Correspondence

Generalized Target Description and Wavelet Decomposition

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***Abstract*—Generalized target description by means of colored bright spots (Altes [2]) is very attractive for recognition or classification tasks in active sonar applications. In this correspondence, we show how such a description can be achieved directly from the impulse response to identify. It turns out that the resulting procedure is in close connection with the recently introduced technique of wavelet decomposition.**

I. Introduction

In active sonar applications, recognition or classification of targets can be achieved by means of parameters related to the target impulse response. These must be extracted from returning echos of some transmitted signal, and for this parameterization to be efficient, a convenient modeling of the impulse response is required. In this respect, it appears that a generalized target description offers great flexibility and that an efficient identification is possible by using suitably chosen transmitted signals.

In this correspondence, we show that the use of a suitable transmitted signal can be made implicit, hence allowing us to identify targets directly from their impulse response. A closer look at the resulting procedure reveals then its close connection with the recently introduced technique of wavelet decomposition.

II. Generalized Target Description

In active sonar situations for which target information is carried by the returning echo of a transmitted waveform, a standard approximation is to consider the target echo as resulting from a number of range-distributed point scatterers. This means that the target impulse response $x(t)$ is supposed to be of the form

$$x(t) = \sum_{k=1}^{K} x_k \delta(t - \tau_k). \tag{1}$$

The transversal filter representation (1) is therefore characterized by a set of weights x_k and delays τ_k (associated to ranges), which are to be identified.

Although (1) is useful as a rough approximation, it appears that it is oversimplified for a proper description of realistic targets [2]. This comes merely from the fact that in (1), all the "glints" or "bright spots" are assumed to be associated to perfect reflections and do not encompass any frequency dependence. This can be overcome by replacing the target description (1) by a *generalized target description* [1], [2] which is based on differentiated and integrated delta functions, and not only on delta functions:

$$x(t) = \sum_{k=1}^{K} \sum_{m=-M}^{M} x_{km} \delta^{(m)}(t - \tau_k), \tag{2}$$

an expression which can be thought of as a decomposition by means of *colored* bright spots [3].

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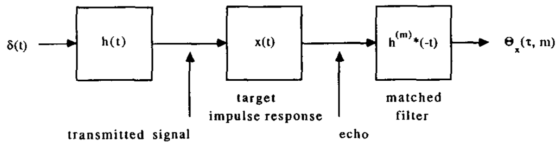

Fig. 1. First receiver configuration using explicitly a transmitted signal $h(t)$.

In order to identify the set of coefficients (x_{km}, τ_k), Altes has designed a receiver structure [2] which, in a first step, makes use of a bank of matched filters, each filter of the bank being matched to some differentiated or integrated version of the transmitted signal: if $h(t)$ stands for this transmitted signal, a first configuration of the receiver is given in Fig. 1 where $\Theta_x(\tau, m)$ is the output of the filter matched to the mth derivative (or integral) of $h(t)$. In such a configuration, identification is achieved by using explicitly a transmitted waveform, which is *a priori* arbitrary.

However, an objective choice of $h(t)$ is possible if we impose some constraint on, e.g., the receiver complexity. Since the bandwidth of the successive derivatives of $h(t)$ increases as the order of differentiation increases, a natural requirement is to impose that their duration decrease correspondingly, in order to keep a time-bandwidth product (and hence, a complexity) constant for all the filters of the bank. A possibility is to deal with signals for which differentiation (respectively, integration) is equivalent to compression (respectively, dilation). This results in very special waveforms whose Fourier transform reads [2]

$$H_{n,g,c}(\omega) = A\omega^n \exp\left(-\frac{1}{2\ln g}\ln^2 \omega\right) \exp\left(j\frac{2\pi c}{\ln g}\ln \omega\right) U(\omega) \tag{3}$$

(where U stands for the unit step function, n, g, and c are real-valued parameters, and A is some constant) or, equivalently [3],

$$H_{\omega_0,g,c}(\omega) = A' \exp\left(-\frac{1}{2\ln g}\ln^2\left(\frac{\omega}{\omega_0}\right)\right) \cdot \exp\left(j\frac{2\pi c}{\ln g}\ln\left(\frac{\omega}{\omega_0}\right)\right) U(\omega) \tag{4}$$

(where A' is some other constant) if emphasis is to be put on the central (angular) frequency

$$\omega_0 = g^n. \tag{5}$$

Such a signal meets the condition of equivalence between differentiation and scaling since [2]

$$H_{n,g,c}(g^{-m}\omega) = \left[g^{-m(n+m/2)} \exp(-j2\pi mc)\right] \omega^m H_{n,g,c}(\omega). \tag{6}$$

