On EMD components

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[1] joint work with Gabriel Rilling

What is a signal component ?

Three main approaches to signal decomposition and/or representation :

1. parametric (ARMA modeling, . . .)
2. non-parametric (Fourier, wavelets, Wigner, . . .)
3. data driven (as above + adaptivity, PCA, ICA, EMD)

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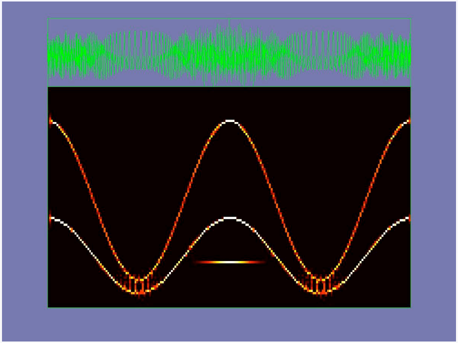
Time-frequency based components

1. non-parametric, transform-based approaches :
   - switching from physical space to some redundant transform domain "offers" to the data the possibility of getting organized
   - components "emerge" as separable, coherent structures
2. time-frequency mostly adapted to AM-FM waveforms



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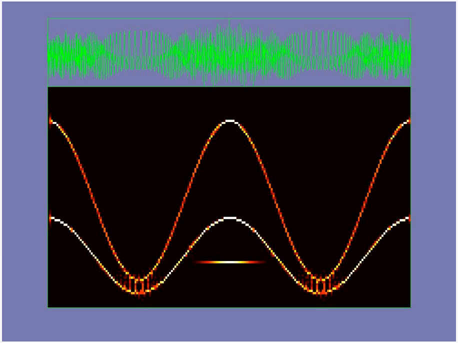

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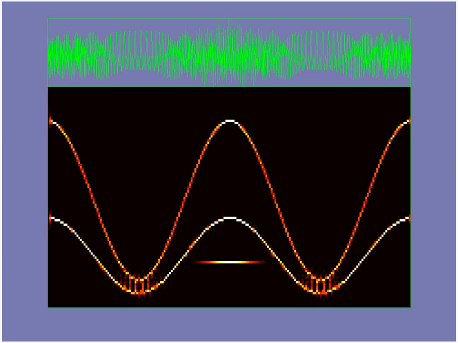

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1. no unique definition :
   - no exact TF disjointness ("uncertainty principle")
   - signal-based criterion of local bandwidth [Cohen, IEEE-ICASSP'92]
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   - signal/measurement interaction
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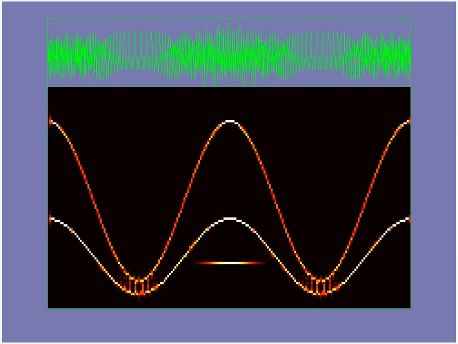
Intrinsic Mode Functions

1. minimal assumption on the existence of AM-FM-like components
2. data-driven extraction from fine to coarse by removing iteratively the locally fastest oscillation



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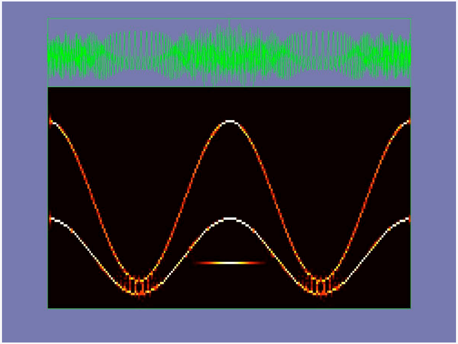

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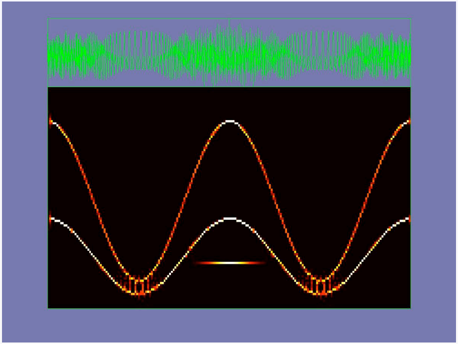

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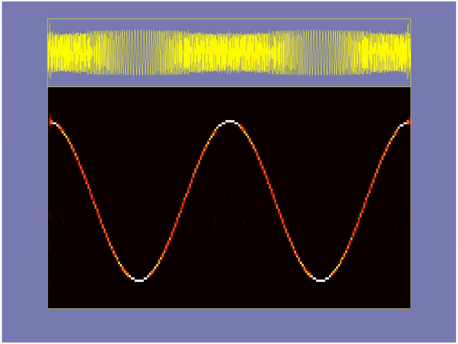

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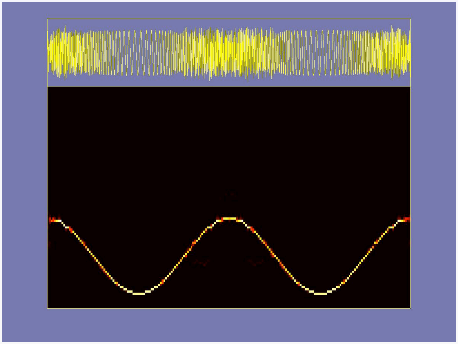

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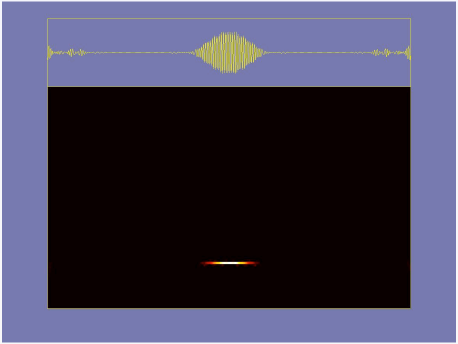

Intrinsic Mode Functions

1. intuitively appealing
2. really intrinsic ? result may depend on the sifting operator $\mathcal{S} : x(t) \mapsto \mathcal{S}x(t)$ (interpolation scheme), boundary conditions, stopping criterion, . . .
3. lack from basic invariances ▸ counter-example

Call for a quantitative evaluation in simple, yet significant, well-controlled situations

$\Rightarrow$ thorough study of the two-tones case

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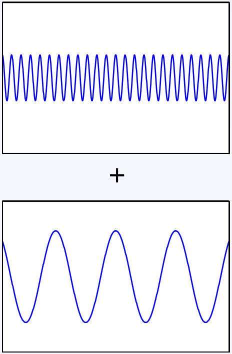

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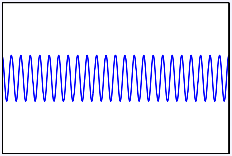

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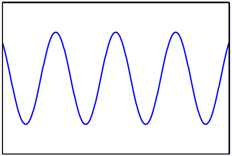

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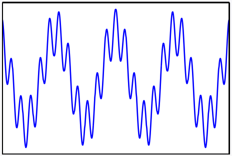

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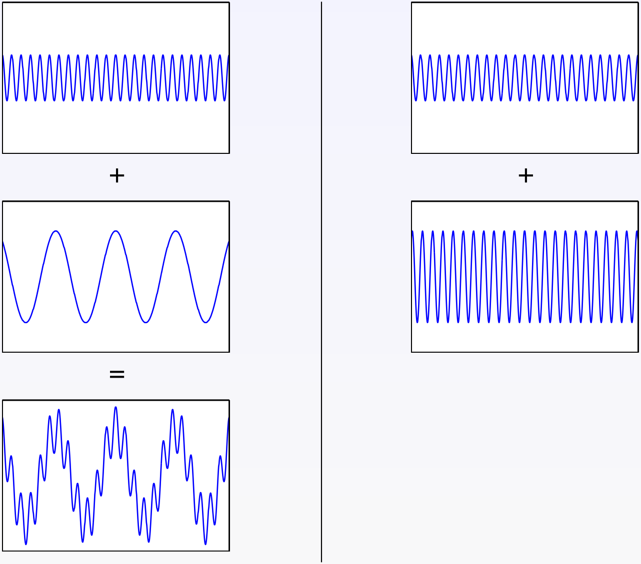

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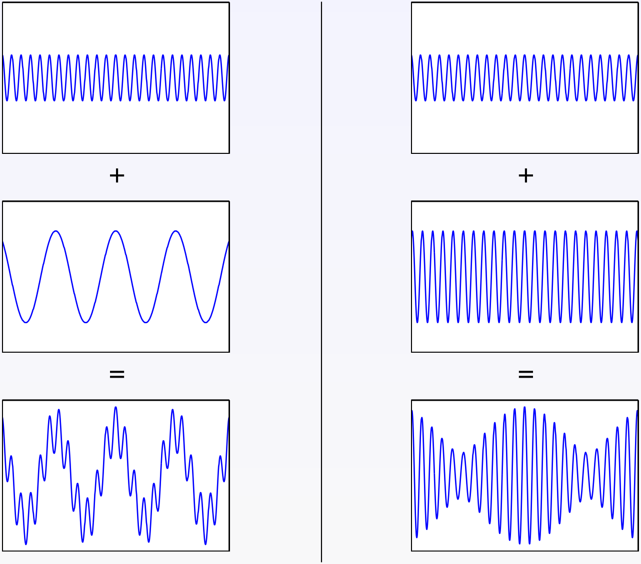

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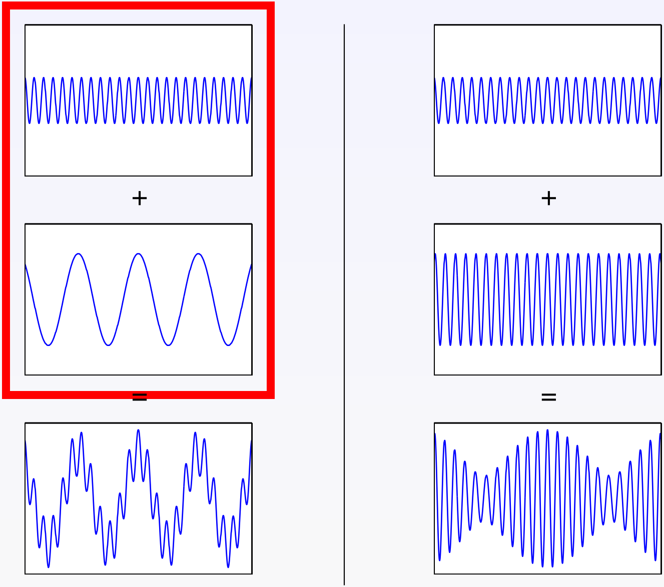

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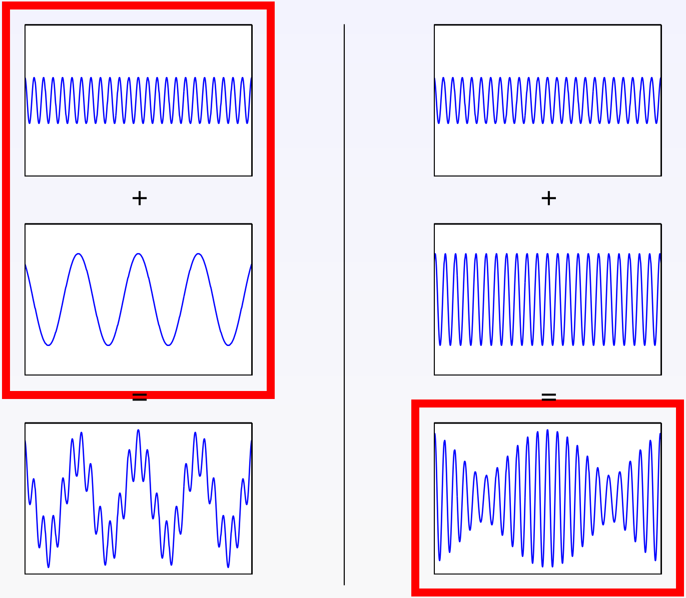

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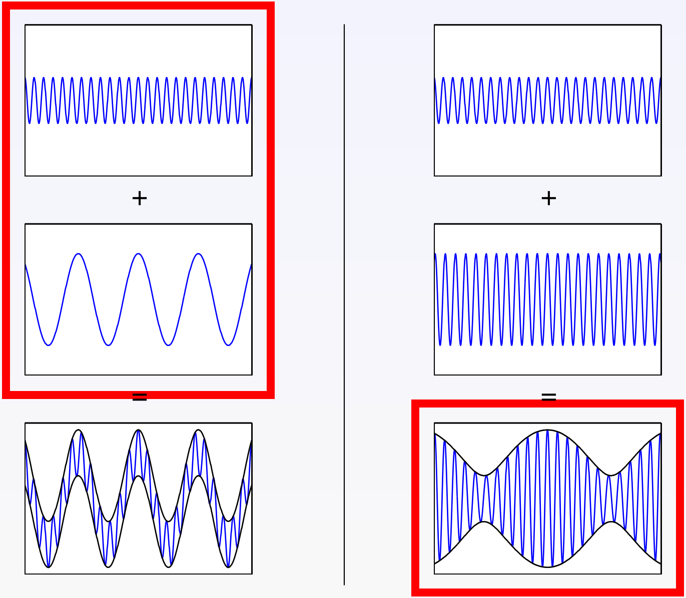

