Fluctuations near a critical point

Fluctuations near a critical point

Motivations

Why is interesting to study these fluctuations near a second order phase transition?

1) The probability density functions of global variables

S. T. Bramwell, P. Holdsworth, J.-F. Pinton, Nature vol. 396, 552 (1998)
Universality of rare fluctuations in turbulence and critical phenomena

E. Bertin, Phys. Rev. Lett, 95 170601 (2005).
Global fluctuations in Gumbel Statistics

2) Aging at critical point

L. Berthier, P. Holdsworth, Europhys. Lett. 58, 35 (2002)
Surfing on a critical line: Rejuvenation without chaos, memory without a hierarchical phase space

P. Calabrese and A. Gambassi, cond-mat/0410357V2
Aging Properties of Critical Systems

Fluctuation Dissipation Ratio (FDR) during aging

In equilibrium

$$\chi(t,t_w) = \frac{1}{k_B T}(C_\theta(t,t) - C_\theta(t,tw))$$

$$\chi(t,t_w) = \frac{<\Delta>}{\Gamma_{ext}} = \int_{t_w}^{t} R(t,t')dt'$$

$$C_\theta(t,t_w) =< \delta\theta(t)\delta\theta(t_w) >$$

Out equilibrium (Cugliandolo and Kurchan 1992) **FDR**

$$\chi(t,t_w) = \frac{X(t,t_w)}{k_B T}(C_\theta(t,t) - C_\theta(t,tw))$$

$$X(t,tw) = \frac{T}{T_{eff}(t,t_w)}$$

Experimentally this idea has been tested in

- **Spin glasses**
- **Colloids**
- **Polymers**

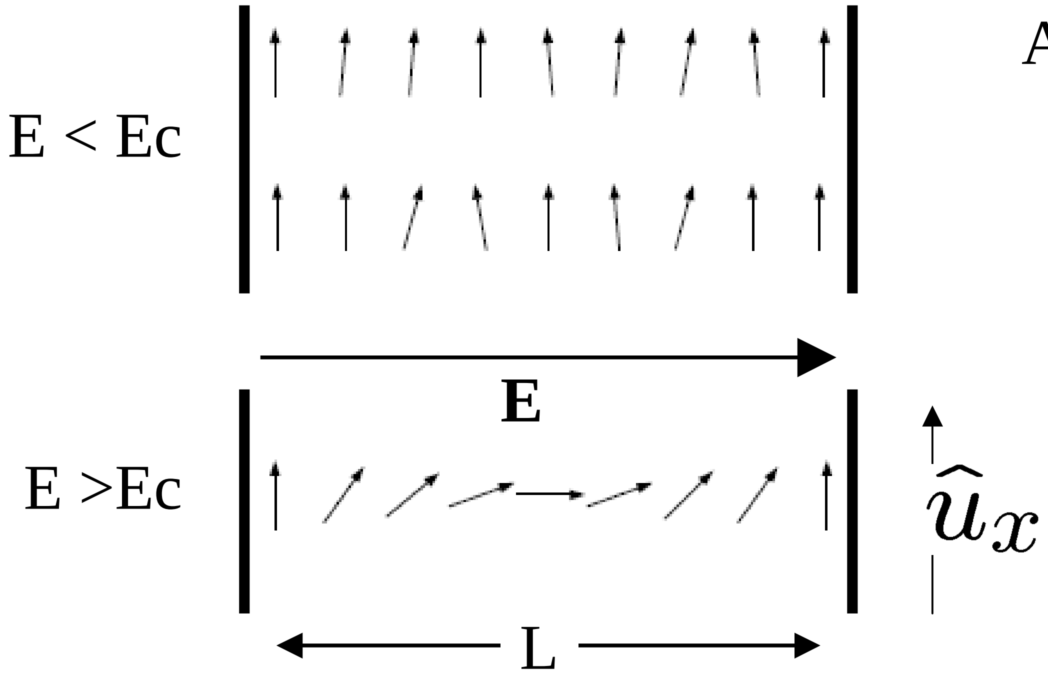
Controversial results

Need for experiments where the results can be directly compared with theoretical models

