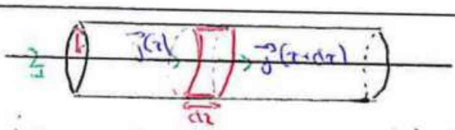
Faire des bilans : récap

Equation de la chaleur :

Bilan d'énergie sur un tronçon :



$dU = \delta Q + \delta W$

$\frac{\partial U}{\partial t} = \phi_e - \phi_s + \cancel{\left(\frac{\partial Q}{\partial t}\right)_{source}} - \cancel{\left(\frac{\partial Q}{\partial t}\right)_{perte}}$ $\quad \delta Q = -j(x+dx)S\,dt\,dx + j(x)S\,dt\,dx$

$= \phi(x) - \phi(x+dx)$

$\frac{\partial U}{\partial t} = c\rho S\cancel{dx}\frac{dT}{dt} = -\frac{\partial \phi(x)}{\partial x}\cancel{\partial x}$ avec $\phi(x) = \iint_S \vec{j}_{th}\, d\vec{S} = j_{th}S = -\lambda \frac{\partial T}{\partial x} S$

$\Rightarrow \mu \cancel{S} dx\, c \frac{\partial T}{\partial t} = \lambda \cancel{S} \frac{\partial^2 T}{\partial x^2}\cancel{dx} \quad \Rightarrow \boxed{\frac{\partial T}{\partial t} = \frac{\lambda}{\mu c}\frac{\partial^2 T}{\partial x^2}}$

Loi de Fourier :

$\vec{j}_{th} = -\lambda\, \vec{grad}\, T$

$[W.m^{-2}]$ $[W.m^{-1}K^{-1}]$ $[K$

Equation locale :

$\frac{\partial \rho c T}{\partial t} + div(\vec{j}_Q) = 0$

Cas du régime stationnaire

a. Géométrie plane

$\frac{dU}{dt} = 0 = \phi_e - \phi_s = \phi(x) - \phi(x+dx) \Rightarrow \phi = c^{ste}$

donc : $\phi(x) = j \times S = -\lambda \frac{dT}{dx} S \Rightarrow \frac{dT}{dx} = -\frac{\phi}{\lambda S}$

$\begin{cases} T(x) - T(0) = -\frac{\phi}{\lambda S} x \\ T(\ell) - T(0) = -\frac{\phi}{\lambda S}\ell \end{cases} \Rightarrow T(x) = T_1 + \frac{(T_2 - T_1)}{\ell}x \Rightarrow \phi = -\lambda \frac{dT}{dx}S = \frac{\lambda_v (T_2 - T_1)}{\ell} S$

$R_{th} = \frac{\ell}{\lambda S}$

b. Géométrie cylindrique

$\phi = -\lambda \frac{dT}{dr} S = -\lambda \frac{dT}{dr} 2\pi r \ell \Rightarrow \frac{dT}{dr} = -\frac{\phi}{\lambda 2\pi r L} \Rightarrow dT = \frac{-\phi}{2\pi\lambda r L}dr$

$-T_1 + T_2 = \frac{+\phi}{2\pi\lambda L}\ln\left(\frac{R_1}{R_2}\right)$ avec $R_{th} = \frac{1}{2\pi\lambda L}\ln\left(\frac{R_1}{R_2}\right)$

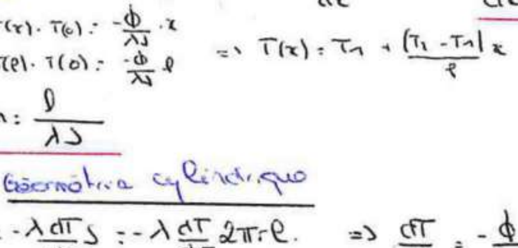
c. Géométrie sphérique

$\phi = -\lambda \frac{dT}{dr}4\pi r^2 \Rightarrow \frac{dT}{dr} = \frac{-\phi}{4\pi\lambda r^2} \Rightarrow dT = \frac{-\phi}{4\pi\lambda}\frac{dr}{r^2}$

$T(r) - T_2 = \frac{\phi}{4\pi\lambda}\left(\frac{1}{r} - \frac{1}{R_2}\right)$

avec $R_{th} = \frac{(T_1 - T_2)}{\phi} = \frac{1}{4\pi\lambda}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Premier principe en écoulement



- à l'instant t, $\Sigma^*(t) = \Sigma \cup d\Sigma_e$ où $d\Sigma_e$ désigne la masse de liquide dm_e entrant dans Σ entre t et $t+dt$.
- à l'instant $t+dt$, $\Sigma^*(t) = \Sigma \cup d\Sigma_s$ où $d\Sigma_s$ désigne la masse de liquide dm_s sortant de Σ entre t et $t+dt$.

On calcule la variation d'énergie interne du système Σ^* entre t et $t+dt$. Comme on est en régime stat. $U_\Sigma = c^{ste}$.