Among different interesting properties of the signal (3), we can mention one which will be useful in the following, and which reads

$$\left|H_{n,g,c}(g^{-m}\omega)\right|^2 = g^{-m(2n+m)} \omega^{2m} H_{2n,\sqrt{g},0}(\omega). \tag{7}$$

The signal defined by (3) or (4) has a structure of *linear period modulation* (hyperbolic frequency modulation) and a log-normal envelope. Thus, in addition to constraining the receiver complexity

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through (6), it also possesses the remarkable feature of being *Doppler-tolerant* [4] and it provides a satisfactory modeling of some natural sonar signals emitted by mammals for echolocation [2].

III. Wavelet Decomposition

In the recent past, sustained interest has been devoted to a new technique of signal decomposition, based on a linear transform which is referred to as the wavelet transform [5]–[7]. The general idea of this approach is to consider a finite energy signal as resulting from the superposition of a number of "building blocks" which all have the same time–bandwidth product, and which can all be deduced from one elementary waveform (the analyzing wavelet) by means of shifts in the time direction and scale changes (dilations or compressions). The spirit is similar to that of classical Gabor–Helström decompositions [8], [9], but when replacing frequency shifts by scale changes, making the wavelet transform a *time-scale* analysis more than a time-frequency analysis [10], [11].

By definition, given an analyzing wavelet $w(t)$, the wavelet transform of $x(t)$ expresses as

$$T_{x,w}(\tau, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\, w^*\left(\frac{t-\tau}{a}\right) dt. \tag{8}$$

For this to make sense, the Fourier transform of $w(t)$ must satisfy the admissibility conditions [5], [6]

$$c_w = \int_{-\infty}^{+\infty} |W(\omega)|^2 \frac{d\omega}{|\omega|} < \infty; \qquad W(0) = 0. \tag{9}$$

The first condition corresponds to a sufficiently fast decay of $W(\omega)$ at high frequencies, whereas the second one imposes $w(t)$ to be zero mean. As a result, the analyzing wavelet can be viewed as the impulse response of a bandpass filter, which possesses at least some oscillations, whence the name.

Provided that the conditions (9) hold, $x(t)$ can be recovered from its wavelet transform as [5], [6]

$$x(t) = \frac{1}{c_w} \int_{-\infty}^{+\infty} \int_0^{\infty} T_{x,w}(\tau, a) \frac{1}{\sqrt{a}}\, w\left(\frac{t-\tau}{a}\right) \frac{d\tau\, da}{a^2}. \tag{10}$$

This inversion formula makes clear the role played by the wavelet transform as a weight associated to each of the scaled and shifted building blocks.

It is known that Gabor–Helström theory is primarily based on Gaussian functions. In the wavelet case, Grossmann and Morlet have justified [5] that a companion function is to be privileged which, in the frequency domain, is just the image of the Gaussian function under a natural map. According to their parameterization, the Fourier transform of this function reads

$$W_\alpha(\omega) = \exp\left(-\frac{\alpha}{2} \ln^2 \omega\right) U(\omega). \tag{11}$$

IV. Generalized Target Description and Wavelet Decomposition

Coming back to the receiver configuration of Fig. 1, it is possible to commute the first two filters in order to directly process the impulse response $x(t)$ without making use explicitly of the transmitted signal $h(t)$. This ends up with the second receiver configuration, which is given in Fig. 2.

This is of special interest when only the impulse response is available. In fact, this procedure allows us to characterize *any* signal considered as the impulse response of some target.

Within this second configuration, the output $\Theta_x(\tau, m)$ expresses as

$$\Theta_x(\tau, m) = \int_{-\infty}^{+\infty} x(t)\, \gamma_h^{(m)*}(t - \tau)\, dt \tag{12}$$

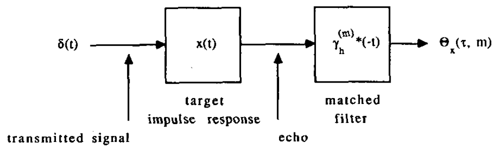

Fig. 2. Second receiver configuration using implicitly the transmitted signal $h(t)$ of Fig. 1.

where

$$\gamma_h(t) = \int_{-\infty}^{+\infty} |H(\omega)|^2 e^{j\omega t} \frac{d\omega}{2\pi} \tag{13}$$

is the autocorrelation function of $h(t)$.