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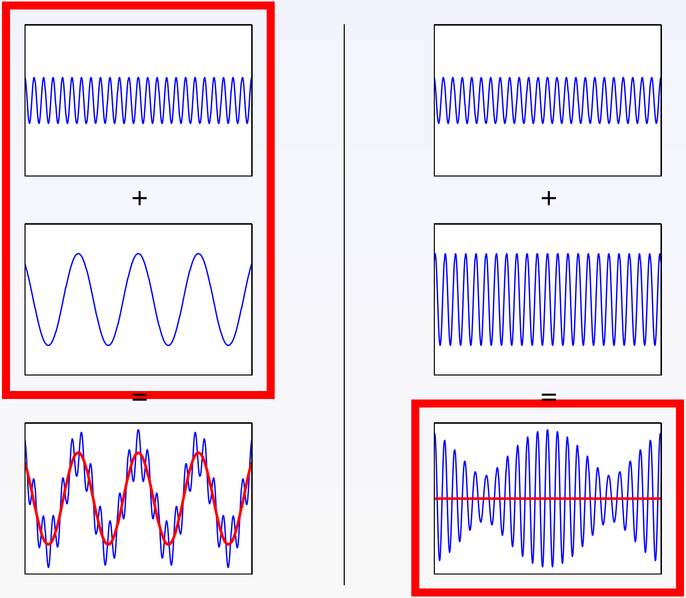

Simulations

Signal

$$x(t) = \underbrace{a_1 \cos(2\pi f_1 t)}_{x_1(t)} + \underbrace{a_2 \cos(2\pi f_2 t + \varphi)}_{x_2(t)}, \quad f_1 > f_2$$

Analysis of its EMD

- only the first IMF is computed : if separation, it should be equal to the highest frequency component $x_1(t)$
- criterion ($= 0$ if separation) :

$$c\left(\frac{a_2}{a_1}, \frac{f_2}{f_1}, \varphi\right) = \frac{\|IMF_1(t) - x_1(t)\|_{\ell_2}}{\|x_2(t)\|_{\ell_2}}$$

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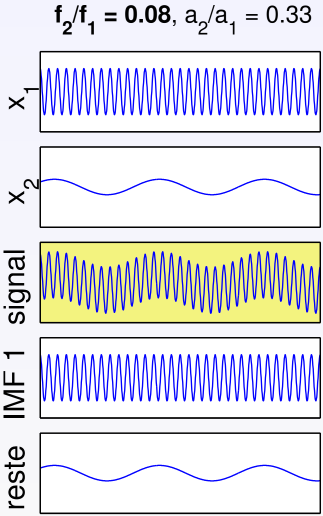

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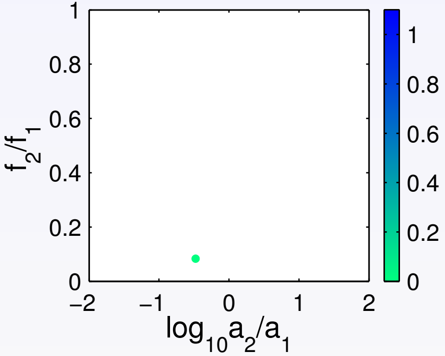

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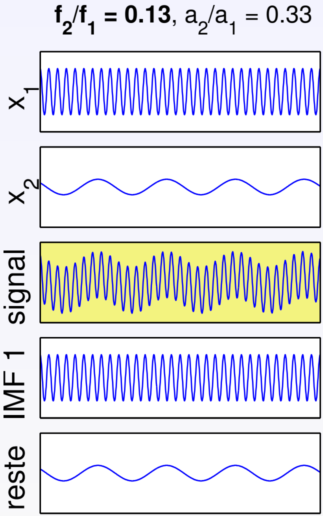

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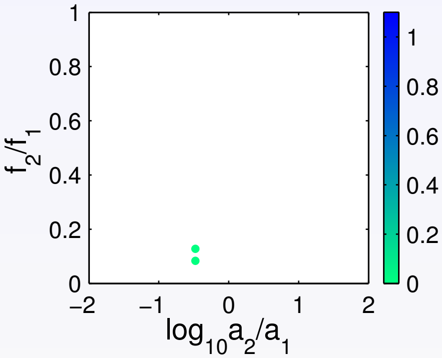

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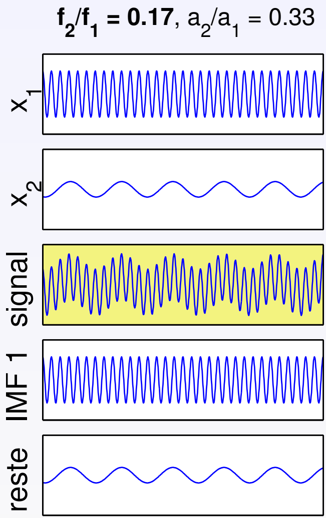

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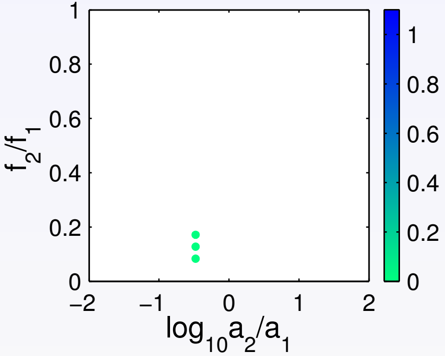

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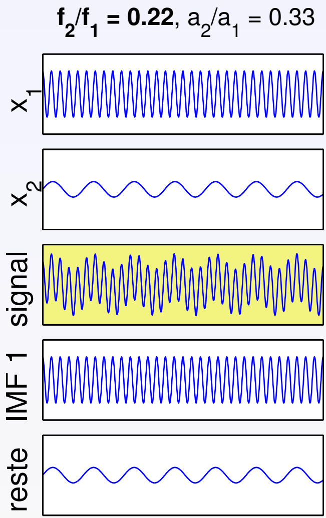

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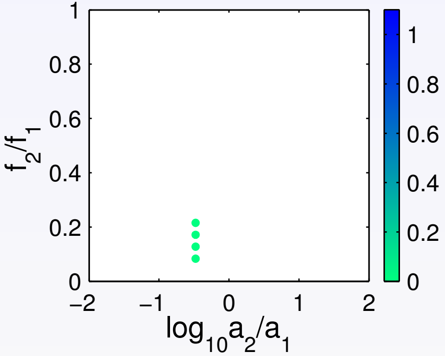

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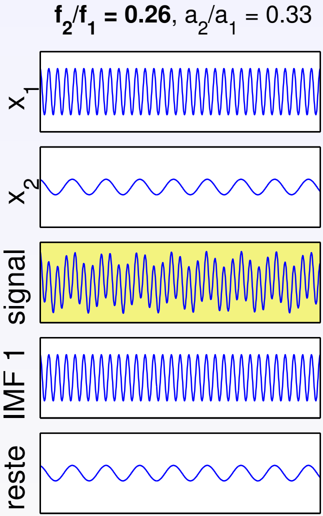

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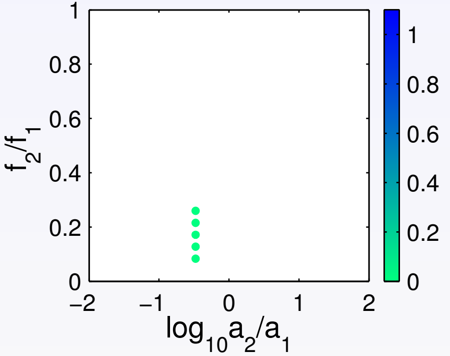

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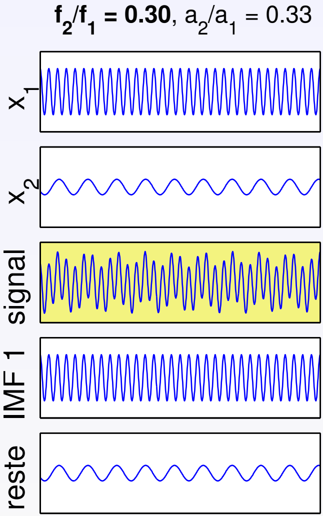

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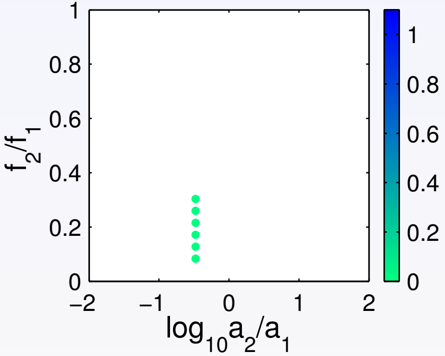

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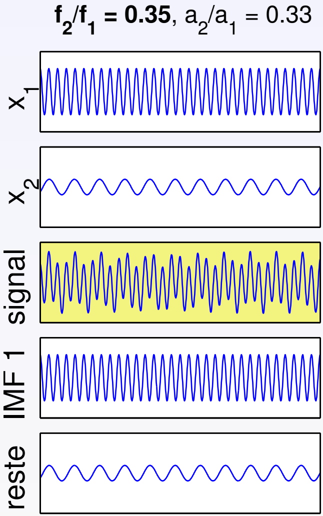

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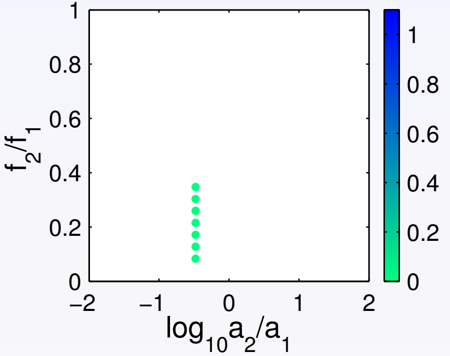

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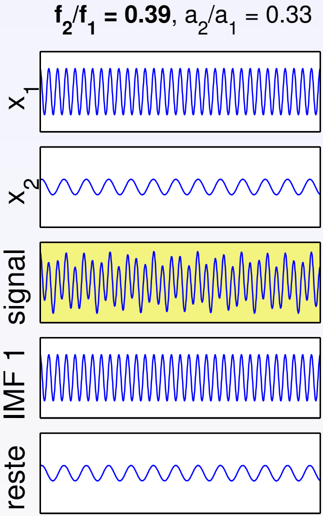

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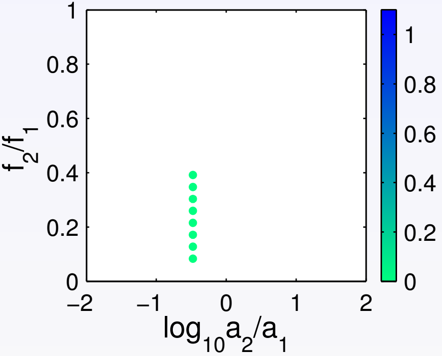

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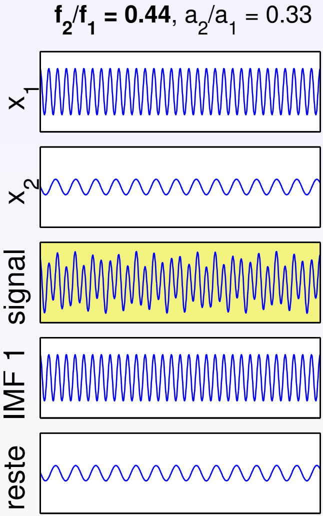

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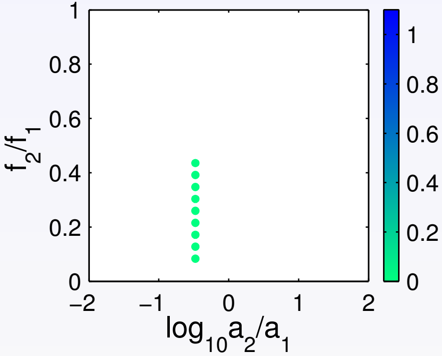

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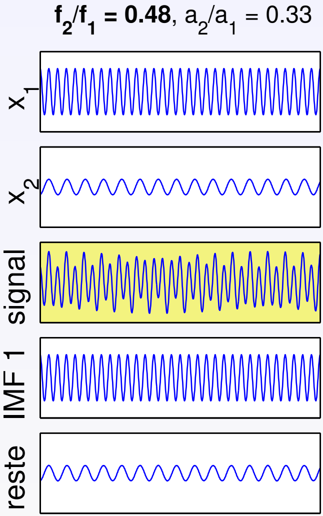

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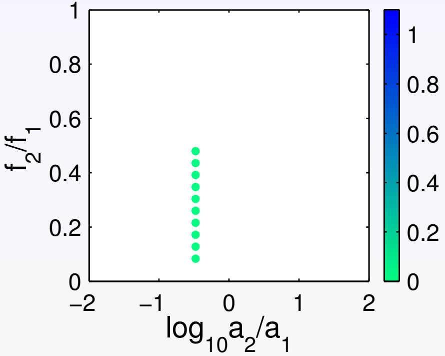

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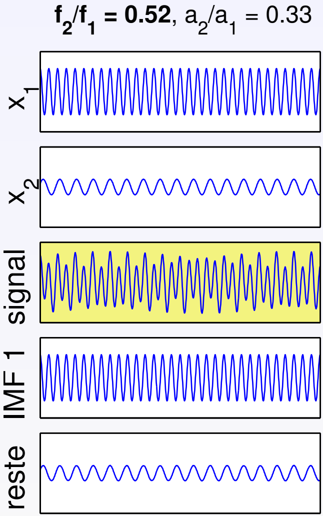

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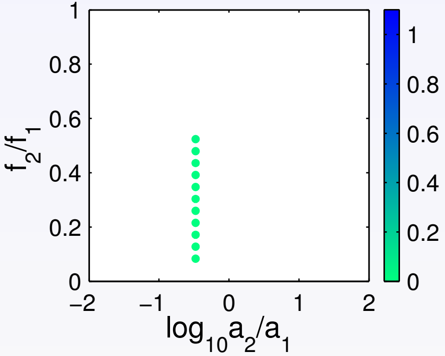

