precises further how the relative confidence intervals can be given a semi-analytical form as a function of the IMF index: the quasi-linear dependences reported in the diagram allow for a parameterization of the curve $T_H[k]$ corresponding to a chosen confidence interval according to a functional relationship of the form

$$\log_2(\log_2(T_H[k]/W_h[k])) = a_H k + b_H, \quad (5)$$

where $W_H[k]$ stands for the H-dependent variation of some IMF mean energy, considered as a variance estimator. In accordance with what has been said previously, the best linear

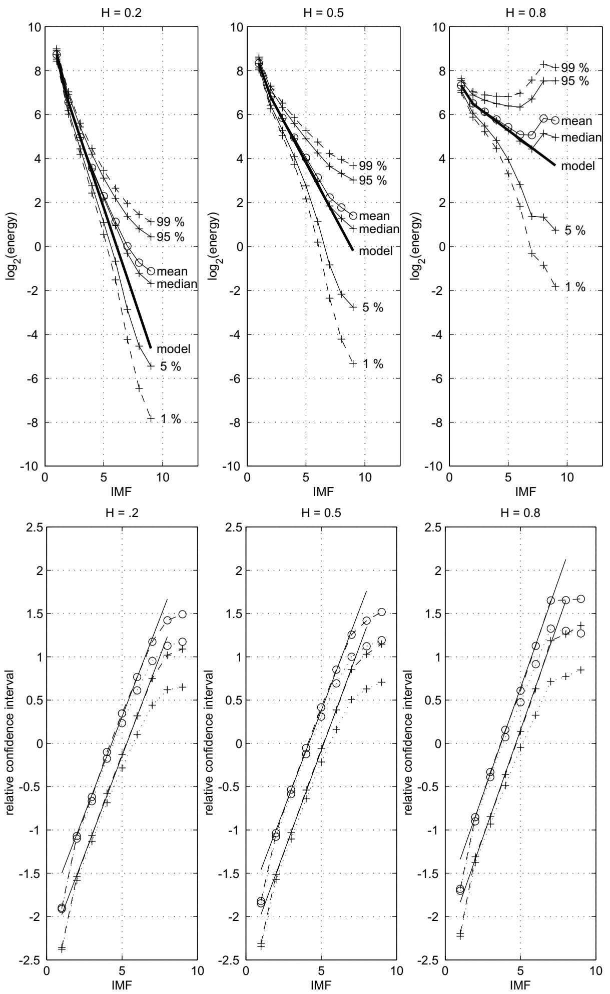

Figure 2: Experimental "modegrams" in the case of fractional Gaussian noise. *Top: for the* 3 *considered values of the Hurst exponent* H*, statistical characteristics (mean, median, confidence intervals) of the logarithm of the estimated EMD variance have been plotted as a function of the IMF index, together with the linear model given by (4). Bottom: the logarithm of the relative confidence intervals, i.e., the quantity* $\log_2(\log_2(T_H[k]/W_H[k]))$*, behaves almost linearly as a function of the IMF index* k*, leading to the semi-analytical form (5). For each of the* 3 *considered values of* H*, crosses (resp., circles) correspond to a confidence interval of* 95% *(resp.,* 99%*), dotted (resp., dashed) lines refer to the cases where the reference* $W[k]$ *is chosen as the mean (resp., median) of the IMF energies over the realizations, and the full lines indicate the corresponding best linear fits.*

fit is obtained when choosing for $W_H[k]$ the median of the IMFs energy over the realizations, which is in this case very close from the model $V_H[k]$. The parameters a_H and b_H that are used as ingredients for modelling the confidence intervals can be deduced from simulation results, and their values are reported in the following Table:

H	$a_H(95\%)$	$b_H(95\%)$	$a_H(99\%)$	$b_H(99\%)$
0.2	0.46	-2.43	0.45	-1.95
0.5	0.47	-2.45	0.46	-1.92
0.8	0.50	-2.33	0.50	-1.83

3.3 Denoising

The considerations above can be used for denoising a signal embedded in fGn of known Hurst exponent H, based on the empirically observed energy $W_H[k]$ of the IMFs $d_k[n]$.

In practice, $W_H[1]$ can be estimated as

$$\hat{W}_H[1] = \sum_{n=1}^{N} d_1^2[n], \tag{6}$$

and the subsequent values of $W_H[k]$ follow as

$$\hat{W}_H[k] = C_H\, \rho_H^{-2(1-H)k},\ k \geq 2, \tag{7}$$

where $C_H = \hat{W}_H[1]/\beta_H$ and β_H values can be found in [6].

Given these results, a possible strategy for denoising a signal corrupted by fGn (with known H) is as follows:

1. assuming that IMF 1 captures mostly noise, estimate the noise level in the signal + noise mixture by computing $\hat{W}_H[1]$ as in (6);
2. estimate the "noise only" model from (6) and (7);
3. estimate the corresponding model for the chosen confidence interval from (5) and the Table;
4. compute the EMD of the signal + noise mixture and compare the IMF energies with the confidence interval used as a threshold;
5. compute a partial reconstruction by keeping only the residual and those IMFs whose energy exceeds the threshold.

A toy example of the EMD approach to denoising is given in Figure 3, in the case of an oscillatory low frequency waveform embedded in fGn with $H = 0.3$.