If $h(t)$ is chosen to be the Altes signal (3), it follows directly from (7) that

$$\gamma_h^{(m)}(t) = j^m g^{(m/2)(1+2n+m/2)}\, \gamma_h(g^{m/2} t). \tag{14}$$

Hence, (12) can be rewritten by means of dilated or compressed versions of the autocorrelation function of $h(t)$. According to the definition (8), this yields to the central result

$$\Theta_x(\tau, m) = j^{-m} g^{(m/2)(1/2+2n+m/2)}\, T_{x,\gamma_h}(\tau, g^{-m/2}). \tag{15}$$

This indicates that the matched filter based processing involved in the identification of a generalized target description is equivalent to the evaluation of (suitably chosen) sections of the wavelet transform of the impulse response, the analyzing wavelet being chosen as the autocorrelation function of the transmitted signal.

It is easily checked that, in the Altes case, this is an admissible wavelet since

$$c_{\gamma_h} = \sqrt{\pi \ln g} < \infty; \; |H_{n,g,c}(0)|^2 = 0. \tag{16}$$

Moreover, the shape itself of the corresponding wavelet is in accordance with that of Grossmann and Morlet since we have

$$W_\alpha(\omega) = |H_{0,e^{1/\alpha},0}(\omega)|^2. \tag{17}$$

Simple physical arguments can be provided for justifying the close connection which exists between generalized target description and wavelet decomposition.

By construction, the wavelet transform (8) depends on a scale parameter and small scales are associated to fine details in a signal, whereas large scales correspond to a gross characterization. From (15), the considered observation scales a are given by

$$a = g^{-m/2} \tag{18}$$

where m is the order of differentiation or integration. It follows from this that, in the generalized target description picture, small scales are associated to high-order differentiations, whereas large scales correspond to high-order integrations: this is a satisfactory physical interpretation.

Evaluation of the wavelet transform at discrete scale values like (18) (and especially on the dyadic grid associated to $g = 4$) approximates the wavelet coefficients involved in a *discrete* wavelet decomposition of the type [7]

$$x(t) = \sum_{k=1}^{K} \sum_{m=-M}^{M} x_{km}\, \gamma_h(g^{m/2} t - k\Delta\tau) \tag{19}$$

where $\Delta\tau$ is some time sampling rate.

In the case of Altes signals (6), (14) holds, which identifies scale changing operations to differentiation or integration. If their time-bandwidth product is furthermore large, the autocorrelation function is sharply peaked (pulse compression effect) and the discrete wavelet decomposition (19) is an approximation of the generalized target description (2).

Nevertheless, in such discrete cases, the wanted coefficients cannot be exactly obtained from either the matched filter output or the wavelet transform, as given by (15): this comes from the fact that the involved projection filters do not constitute an orthonormal basis. Some postprocessing is therefore necessary and a least squares solution has been proposed [2]. However, exact approaches have been introduced in the wavelet theory [7], [12], and it is believed that they can be useful in the generalized target description case as well.

V. Conclusion

Two different and apparently unrelated approaches, generalized target description and wavelet decomposition, have been considered. In fact, it has been shown that they share important common features concerning both the structure of their privileged analysis tools (transmitted signal or analyzing wavelet) and the way by which they get relevant information on a system under investigation.

The interest in such a comparative perspective is believed to be twofold since 1) it provides new insights in the physical meaning of the wavelet transform by relating it to the extraction of physical target parameters, and 2) it allows generalized target description to make a profit of the mathematical body of knowledge concerning the wavelet transform (cf., e.g., [7], [12]).

References

[1] A. Freedman, "A mechanism of acoustic echo formation," *Acustica*, vol. 12, pp. 10-21, 1962.

[2] R. A. Altes, "Sonar for generalized target description and its similarity to animal echolocation systems," *J. Acoust. Soc. Amer.*, vol. 59, no. 1, pp. 97-105, 1976.

[3] M. Zakharia and J. P. Sessarego, "Sonar target classification using a coherent echo processing," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing (ICASSP'82)*, Paris, France, 1982, pp. 331-334.

[4] R. A. Altes, "Some invariance properties of the wide-band ambiguity function," *J. Acoust. Soc. Amer.*, vol. 53, no. 4, pp. 1154-1160, 1973.