Outline

1) The Fréedericksz transition in liquid crystals
2) Second order phase transition and the global variable of interest
3) Experimental system
4) Experimental results on PDF: the universal PDF for global variables
5) Aging at critical point
6) Experimental results on aging
7) Conclusions

Liquid Crystals and Fréedericksz transition (I)



A liquid crystal consists of elongated molecules

$\hat{n}$ is the director

Surface treatement.
Parallel anchored
(planar allignement)

Competition between :

- **Elastic energy** $\widehat{n} // \widehat{u}_x$
- **Electrostatic energy** $\widehat{n} // \overrightarrow{E}$

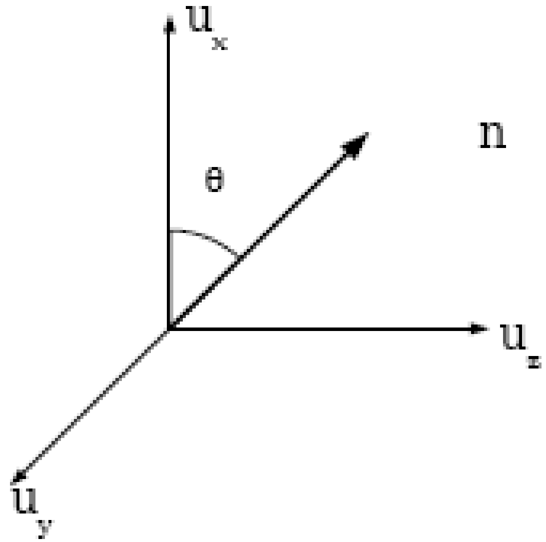
Control parameter :
voltage difference U

Liquid Crystals and Fréedericksz transition (II)

$$\hat{n} = \cos(\theta)\ \hat{u}_x + \sin(\theta)\ \hat{u}_z$$

With boundary conditions

$$\theta(z=0) = \theta(z=L) = 0$$



Solution of the form: $\theta(z) = \theta_0(x,y)\sin(\frac{\pi z}{L})$

If $\theta_0 << 1$ remains small, the equation of motion of θ_0 is :

$$\tau_0 \frac{\mathrm{d}\theta_0}{\mathrm{d}t} = \epsilon\theta_0 - \left(\kappa + \frac{\epsilon+1}{2}\right)\theta_0^3$$

$$\tau_0 = \frac{\gamma}{\epsilon_0\epsilon_a E_c^2} \quad U_c = \pi\sqrt{\frac{K_1}{\epsilon_0\epsilon_a}} \quad \kappa = \frac{K_3 - K_1}{K_1} \quad \epsilon = \frac{U^2}{U_c^2} - 1$$

Correlation length in the xy plane : $\xi_r = \frac{L}{\pi\sqrt{\epsilon}}$

(San Miguel, Phys. Rev. A, 32, 3811, 1985)

Fréedericksz transition

- The Fréederick transition is a second order phase transition
- The order parameter is $\theta_0(x, y)$
- The control parameter is $\epsilon = U^2/U_c^2 - 1$
- The relaxation time is $\tau_{relax} = \tau_o/\epsilon$
- The correlation length $\xi_r = \frac{L}{\pi\sqrt{\epsilon}}$