$dU^* = dm_s\, u_s + U(t+dt) - (dm_e u_e + U(t))$

$= D_m' dt\,(u_s - u_e)$ $\quad$ Donc : $D_m' dt\left(u_s + \frac{P_s}{\rho_s} - \left(u_e + \frac{P_e}{\rho_e}\right)\right) = \delta W_u + \delta Q \Rightarrow \boxed{h_s - h_e = w_u + q}$

1er principe

$dU^* = \delta W_p + \delta W_u + \delta Q \quad : P_e dV_e - P_s dV_s = D_m' dt\left(\frac{P_e}{\rho_e} - \frac{P_s}{\rho_s}\right) + \delta W_u + \delta W_q$ transfert thermique mécanique

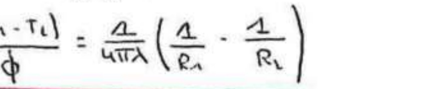
Conduction électrique

Loi d'Ohm locale

$[A.m^{-2}]\ \vec{j} = \sigma \vec{E}$ $[V.m^{-1}]$

conductivité

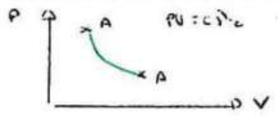
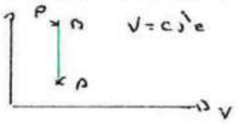
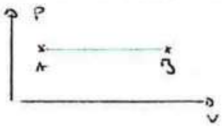
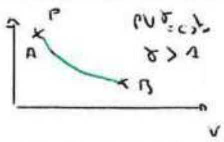
$[A.V^{-1}.m^{-1}] = [\Omega^{-1}.m^{-1}] = [S.m^{-1}]$



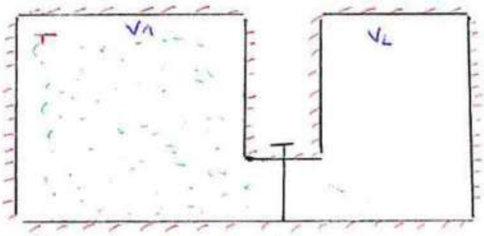
$I = \iint_{M\in S} \vec{j}(M)\cdot d\vec{S}_M = jS$

$V(A_2) - V(A_1) = \int_{P\in[A_1,A_2]} \vec{grad}(V(P))\cdot d\vec{P}_P = -\int_{P\in[A_1,A_2]} \vec{E}(P)\cdot d\vec{P}_P = -\int_{P\in[A_1,A_2]} \vec{E}(P)\cdot d\vec{P}_P$ $\quad U = \frac{\ell}{\sigma S} I$

Thermodynamique

Transformation	isotherme	isochore	isobare	isentropique
Diag. de Clapeyron				
ΔU	0	$nC_{vm}\Delta T = \frac{nR}{\gamma-1}\Delta T$	$nC_{vm}\Delta T = \frac{nR}{\gamma-1}\Delta T$	$nC_{vm}\Delta T$
ΔH	0	$nC_{pm}\Delta T = \frac{n\gamma R}{\gamma-1}\Delta T$	$nC_{pm}\Delta T = \frac{n\gamma R}{\gamma-1}\Delta T$	$nC_{pm}\Delta T$
W	$-nRT\ln\left(\frac{V_f}{V_i}\right) = -nRT\ln\left(\frac{P_i}{P_f}\right)$	0	$-P\Delta V$	$W = \Delta U = nC_{vm}\Delta T$
Q	$-W$	$\frac{nR}{\gamma-1}\Delta T$ $(Q_v = \Delta U)$	$\frac{n\gamma R}{\gamma-1}\Delta T$	0
ΔS	$-nR\ln\left(\frac{P_f}{P_i}\right) = -nR\ln\left(\frac{V_i}{V_f}\right)$	$\frac{nR}{\gamma-1}\ln\left(\frac{P_f}{P_i}\right) = \frac{nR}{\gamma-1}\ln\left(\frac{T_f}{T_i}\right)$	$\frac{nR\gamma}{\gamma-1}\ln\left(\frac{V_f}{V_i}\right) = \left(\frac{nR\gamma}{\gamma-1}\right)\ln\left(\frac{T_f}{T_i}\right)$	$\Delta S = 0$

Détente de Joule Gay Lussac



→ Transformation adiabatique à énergie interne constante $\Delta U = 0$

1ère loi de Joule Pour un GP : $U(T,V) \Rightarrow U(T)$

2nde loi de Joule Pour un GP : $H(T,V) \Rightarrow H(T)$

Détente de Joule Thomson : → isenthalpique $\Delta H = 0$

Relation de Mayer pour un GP :

$$H = U + PV$$
$$= U + nRT$$
$$= U + RT \quad \text{pour une mole.}$$

$$\Rightarrow \frac{dH}{dT} = \frac{dU}{dT} + R \Rightarrow C_{pm} - C_{vm} = R$$

Or : $C_{vm} = \frac{3}{2}RT$ $\quad C_{pm} = \frac{5}{2}RT$

Coefficient γ d'un gaz

$$\gamma = \frac{C_{pm}}{C_{vm}} = \frac{C_p}{C_v}$$

Ainsi, $C_{vm} = \frac{R}{\gamma-1}$ $\quad C_{pm} = \frac{\gamma R}{\gamma-1}$

Calculs de variation d'entropie

$$dS = \frac{\delta Q}{T_{ext}} + \delta S_{cr}$$