A finite presentation of graphs of treewidth at most 3

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LIP, ENS de Lyon

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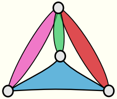
Outline

Two parts:

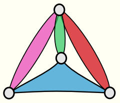
1. Basic notions and the main result: graphs, terms, and axioms.
2. Main tools of the proof, or how to decompose graphs.

Graphs, terms, treewidth, and the main result

Graphs



Graphs



Specificities:

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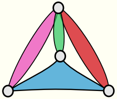

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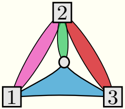

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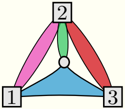

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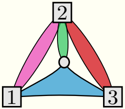

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Arity of a graph: number of sources.

Graph operations

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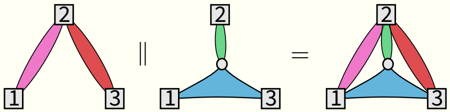

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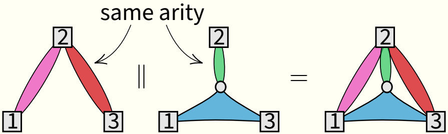

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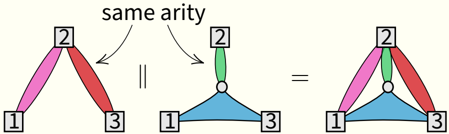

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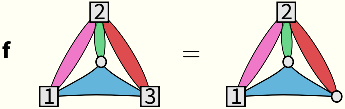

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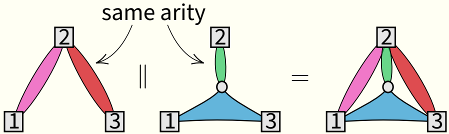

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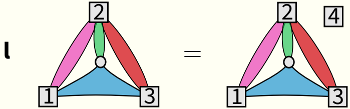

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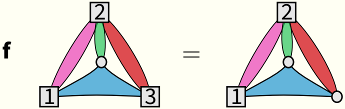

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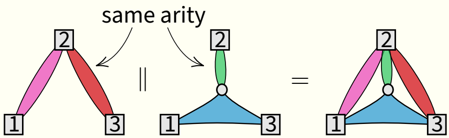

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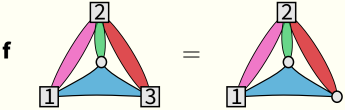

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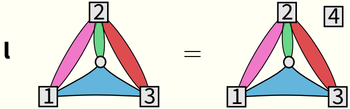

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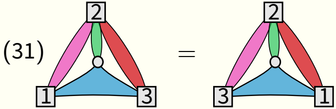

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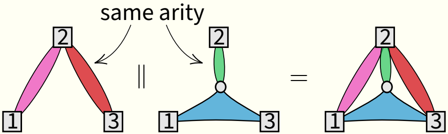

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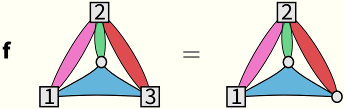

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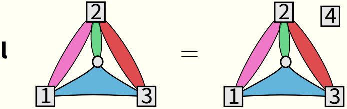

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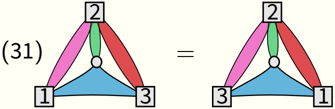

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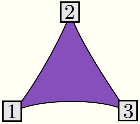

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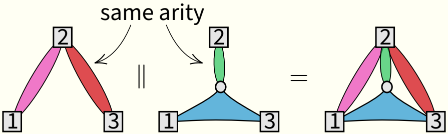

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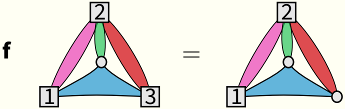

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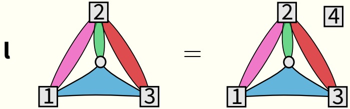

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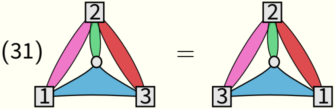

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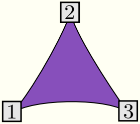
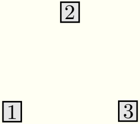


Empty graph of arity k, $\emptyset$:



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Corresponding grammar:

$$t, u ::= t \parallel u \mid \mathbf{l}\, t \mid \mathbf{f}\, t \mid \mathrm{p}\, t \mid a \mid \emptyset,$$

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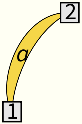

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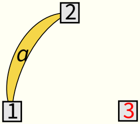

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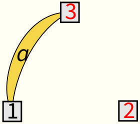

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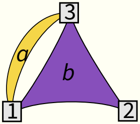

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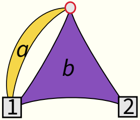

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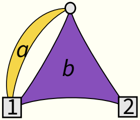

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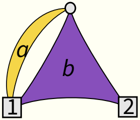

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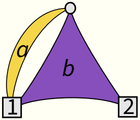

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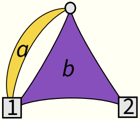
- Function giving the **graph of a term** noted $\mathbf{g}\, t$.
- **Parsing** of a graph G: term t such that $\mathbf{g}\, t \simeq G$.