Shadowgraph image

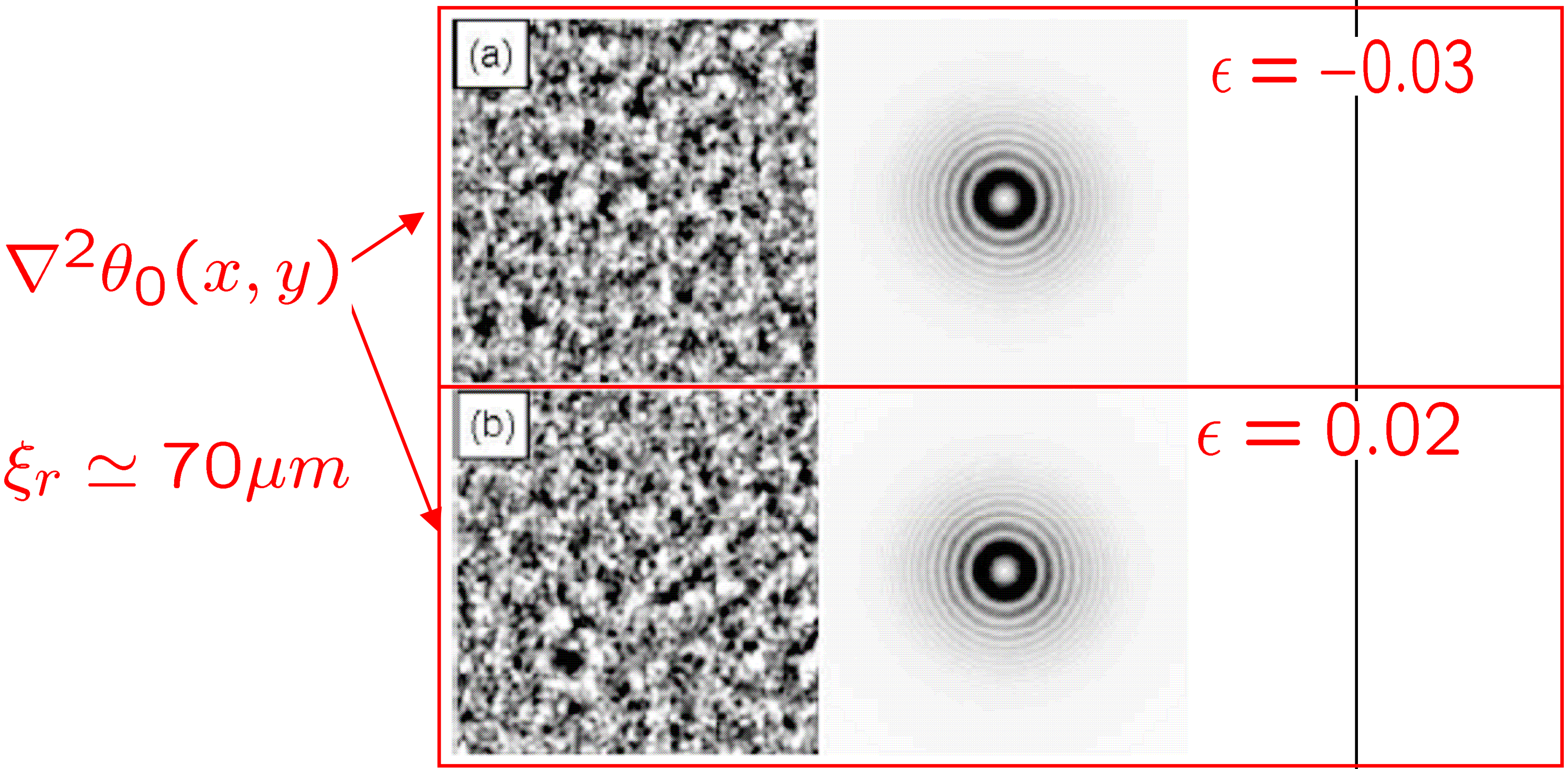


FIG. 1: Left column: shadowgraph snapshots of an area of 1.03×1.03 mm^2. Right column: corresponding structure factors, averaged over 256 images. (a): $V_0 = 3.170$, (b): $V_0 = 3.256$ Volt.

$U_c = 3.22V$ and $L = 27\mu m$

from Zhou,Ahlers, arXiv: nlin/0409015v2

Fréedericksz transition

- The Fréederick transition is a second order phase transition
- The order parameter is $\theta_0(x, y)$
- The control parameter is $\epsilon = U^2/U_c^2 - 1$
- The relaxation time is $\tau_{relax} = \tau_o/\epsilon$
- The correlation length $\xi_r = \frac{L}{\pi\sqrt{\epsilon}}$

Global variable:

$$\zeta = \frac{2}{L}\int_0^L < (1 - n_x^2) >_{xy} dz \simeq \iint_A \theta_0^2 \, \frac{dxdy}{A}$$