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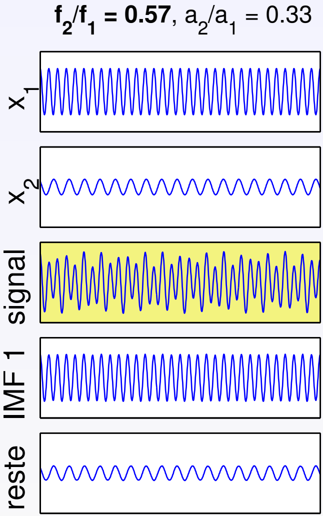

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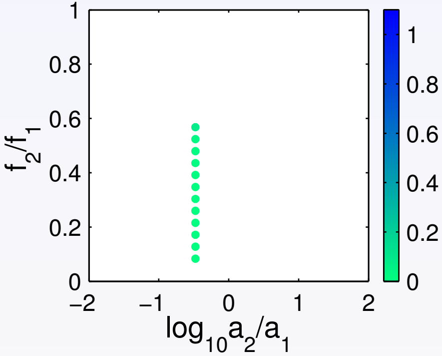
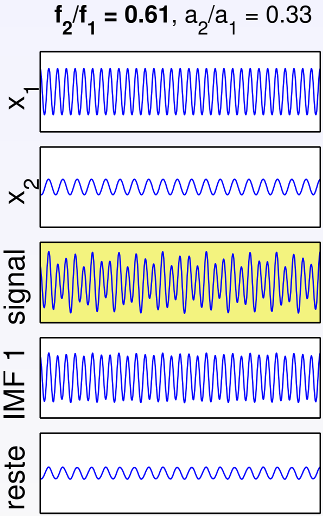
$$= 0 \quad \text{if separation}$$



Sum of two tones



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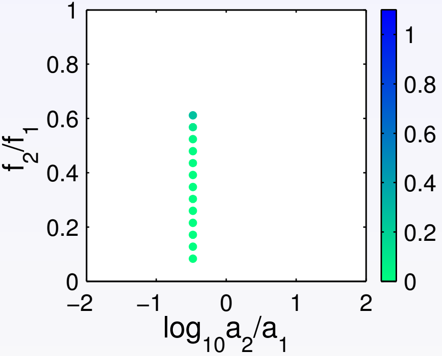

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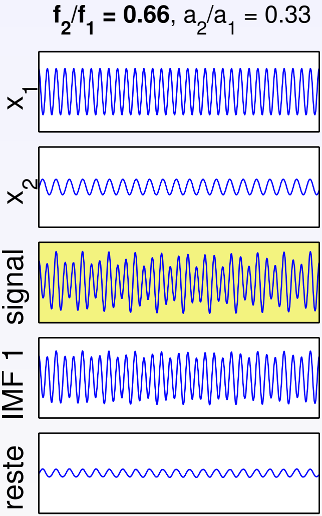

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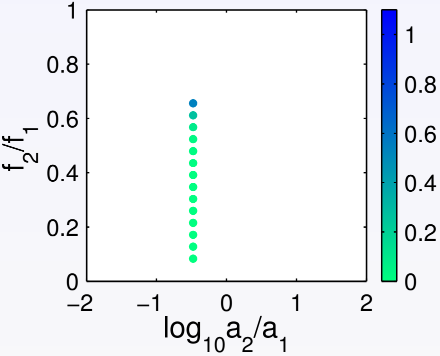

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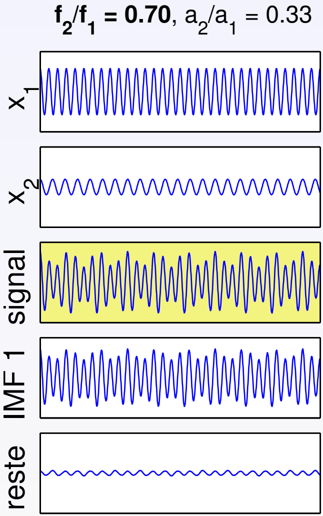

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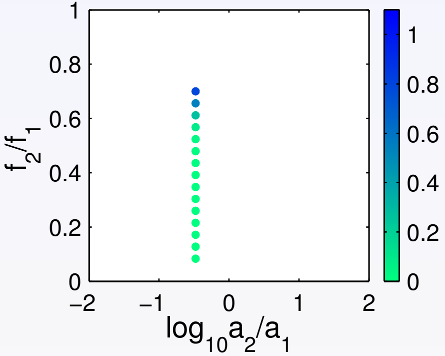

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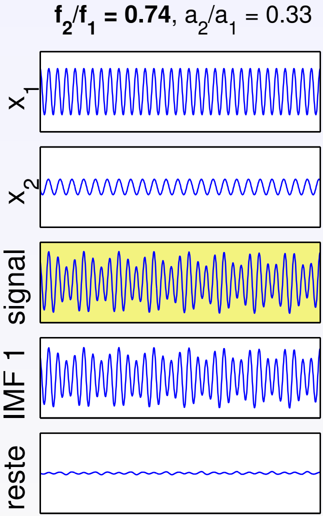

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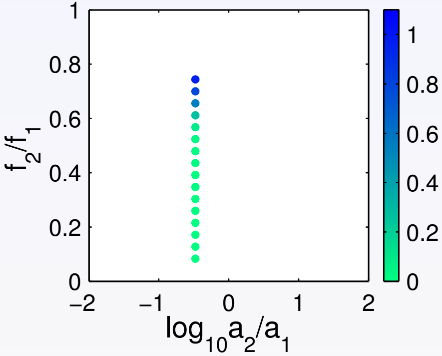

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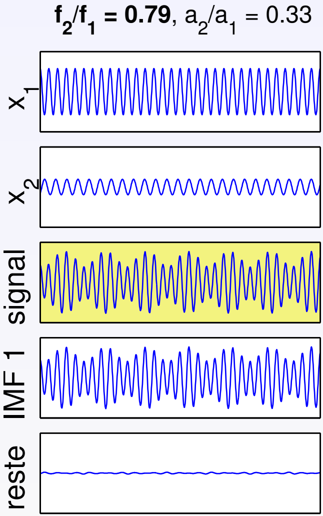

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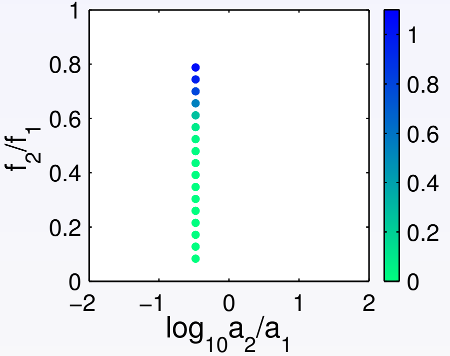

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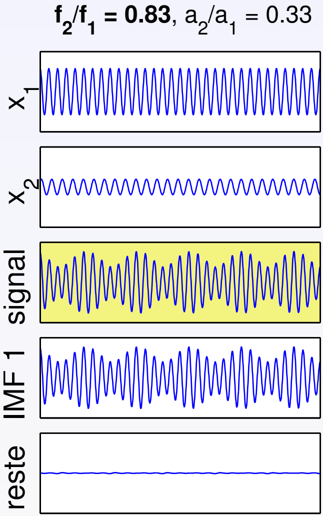

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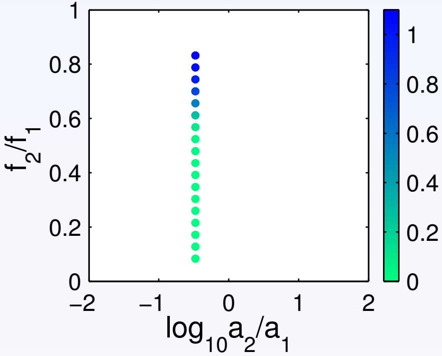

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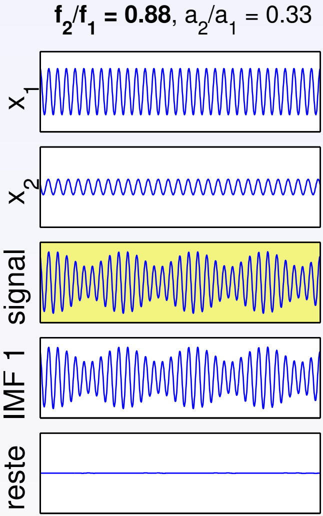

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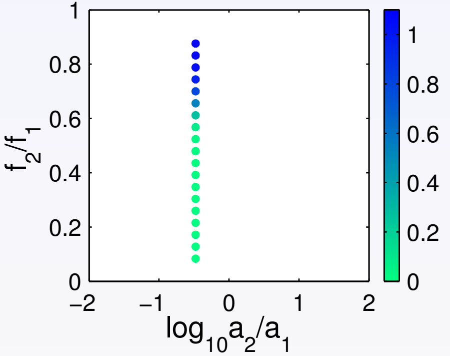
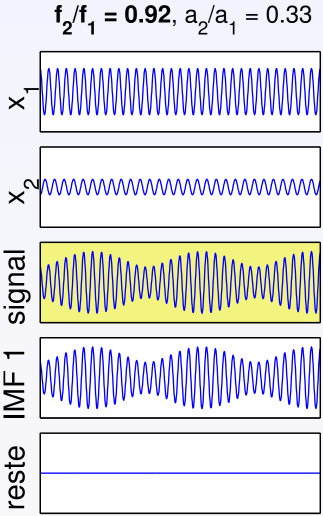


Sum of two tones



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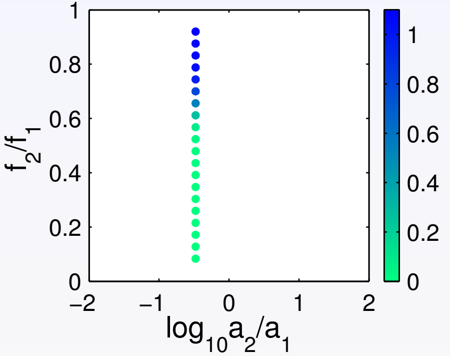

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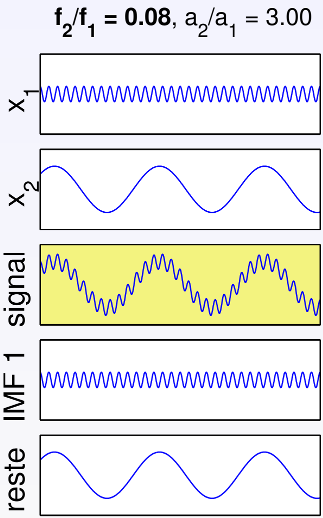

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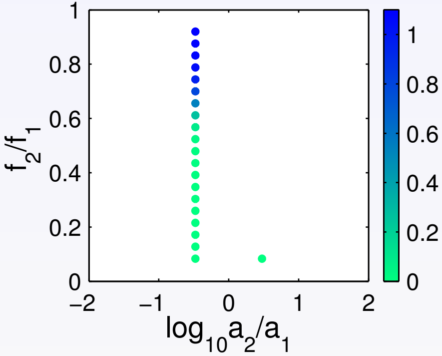

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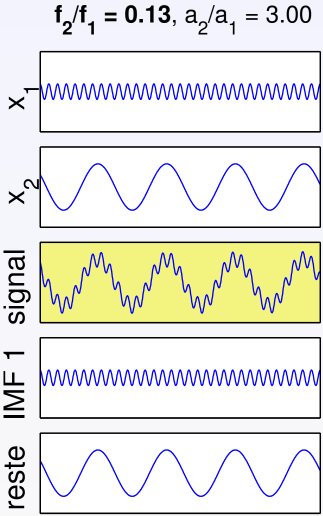

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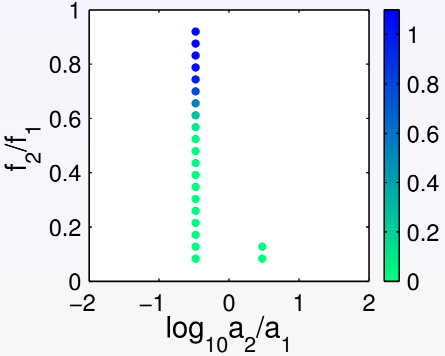

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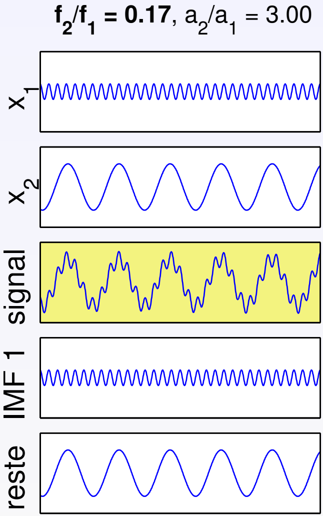

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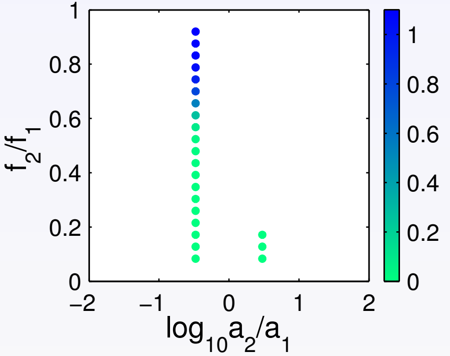