Multiple parsings and goal

When do terms parse the same graph ?



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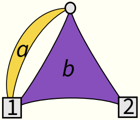

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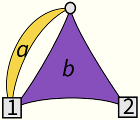

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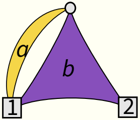

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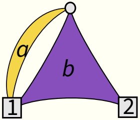

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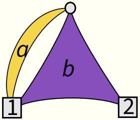

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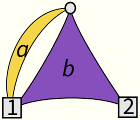

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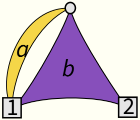

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We focus on graphs of treewidth at most 3.

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Proof idea: we use axioms to recursively decompose terms following structural decompositions adapted to our graphs,

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Axioms Ax given by:

1. $a \parallel (b \parallel c) = (a \parallel b) \parallel c, \quad a \parallel b = b \parallel a$, and $\quad a \parallel \emptyset = a$,
2. $pqa = (p \circ q)a$, and $\quad \mathrm{id}\, a = a$,
3. $p(a \parallel b) = pa \parallel pb$ and $\quad p\emptyset = \emptyset$,
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8. + 4 other axioms not detailed here.

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For all terms t, u of treewidth at most 3 $\mathrm{Ax} \vdash t = u \Leftrightarrow \mathbf{g}\, t \simeq \mathbf{g}\, u$.

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Recursively decomposing graphs of treewidth at most 3

A usual decomposition in graph theory

Reduce to more and more connected graphs by identifying separators

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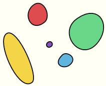

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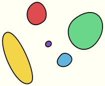

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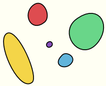

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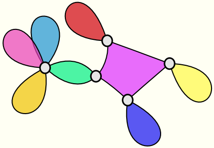

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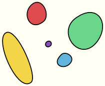

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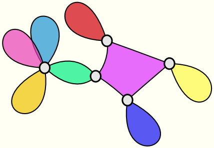

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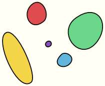

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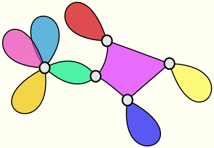

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We adapt these decompositions to sourced graphs.

Decomposing graphs: full prime decompositions

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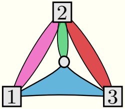

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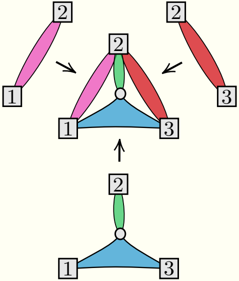

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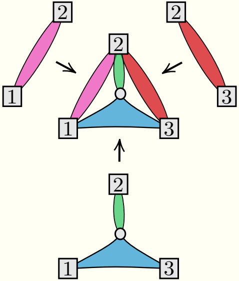

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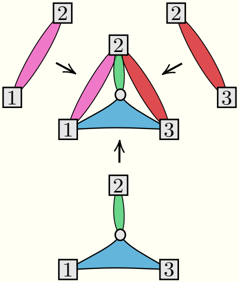

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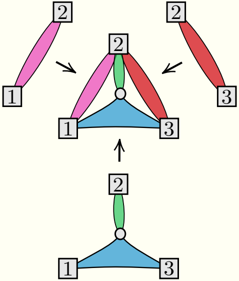

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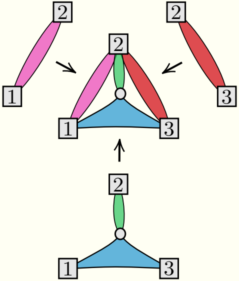

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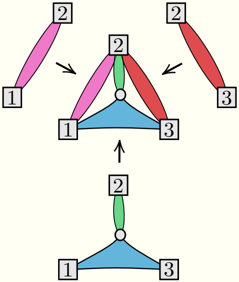

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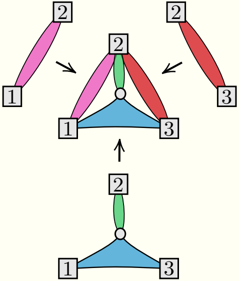