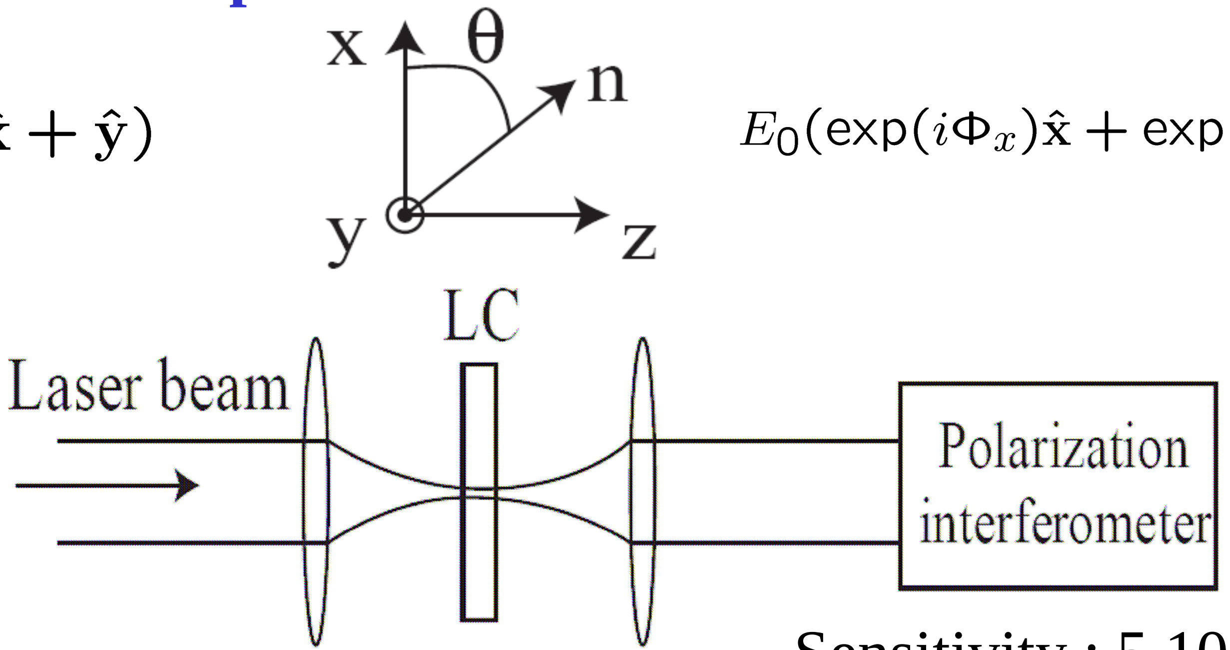
where $A = \pi D^2/4$ and $< . >_{xy}$ stands for mean on A.

We measure de fluctuations of ζ

as a function of D/ξ and ϵ

Experimental set-up

$E_0(\hat{\mathbf{x}} + \hat{\mathbf{y}})$ $\quad$ $E_0(\exp(i\Phi_x)\hat{\mathbf{x}} + \exp(i\Phi_y)\hat{\mathbf{y}})$



Sensitivity : 5 10^{-14} m/$\mathrm{Hz}^{1/2}$

- Two cells with different thickness: $L = 6\mu m$ and $L = 25\mu m$. Surface $S = 1cm^2$
- The liquid crystal cell is a birifrengent plate. Optical axis $//\ \hat{n}$
- Measurement of the dephasing $\Phi = \Phi_x - \Phi_y$ between the two polarisations.

$$\Phi = a + b\,\zeta = a + b \iint_A \theta_0^2\, \frac{dxdy}{A}$$

- Laser diameter inside the cell $38\mu m$ (limited by diffraction)

Measure of the dephasing

The dephasing between the Ex and Ey is :

$$\Phi = \left\langle \frac{2\pi}{\lambda} \int_0^L \left(\frac{n_o n_e}{\sqrt{n_0^2 \cos(\theta)^2 + n_e^2 \sin(\theta)^2}} - n_0 \right) \mathrm{d}z \right\rangle_{xy}$$