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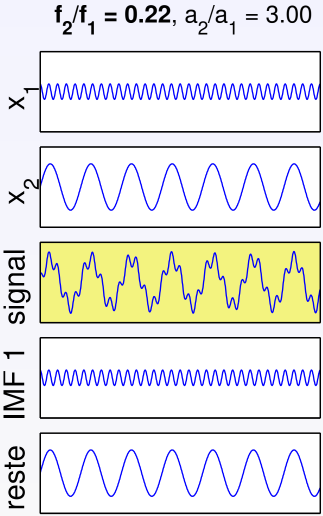

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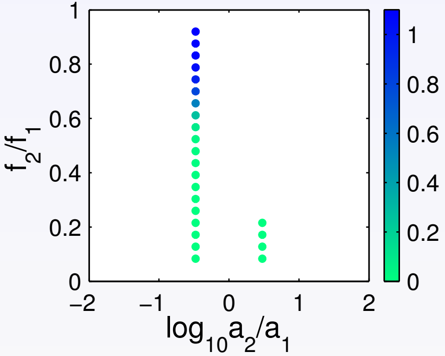

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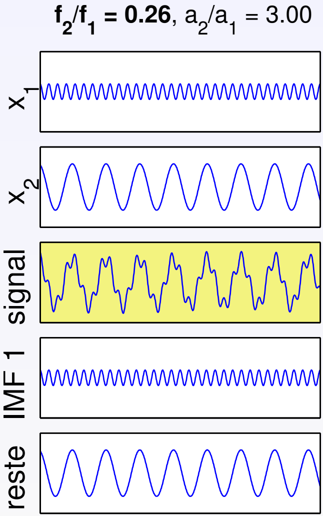

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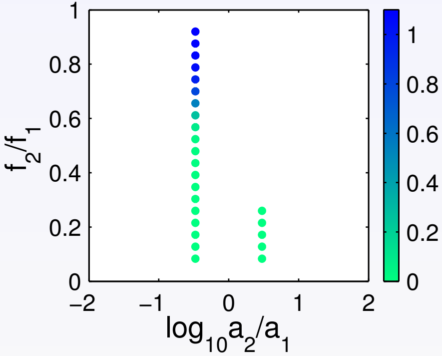

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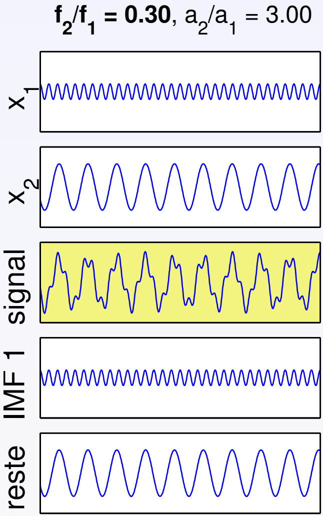

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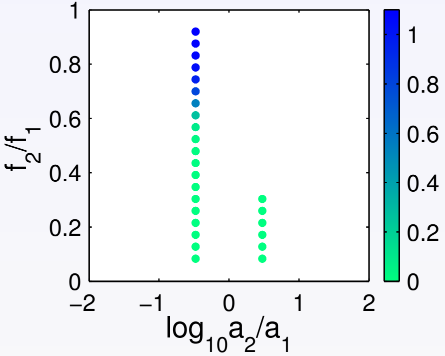

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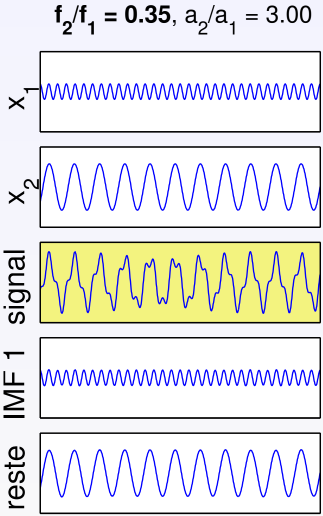

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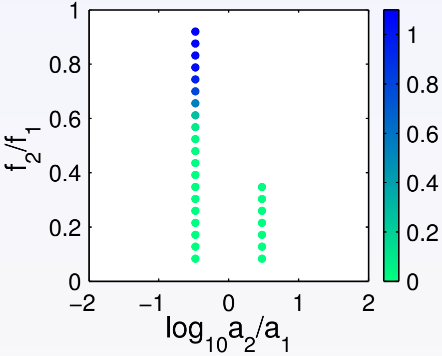

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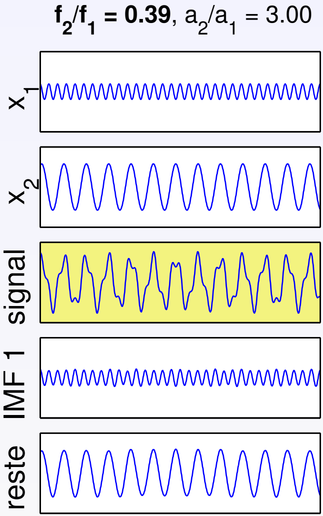

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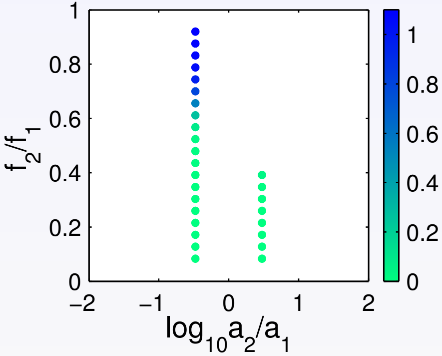

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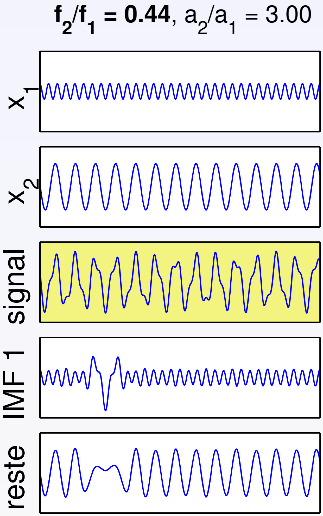

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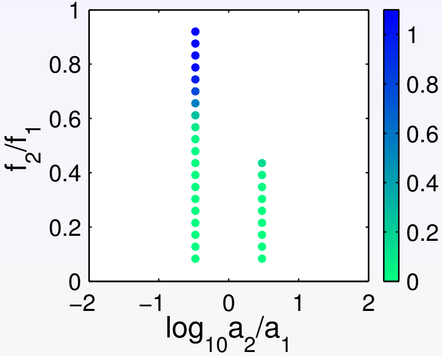

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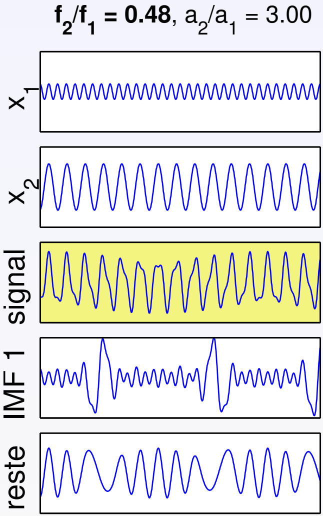

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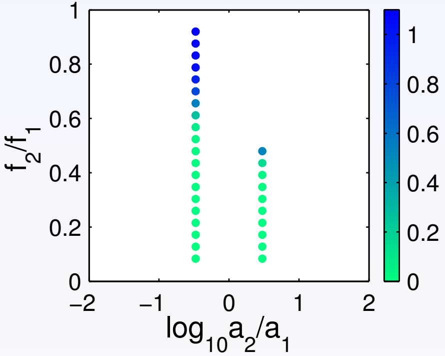

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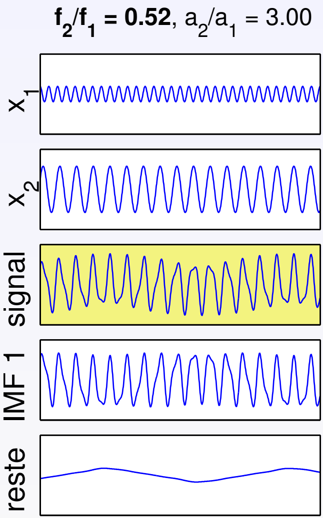

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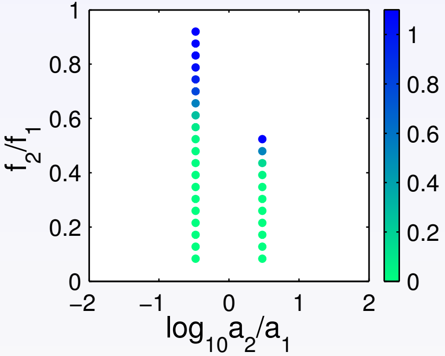

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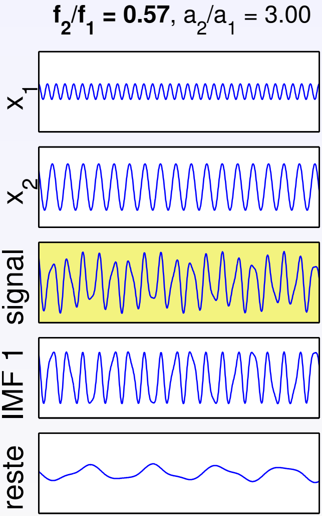

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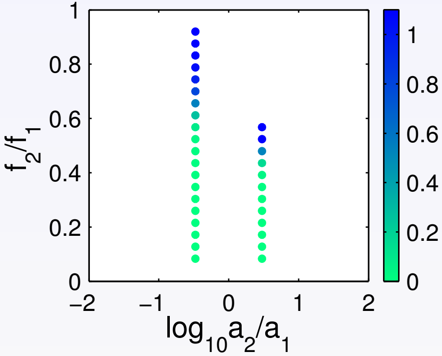

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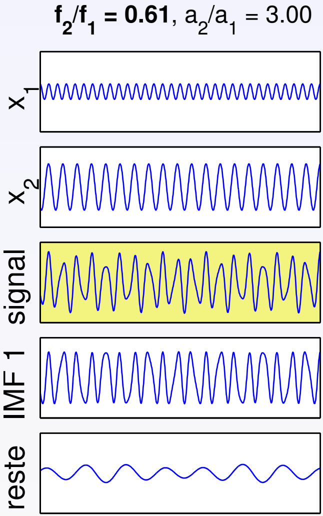

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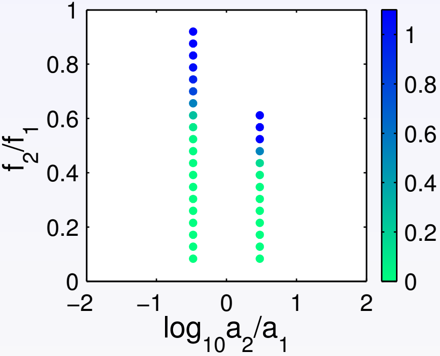

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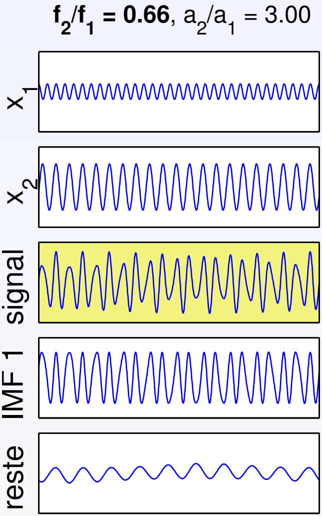

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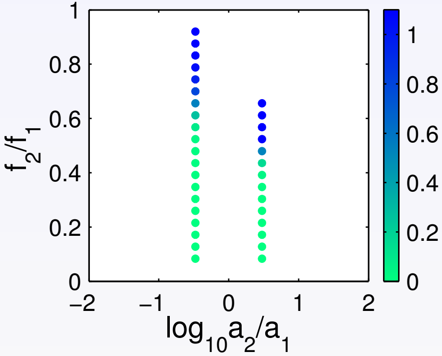

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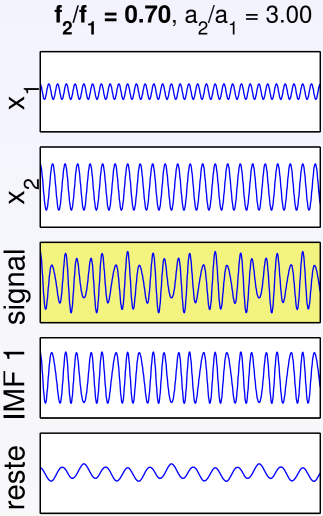

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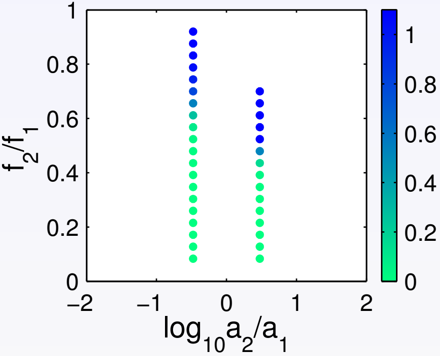

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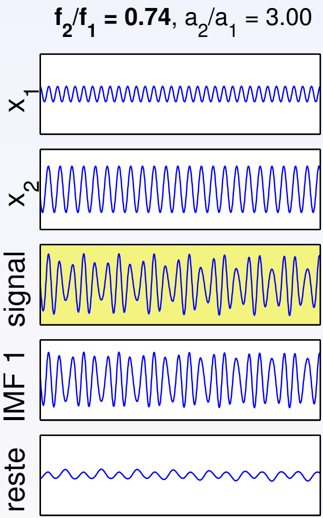