This Figure suggests of course that a dual strategy can be used for detrending a fGn-type noise process by computing the complementary partial reconstruction based on only those IMFs whose energy is below the threshold. In this respect, the HRV example used in Section 3.1 can be revisited from a more quantitative perspective. Indeed, the inspection of the signal spectrum (top of Figure 4) suggests that a fGn model is qualitatively admissible in the mid-frequency range, with a spectral exponent $2H - 1 \approx 0.79$, leading to $H \approx 0.9$. The resulting detrending (which can be compared with profit to Figure 4) has been obtained by letting $H = 0.9$ and using a confidence interval of only 95% because of the approximate relevance of the fGn model.

4. CONCLUDING REMARKS

Empirical Mode Decomposition (EMD) is an appealing new technique for adaptively decomposing signals in a sum of AM-FM modes. Because the selection of these modes is fully data-driven and very local in time, EMD paves the way for new automatic approaches to detrending and denoising in nonstationary situations. In this perspective, we have reported here on exploratory quantitative results that demonstrate effective and potential usefulness of EMD-based techniques. The current status of EMD, which still lacks from

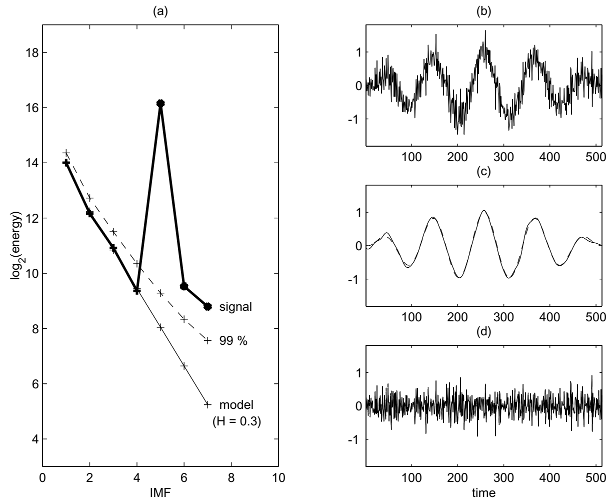

Figure 3: Denoising. *An example of an amplitude modulated low frequency oscillation embedded in fractional Gaussian noise of Hurst exponent $H = 0.3$ is plotted in (b). The estimated energies of the 7 IMFs are plotted in (a) as the thick line, together with the "noise only" model corrsponding to $H = 0.3$ (thin line) and the 99% confidence interval (dotted line). The partial reconstruction obtained by adding the EMD residual and IMFs 5 to 7 (the only ones whose energy exceeds the threshold in (a)) is plotted in (c) as the full line, and this denoised signal is superimposed to the actual signal component (dotted line). The partial reconstruction of IMFs 1 to 4 (noise estimate) is plotted in (d).*

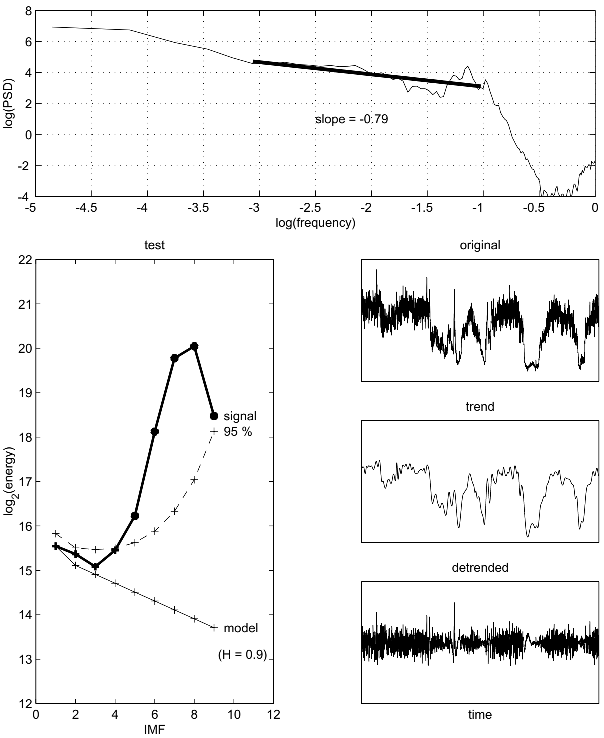

Figure 4: Detrending of the Heart-Rate Variability signal of Figure 1. *Top diagram: Signal spectrum in log-log coordinates (thin line), with a linear fit in the mid-frequency range (thick line). Bottom diagram: model-based detrending. Left: estimated energy of the 9 IMFs, plotted as the thick line, together with the "noise only" model (thin line) and the 95% confidence interval (dotted line). Top right: original signal. Middle right: estimated trend obtained from the partial reconstruction with IMFs* 5 *to* 9 *(the only ones whose energy exceeds the threshold in the left diagram) and the residual. Bottom right: detrended signal obtained from the partial reconstruction with IMFs* 1 *to* 4.

solid theoretical grounds, imposed to conduct the present study on the basis of extended numerical experiments. In order to elaborate on our present findings, extended studies are necessary, both in terms of experiments (in particular, comparisons with existing competing approaches), and theory developments.

Software. The Matlab codes used in this study are available, and they can be downloaded for free from the URL: `perso.ens-lyon.fr/patrick.flandrin/emd.html`.

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