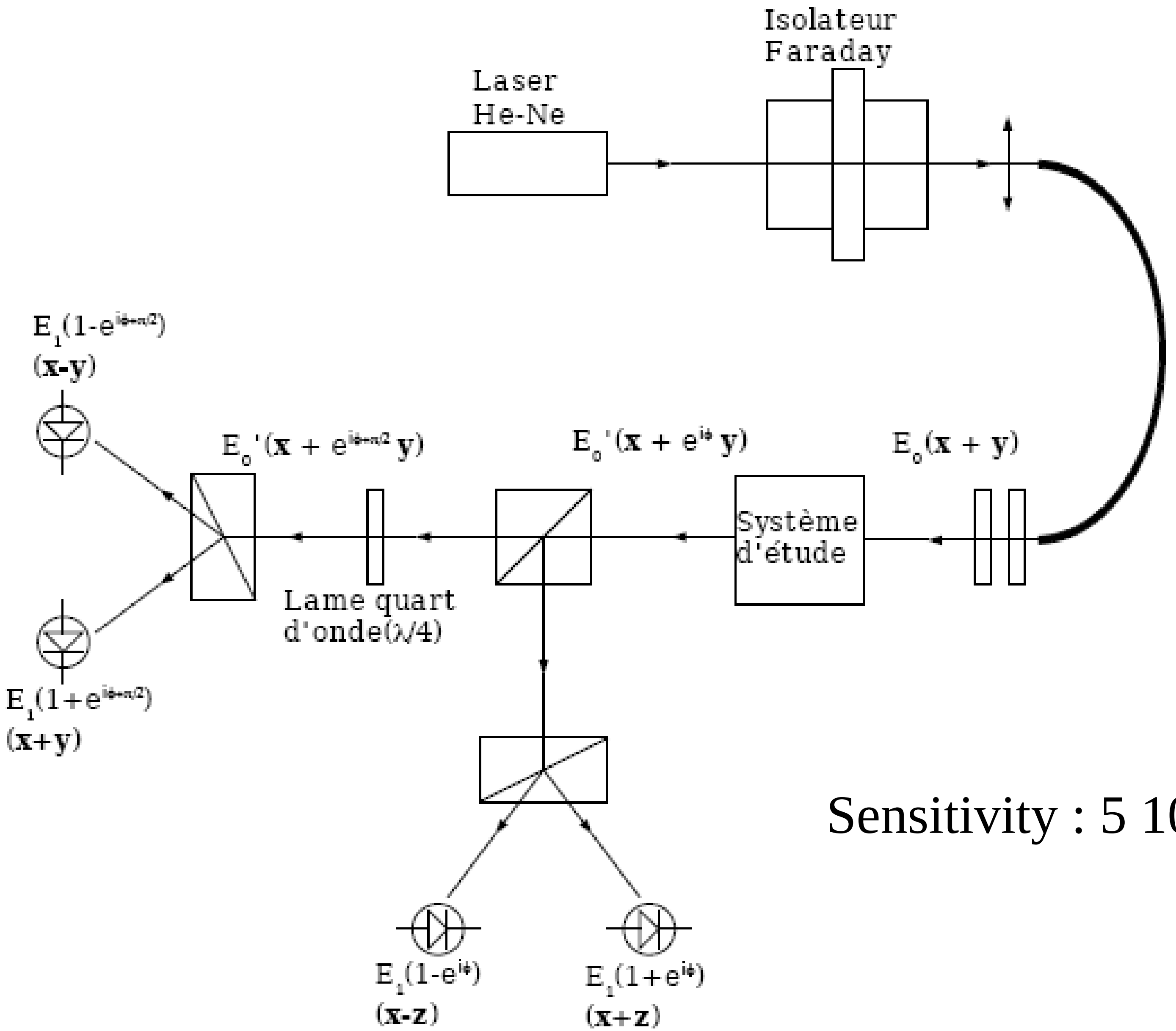
with (n_o, n_e) the two anistotropic refractive indices.