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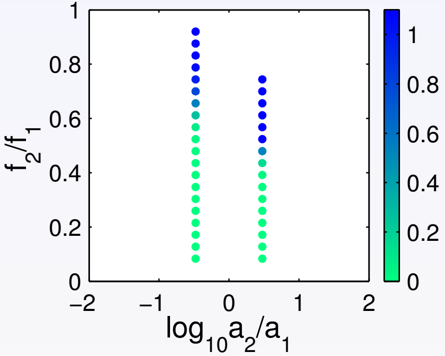

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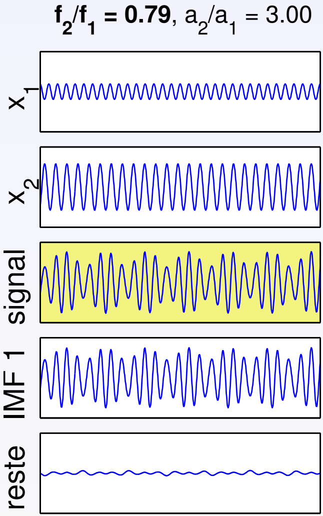

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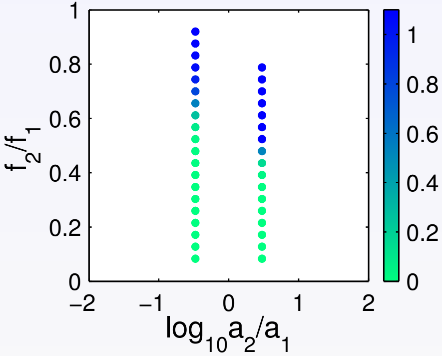

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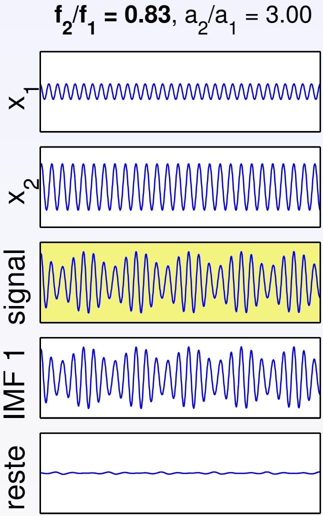

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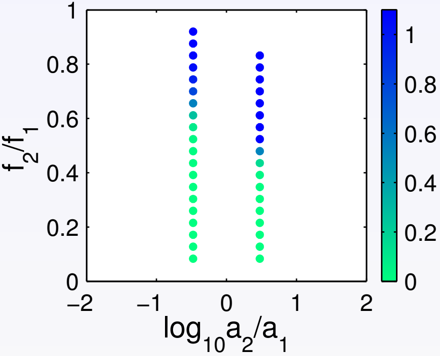

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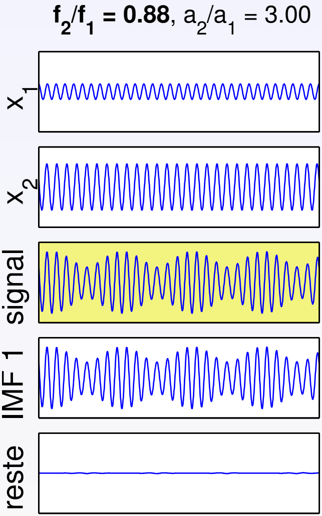

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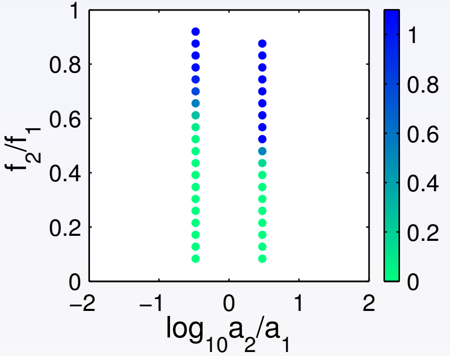
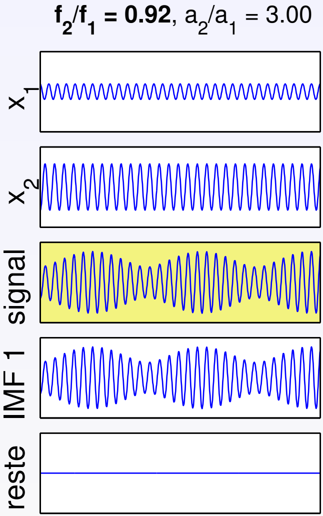


Sum of two tones



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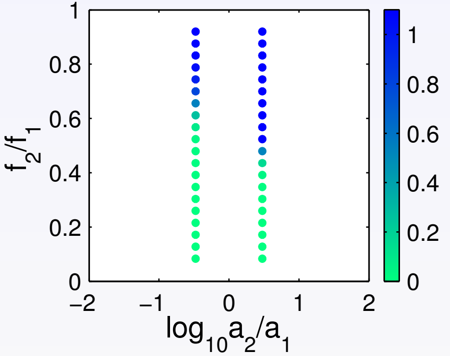
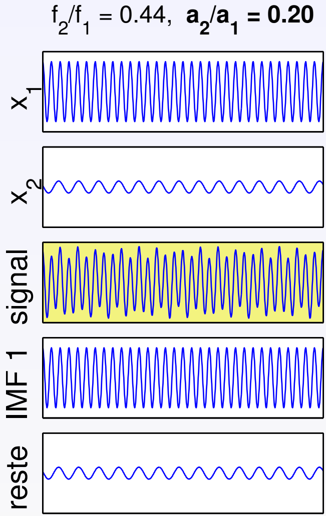


Sum of two tones



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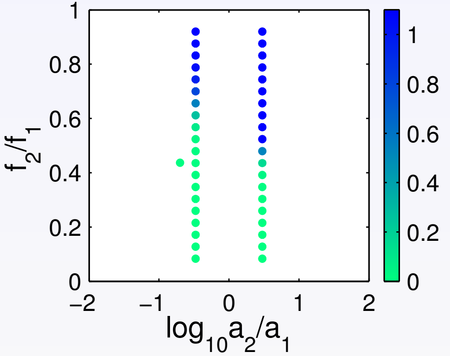
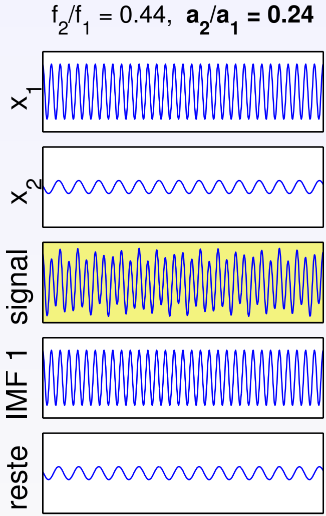


Sum of two tones



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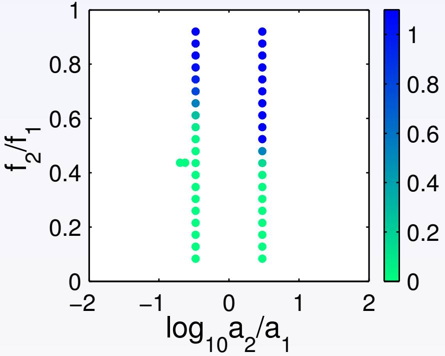
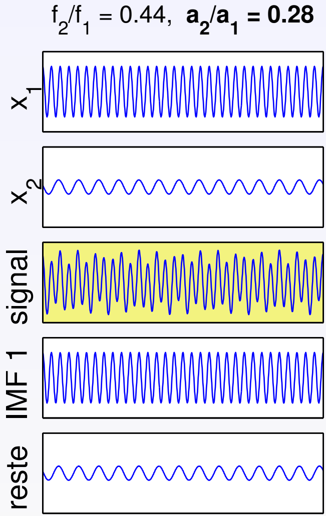


Sum of two tones



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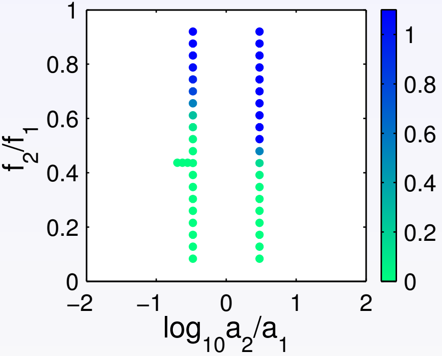
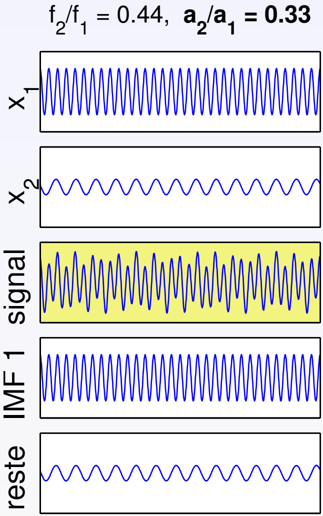


Sum of two tones



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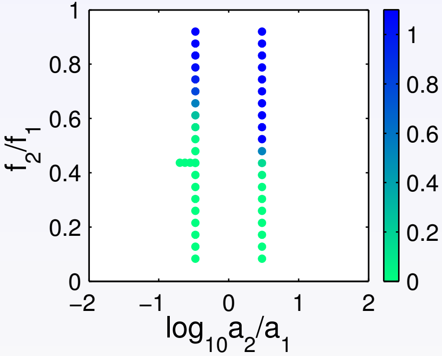
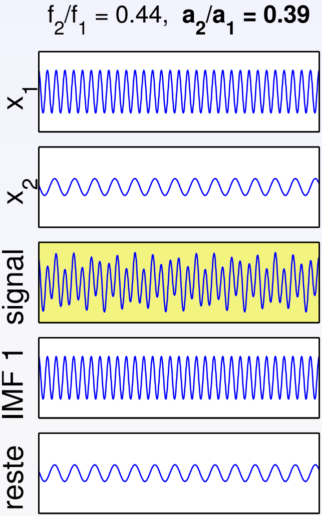


Sum of two tones



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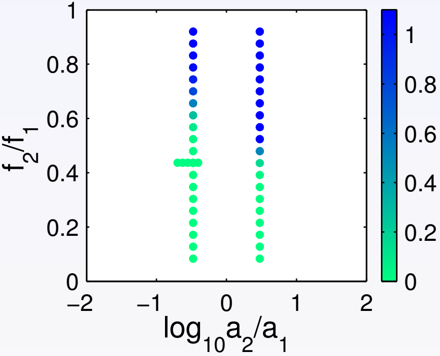

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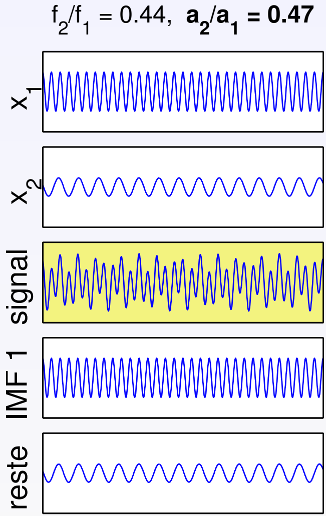

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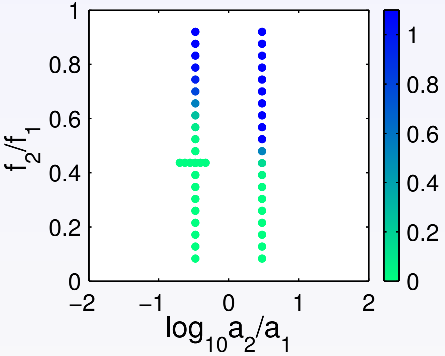
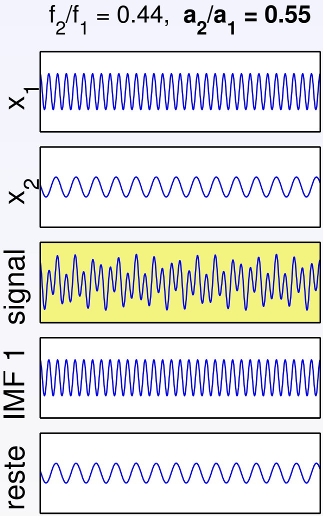


Sum of two tones



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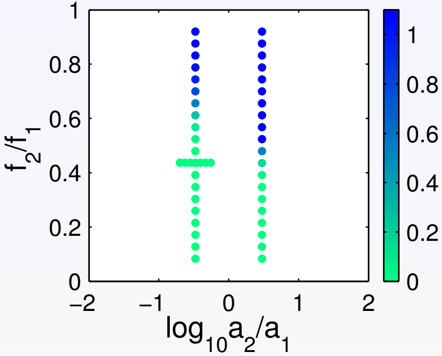
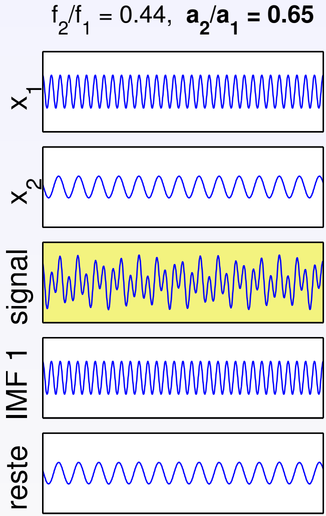