[5] A. Grossmann and J. Morlet, "Decomposition of Hardy functions into square integrable wavelets of constant shape," *SIAM J. Math. Anal.*, vol. 15, no. 4, pp. 723-736, 1984.

[6] R. Kronland-Martinet, J. Morlet, and A. Grossmann, "Analysis of sound patterns through wavelet transforms," *Int. J. Pattern Recognition Artificial Intell.*, vol. 1, no. 2, pp. 273-302, 1987.

[7] I. Daubechies, "The wavelet transform, time-frequency localization and signal analysis," Preprint, AT&T Bell Lab., Murray Hill, NJ, 1987.

[8] D. Gabor, "Theory of communication," *J. IEE*, vol. 93, no. III, pp. 429-457, 1946.

[9] C. W. Helström, "An expansion of a signal in Gaussian elementary signals," *IEEE Trans. Inform. Theory*, vol. IT-12, pp. 81-82, 1966.

[10] P. Flandrin, "Some aspects of non-stationary signal processing with emphasis on time-frequency and time-scale methods," Rapport Interne ICPI TS-8802; also, in *Wavelets*, J. M. Combes, A. Grossmann, and P. Tchamitchian, Eds. Berlin: Springer-Verlag, 1989, pp. 68-98.

[11] —, "Time-frequency and time-scale," in *Proc. IEEE 4th ASSP Workshop Spectrum Estimation and Modeling*, Minneapolis, MN, 1988, pp. 77-80.

[12] I. Daubechies, A. Grossmann, and Y. Meyer, "Painless non-orthogonal expansions," *J. Math. Phys.*, vol. 27, no. 5, pp. 1271-1283, 1986.

A Novel Implementation of a Chirp Z-Transform Using a CORDIC Processor

YU HEN HU AND S. NAGANATHAN

***Abstract*—In this correspondence, an efficient implementation of the chirp Z transform (CZT) using a CORDIC (coordinate rotation digital computer) processor is presented. In particular, it is shown that a scaling operation in the CZT algorithm can be conveniently implemented with a *norm correction (normalization)* computation, which is often considered as an overhead in the CORDIC algorithm. Furthermore, since the desired frequencies of CZT are specified before computation, it is possible to reduce the total number of CORDIC iterations by finding a most economic representation of the angle in terms of the elementary CORDIC rotation angles. A simple suboptimal solution is proposed to solve this difficult optimization problem. This implementation is most effective when very few complex frequencies on the Z plane are to be evaluated via CZT.**

I. Introduction

In the formulation of several orthogonal transformation-based digital signal processing algorithms such as fast (discrete) Fourier transformation and QR factorization for least square estimation, a major portion of operations involves the rotation of a two by one vector through a certain angle [1]. Such an operation admits an efficient hardware implementation using a rotation-based arithmetic algorithm known as CORDIC [2]-[3]. Recently, a number of research efforts have been made to design VLSI (very large scale integration)-based pipelined CORDIC array processors circuits [5]-[8] for highly concurrent computation. For example, the FFT can easily be implemented with a CORDIC processor [9] since each butterfly computation is essentially a rotate-and-accumulate operation. In this correspondence, the implementation of the chirp Z-transformation algorithm [4] using a CORDIC processor is considered.

The chirp Z-transform [4] evaluates the Z transformation of a discrete sequence $\{x(n)\}$ at the points $z_n = AW^{-m}$ for $m = 0, 1, \cdots, M-1$ where $W = W_0 e^{-j\phi_0}$ and $A = A_0 e^{j\theta_0}$ with W_0, A_0, ϕ_0, and θ_0 being real numbers. That is,

$$X(z_m) = \sum_{n=0}^{N-1} x(n) z_m^{-n} = \sum_{n=0}^{N-1} x(n) A^{-n} W^{mn}. \quad (1)$$

Traditionally, CZT may be evaluated using fast Fourier transformation using $O((N + M - 1) \log_2 (N + M - 1))$ operations [4]. However, when only a few chirp frequencies are needed ($M \ll N$), it would be more efficient to evaluate (1) directly using a recurrence formulation:[1] for $n = 0$ to $N - 1$,

$$X(z_m) = x(0) + A^{-1}W^m\big[x(1) + A^{-1}W^m\big[x(2) \cdots + A^{-1}W^m\big[x(N-2) + A^{-1}W^m x(N-1)\big] \cdots \big]\big]. \quad (2)$$

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[1]The numerical properties of (1) and (3) are different. However, a thorough numerical analysis on using (3) with a CORDIC processor is beyond the scope of this correspondence.

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