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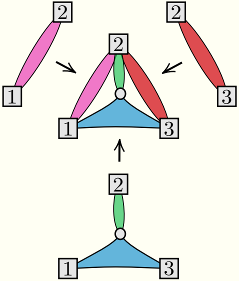

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The main difficulty: forget points non-determinism

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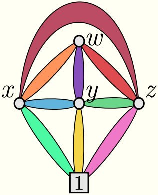

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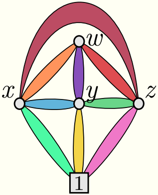

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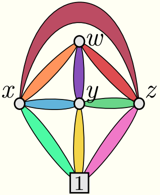

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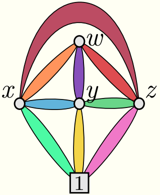

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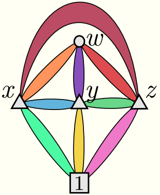

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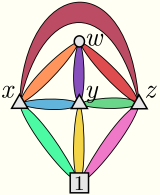

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The difficulty comes from graphs having many forget points: there are no canonical choices.

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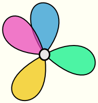

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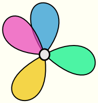

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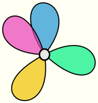

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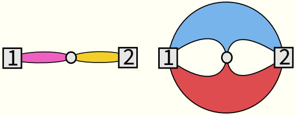

+1 case

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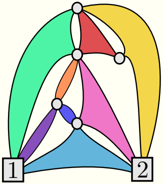

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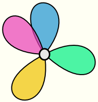

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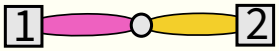

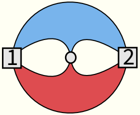

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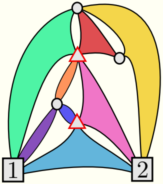

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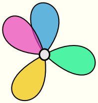

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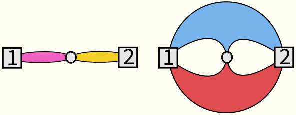

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Decomposing graphs: separation pairs

Hard graph: a full prime graph without anchors.

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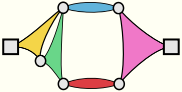

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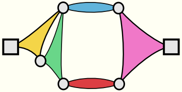

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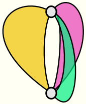

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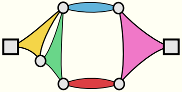

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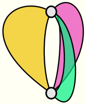

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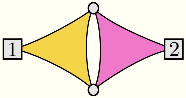

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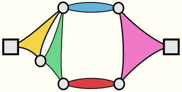

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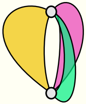

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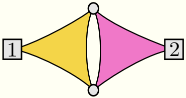

Proposition

Every hard graph of treewidth at most 3 has at least one (minimal) separation pair consisting of forget points.

Classifying hard graphs of treewidth at most 3

What happens if a hard graph has several separation pairs ?

Classifying hard graphs of treewidth at most 3

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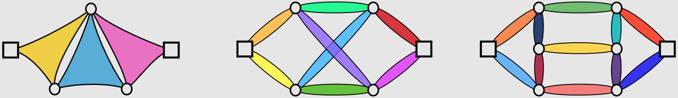
Theorem

Whenever a hard graph has two distinct (minimal) separation pairs whose vertices are forget points, then it must have one of the following shapes:



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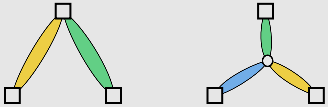
A graph with 3 sources either has the triangle K_3 as a minor on its sources, or it has one of the following shapes:



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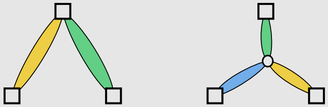

Graph minors: a way to see patterns in graphs, generalising the notion of subgraphs.

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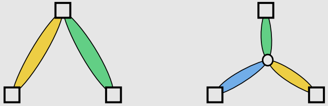

Graph minors: a way to see patterns in graphs, generalising the notion of subgraphs.
This is reminiscent of the proposition stating that a graph without a cycle (i.e. without K_3 as a minor) is a tree. Our lemma is stronger.

Conclusion

Generalising ?

Generalising to treewidth at most 4 ?

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The classification result might be usefull to get heuristic to compute treewidth in general.

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- graphs with sources and hyperedges,
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- two research lines: generalisation and algorithmic aspects (testing treewidth at most 3 and the isomorphism on graphs of treewidth at most 3).

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Thank you !