Sum of two tones



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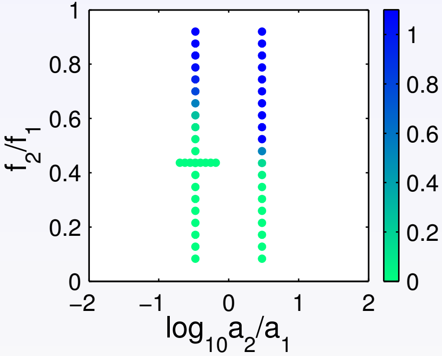
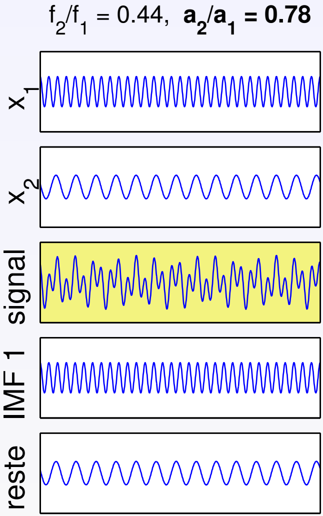


Sum of two tones



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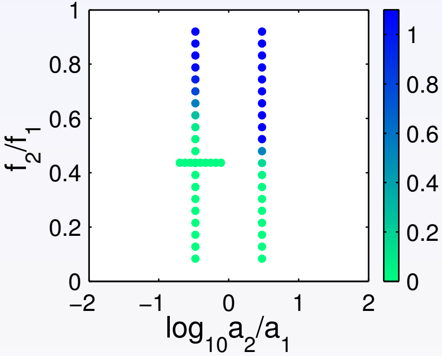
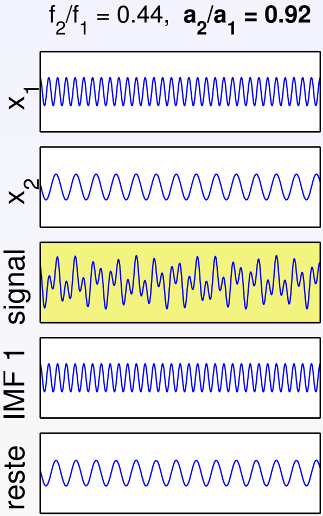


Sum of two tones



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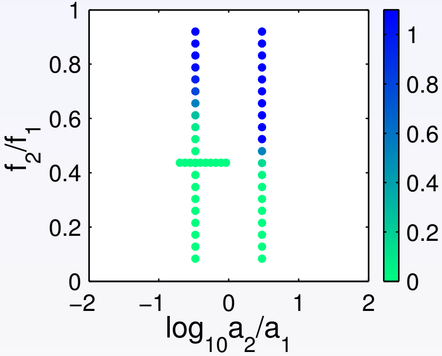
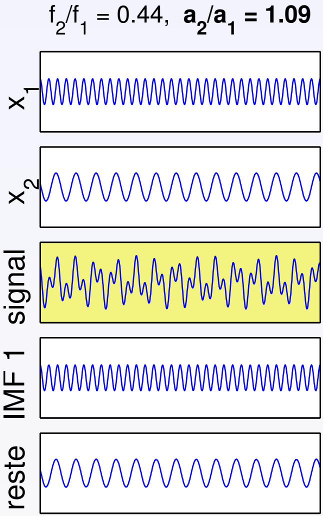


Sum of two tones



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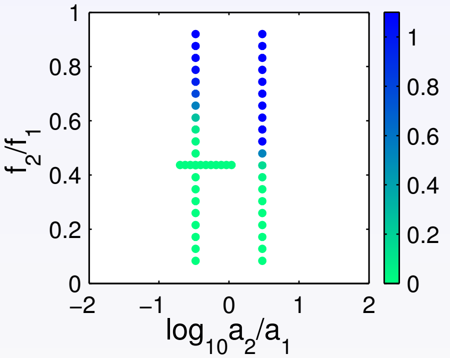

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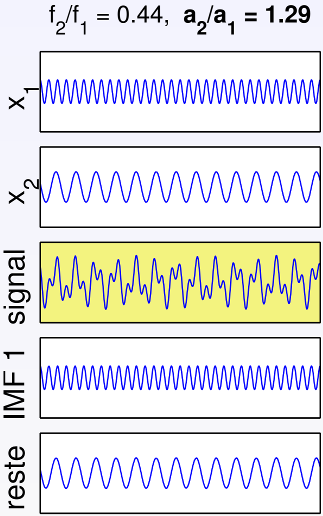

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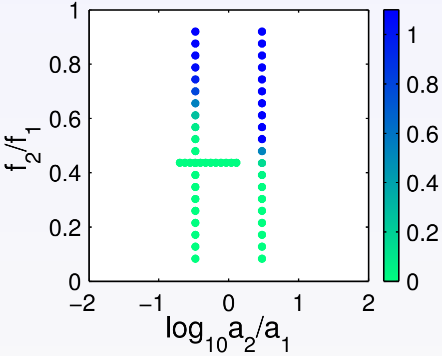
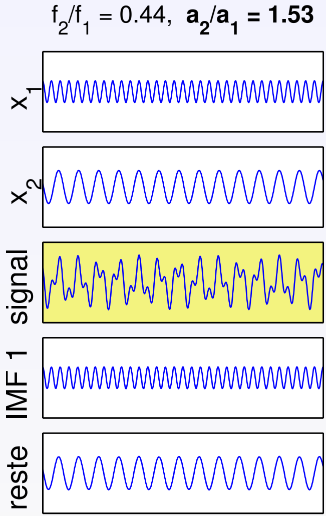


Sum of two tones



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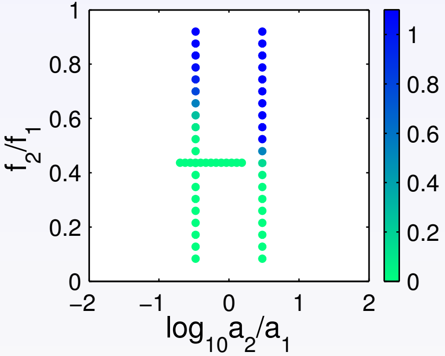
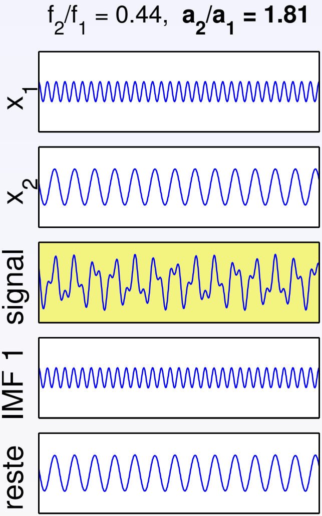


Sum of two tones



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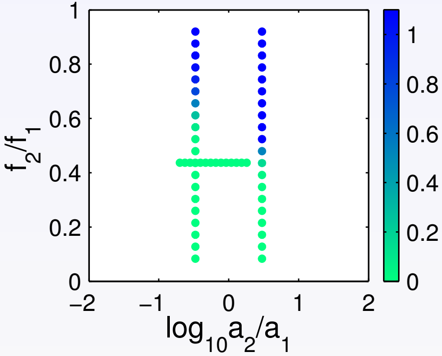
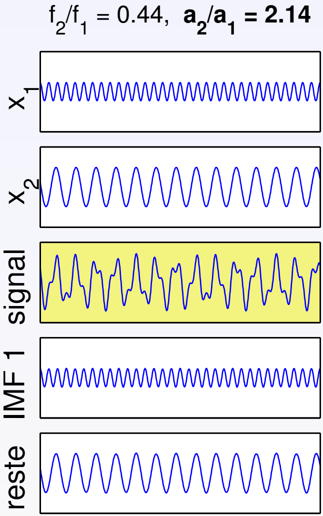


Sum of two tones



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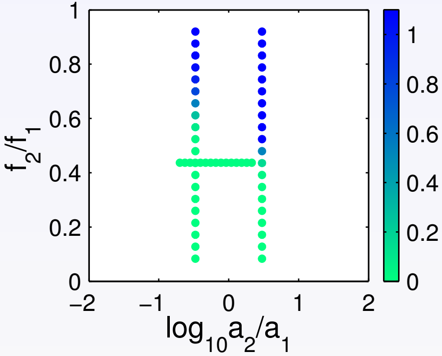
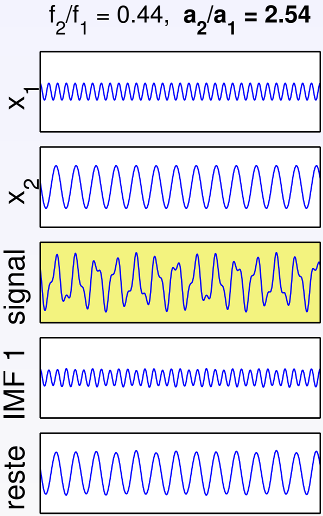


Sum of two tones



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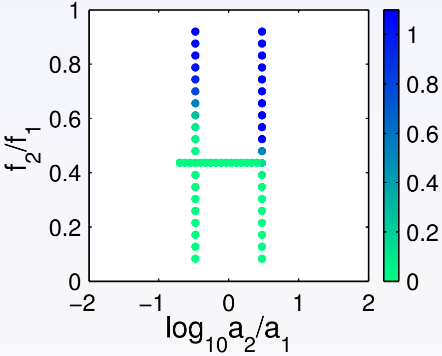
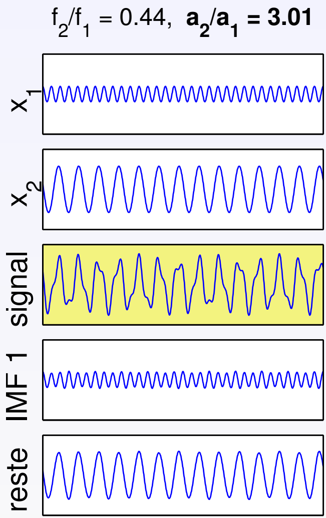


Sum of two tones



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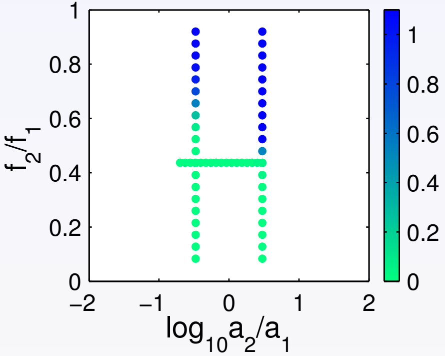
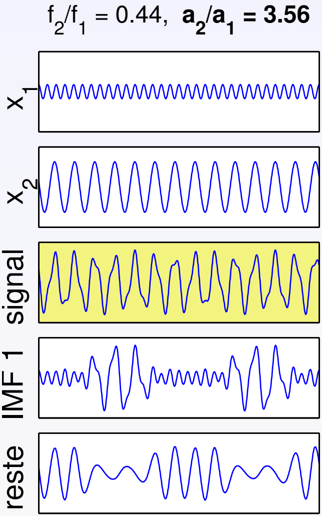


Sum of two tones



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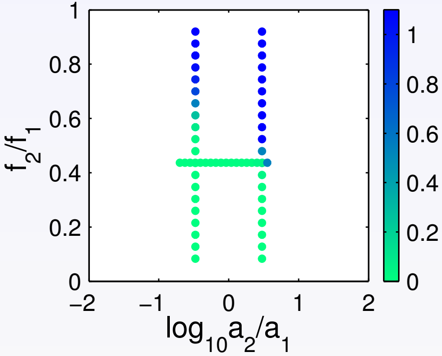
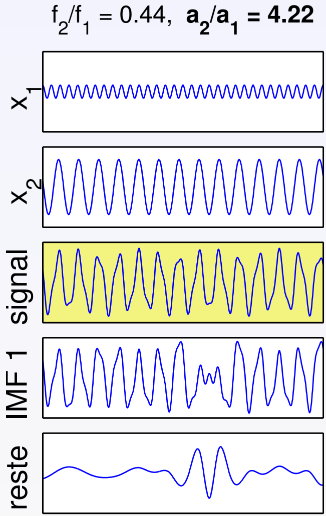


Sum of two tones



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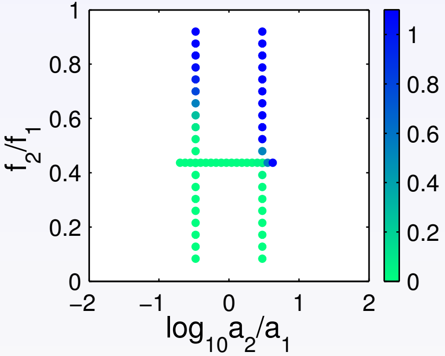

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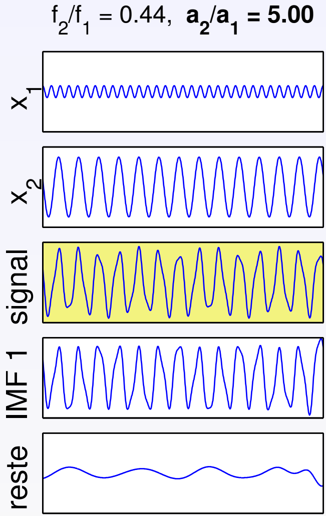

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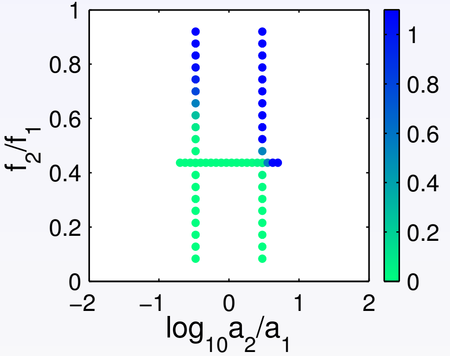
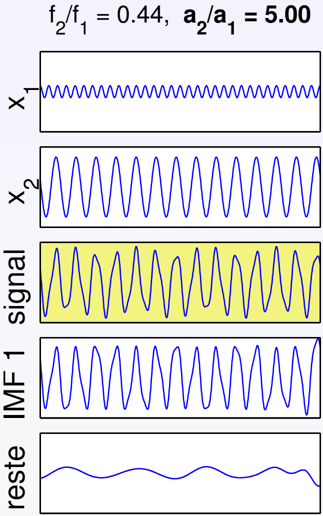


Sum of two tones



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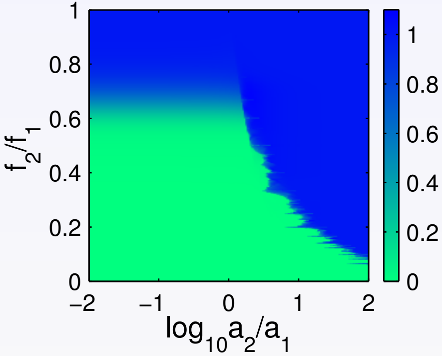
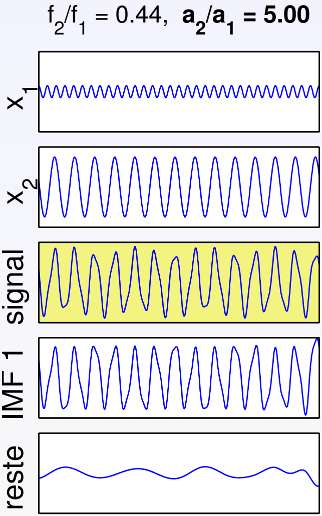


Sum of two tones



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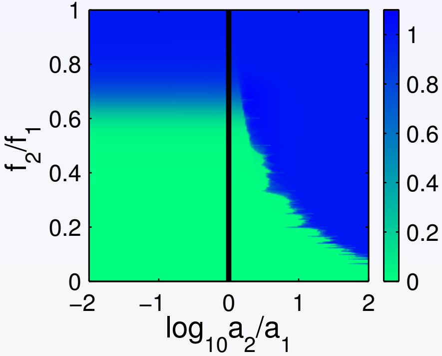

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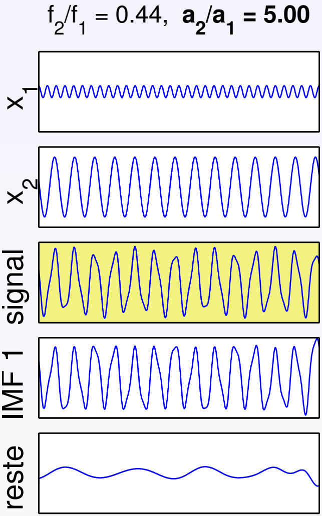

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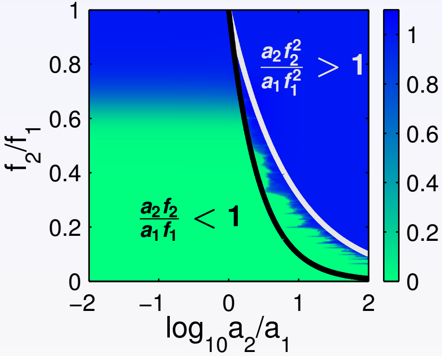
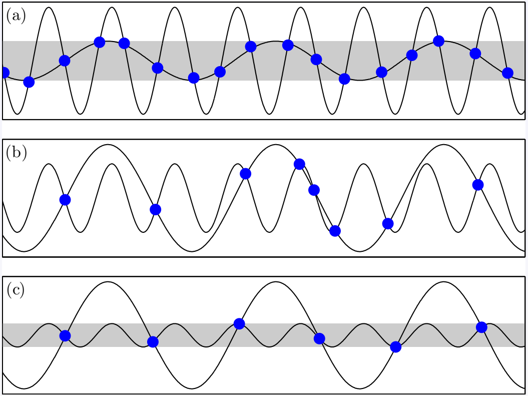


Three regimes

extrema of HF + LF as $\partial_t \mathrm{HF} \cap -\partial_t \mathrm{LF}$



(a) $a_2 f_2 < a_1 f_1$; (b) $a_2 f_2 > a_1 f_1$ & $a_2 f_2^2 < a_1 f_1^2$; (c) $a_2 f_2^2 > a_1 f_1^2$

Case of regularly spaced extrema

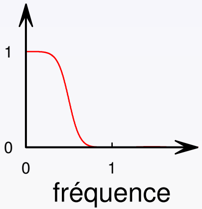
selection of extrema = uniform sampling

$$\sum_{n\in\mathbb{Z}} \delta(t - nT) \quad \xrightarrow{\mathcal{FT}} \quad \frac{1}{T}\sum_{n\in\mathbb{Z}} \delta\left(\nu - \frac{n}{T}\right)$$

interpolation = linear filtering

transfer function of cubic spline interpolator for unit spaced samples

$$\hat{I}(\nu) = \left(\frac{\sin \pi\nu}{\pi\nu}\right)^4 \frac{3}{2 + \cos 2\pi\nu}$$



Case 1 : $a_2 f_2 < a_1 f_1$

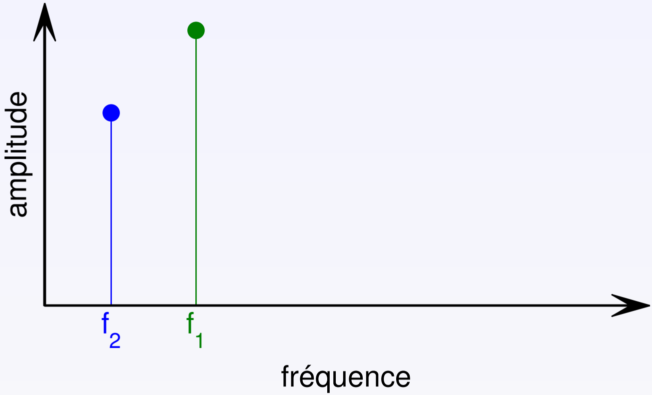
Model

$$\widehat{\mathcal{S}x}(\nu) = \hat{x}(\nu) - \hat{l}\left(\frac{\nu}{f_1}\right)\left(\sum_{n\in\mathbb{Z}} \hat{x}(\nu + 2nf_1)\right)$$

Properties

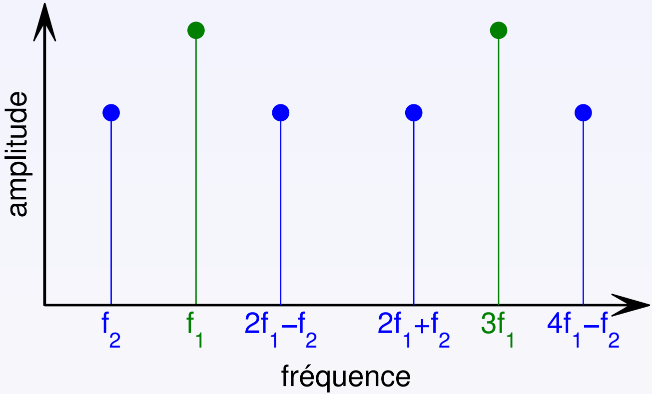
- mixture of « simple » elements :
  sampling and linear filtering
- fixed linear model for the sampling operator

Case 1 : $a_2 f_2 < a_1 f_1$



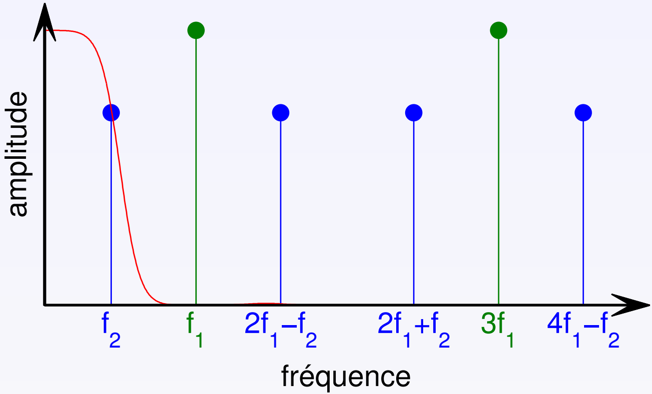
$\hat{x}(\nu)$

Case 1 : $a_2 f_2 < a_1 f_1$



$$\hat{x}(\nu) * \sum_{n \in \mathbb{Z}} \delta(\nu - 2nf_1)$$

Case 1 : $a_2 f_2 < a_1 f_1$



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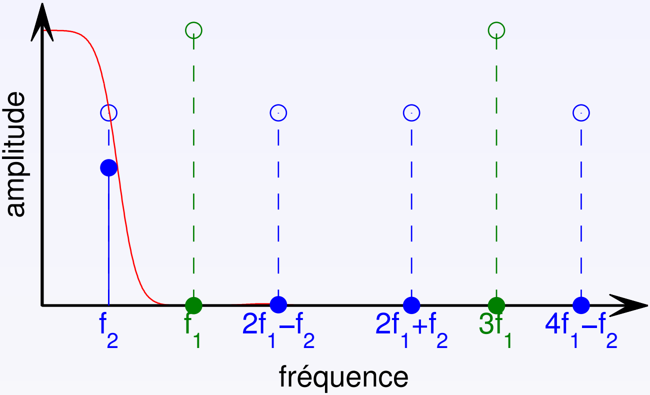

$$\hat{l}\left(\frac{\nu}{f_1}\right) \cdot \left(\hat{x}(\nu) * \sum_{n\in\mathbb{Z}} \delta\left(\nu - 2nf_1\right)\right)$$

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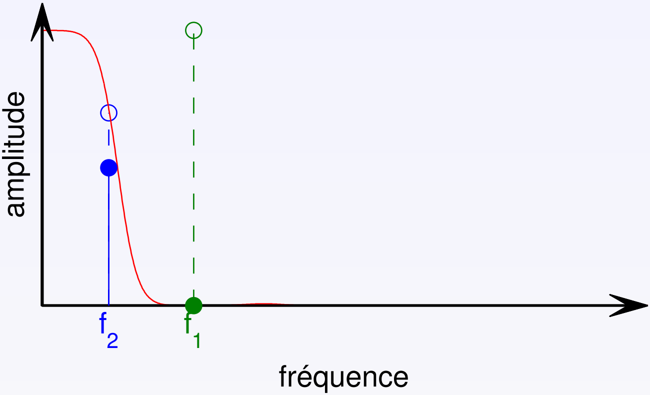

$$\hat{I}\left(\frac{\nu}{f_1}\right) \quad \cdot \quad \hat{x}(\nu)$$

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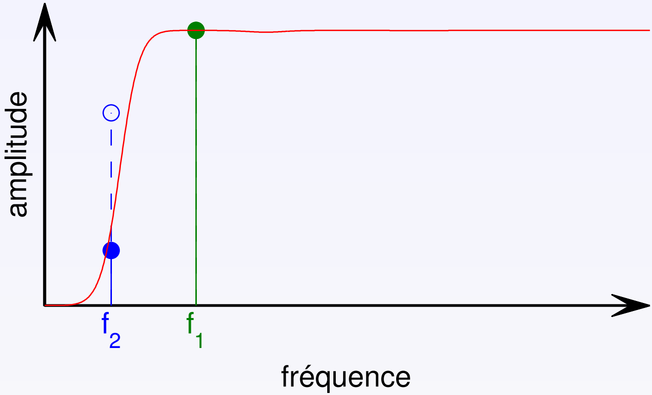

$$\widehat{\mathcal{S}\ x}(\nu) \approx \left(1 - \hat{I}\left(\frac{\nu}{f_1}\right)\right) \quad \cdot \quad \hat{x}(\nu)$$

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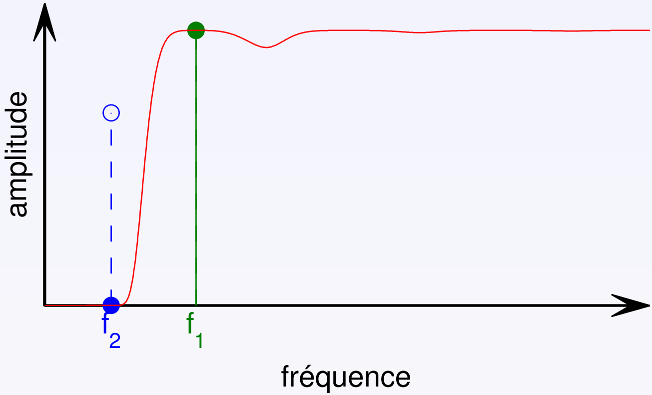


$$\widehat{\mathcal{S}^n x}(\nu) \approx \left(1 - \hat{l}\left(\frac{\nu}{f_1}\right)\right)^n \cdot \quad \hat{x}(\nu)$$

Case 1 : $a_2 f_2 < a_1 f_1$

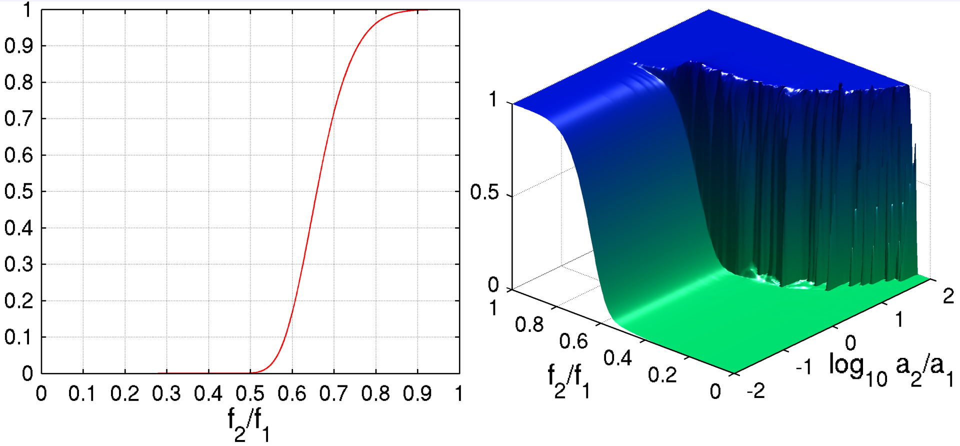
IMF 1 for n iterations

$$IMF_1(t) \approx x_1(t) + \left(1 - \hat{l}\left(\frac{f_2}{f_1}\right)\right)^n x_2(t)$$

Consequence on criterion

$$c\left(\frac{a_2}{a_1}, \frac{f_2}{f_1}, \varphi\right) = \frac{\|IMF_1(t) - x_1(t)\|_{\ell_2}}{\|x_2(t)\|_{\ell_2}} \approx \left(1 - \hat{l}\left(\frac{f_2}{f_1}\right)\right)^n$$

Comparison theory/experiment (10 iterations)



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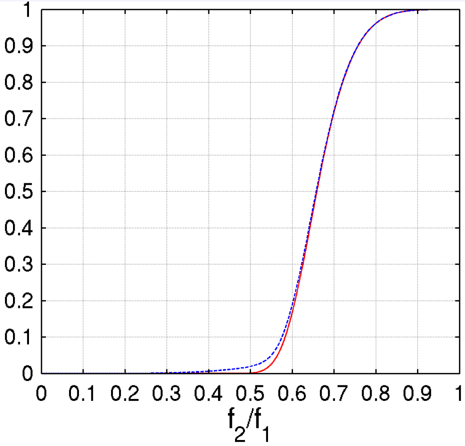
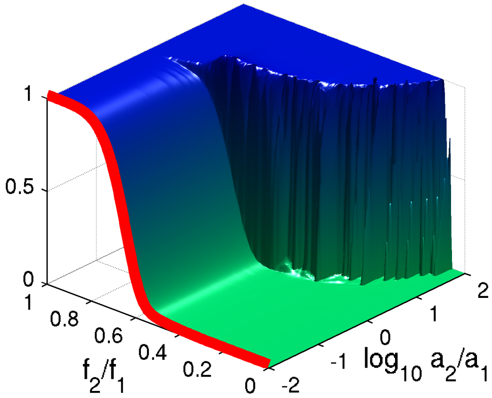
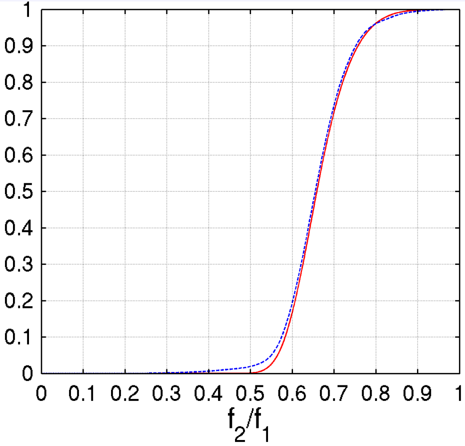
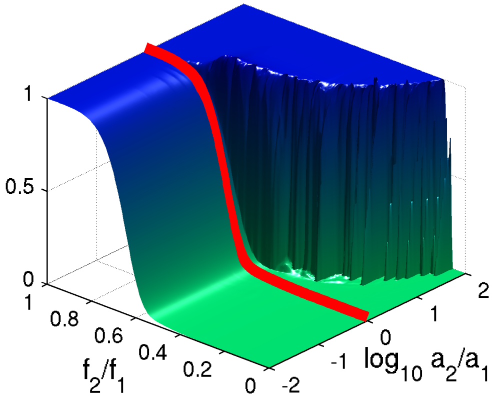




Comparison theory/experiment (10 iterations)





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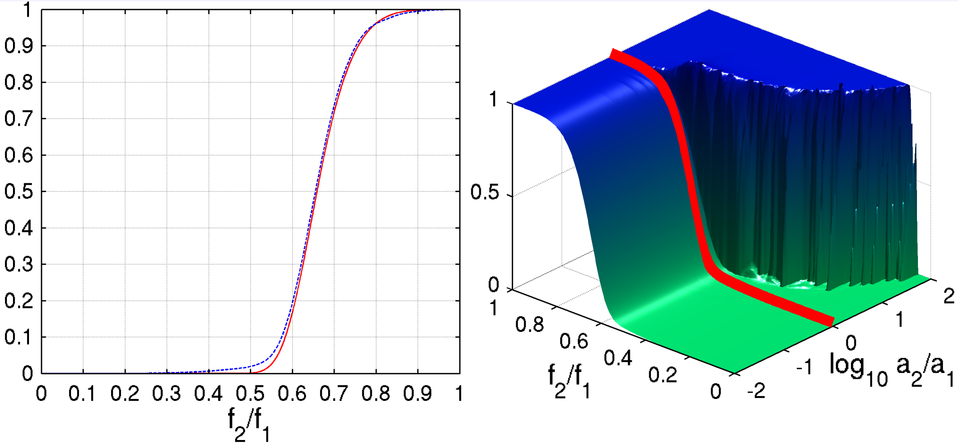
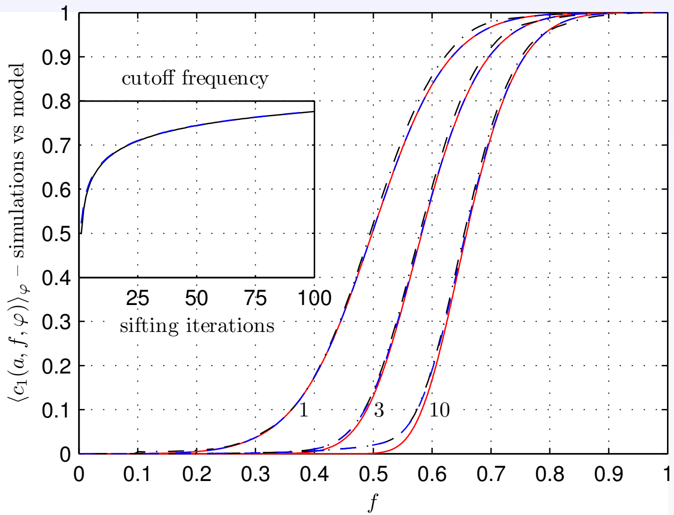


Important conclusion

model still valid even far from the idealized case where extrema are uniformly spaced

Equivalent filter model and sifting iterations



Case 2 : $a_2 f_2^2 > a_1 f_1^2$

IMF 1 for n iterations

$IMF_1 = x(t) - \lambda_n \cos(2\pi f_a t + \varphi_a)$

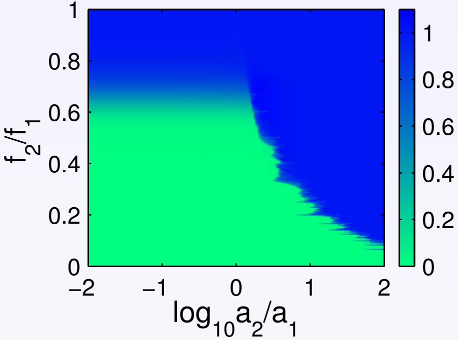
New component

frequency : $f_a = 2kf_2 - f_1$, with k closest integer to $f_1/2f_2$

phase : $\varphi_a = 2k\varphi$

amplitude : $\lambda_n = a_1 \left(1 - \left(1 - \hat{I}\left(\frac{2kf_2 - f_1}{f_2}\right)\right)^n\right)$

Case 2 : $a_2 f_2^2 > a_1 f_1^2$



Criterion

$$c\left(\frac{a_2}{a_1}, \frac{f_2}{f_1}, \varphi\right) = \frac{\|IMF_1(t) - x_1(t)\|_{\ell_2}}{\|x_2(t)\|_{\ell_2}}$$

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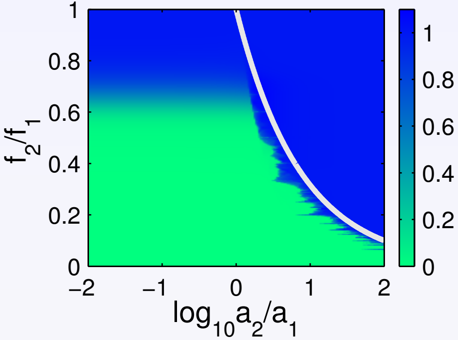

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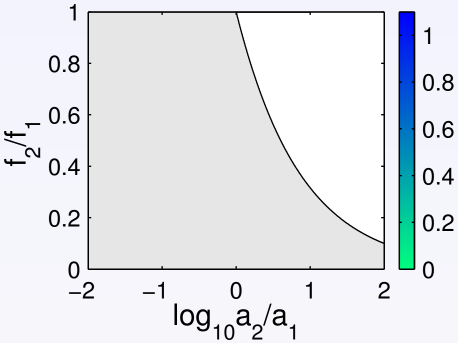

Criterion

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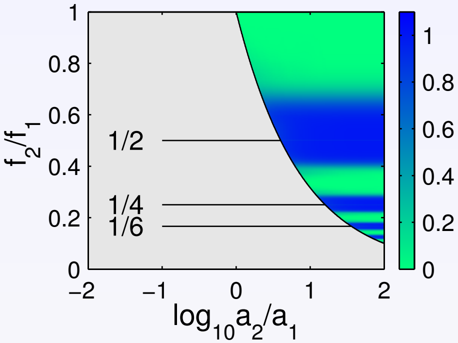
Case 2 : $a_2 f_2^2 > a_1 f_1^2$, 1 iteration



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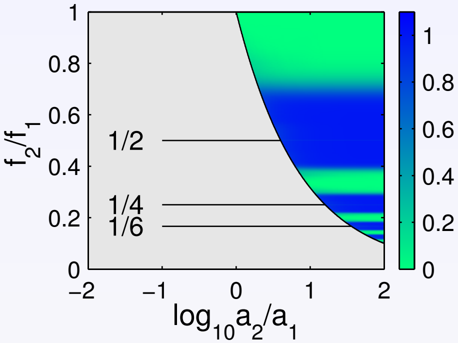
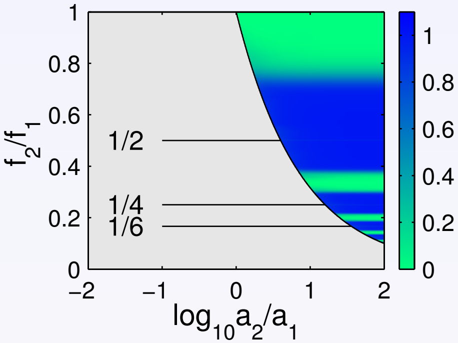
Case 2 : $a_2 f_2^2 > a_1 f_1^2$, 3 iterations



Criterion

$$c'\left(\frac{a_2}{a_1}, \frac{f_2}{f_1}, \varphi\right) = \frac{\| IMF_1(t) - x(t) \|_{\ell_2}}{\| x_1(t) \|_{\ell_2}}$$

Case 2 : $a_2 f_2^2 > a_1 f_1^2$, 10 iterations



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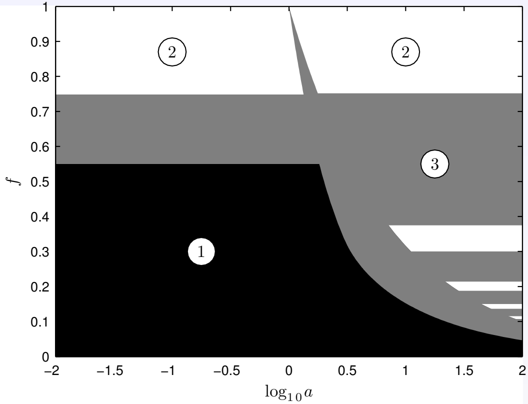
To summarize



1. 2 components
2. 1 single waveform
3. ? ? ?

To summarize

- Essentially three regimes, with
  1. a "$\sim$ half-band splitting" (to be compared with the stochastic case *[F. et al., IEEE Sig. Proc. Lett., 2004]*
  2. a well-defined phase transition

- Spontaneous emergence of distinct components, in accordance with physical interpretation ("beating effect") and with possible relevance to perception (auditory system)

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To conclude

- EMD offers a new tool for exploratory data analysis, with IMF identification as a pre-processing step prior further (e.g., TF) analysis
- IMFs are expected to be “natural” signal components, in between representation-only transforms (spectrograms, Wigner, etc.) and dictionary-based approaches (Gabor, matching pursuit, etc.) but. . . how much “intrinsic” ?
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  - data fluctuations (EEMD ?)
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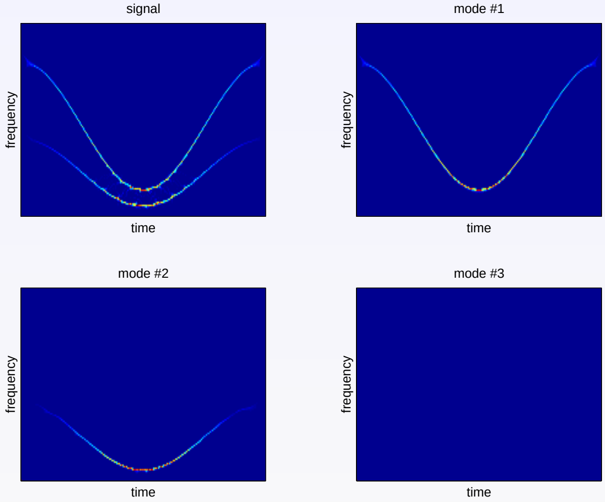
More

(p)reprints and Matlab codes available at

`http ://perso.ens-lyon.fr/patrick.flandrin`

Counter-example

back



Counter-example

◂ back