If $\theta << 1$ in terms of ζ we get:

$$\Phi = \Phi_0 \left(1 - \frac{n_e(n_e + n_o)}{4n_o^2} \zeta \right) \qquad \Phi_0 \equiv \frac{2\pi}{\lambda}(n_e - n_o)L$$

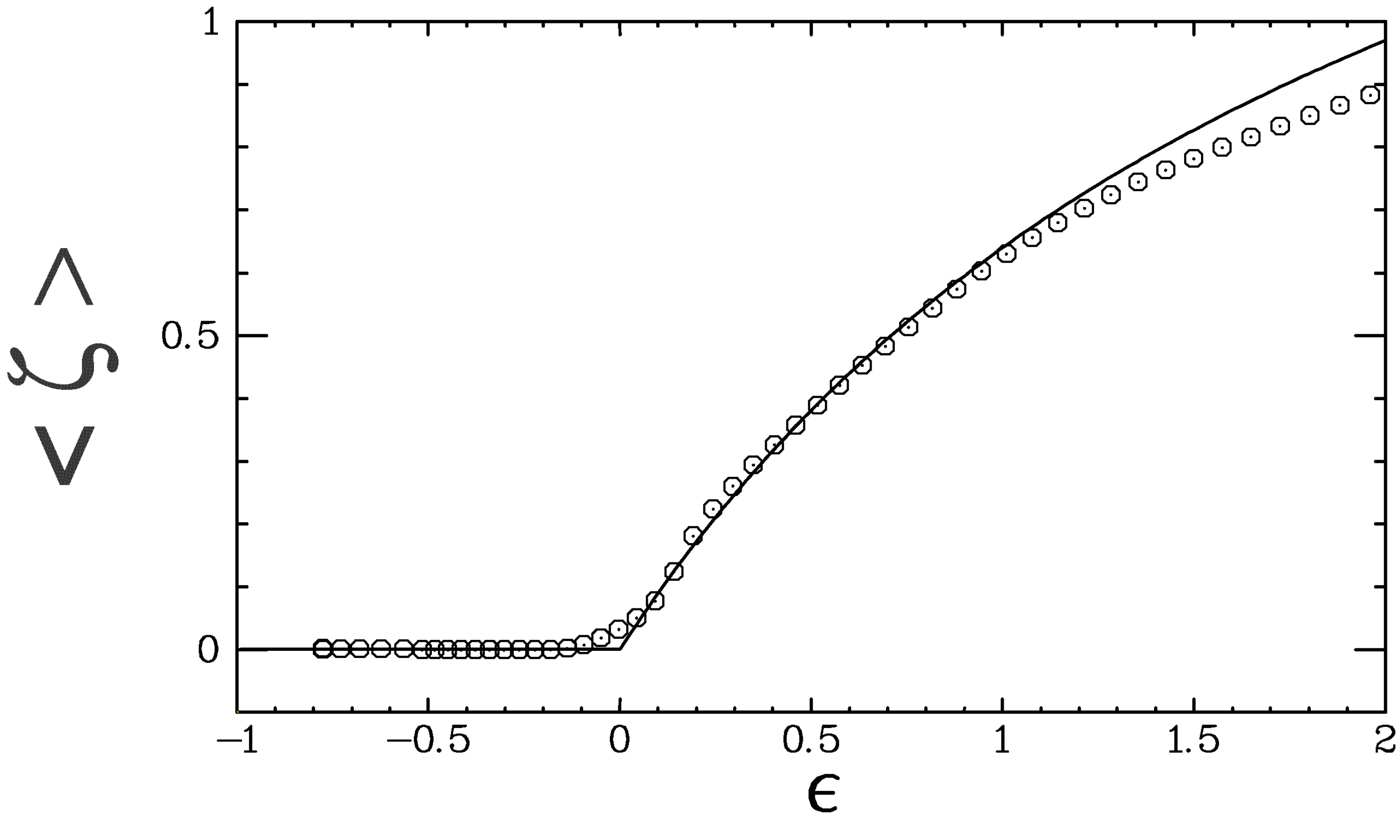
Interferometer noise : $\dfrac{(\Phi - \Phi_o)}{\Phi_o} \simeq 6\ 10^{-8} Hz^{-1/2}$

Experimental system: polarization interferometer



Sensitivity : 5 10^{-14} m/$Hz^{1/2}$

Mean Amplitude of $<\zeta>$



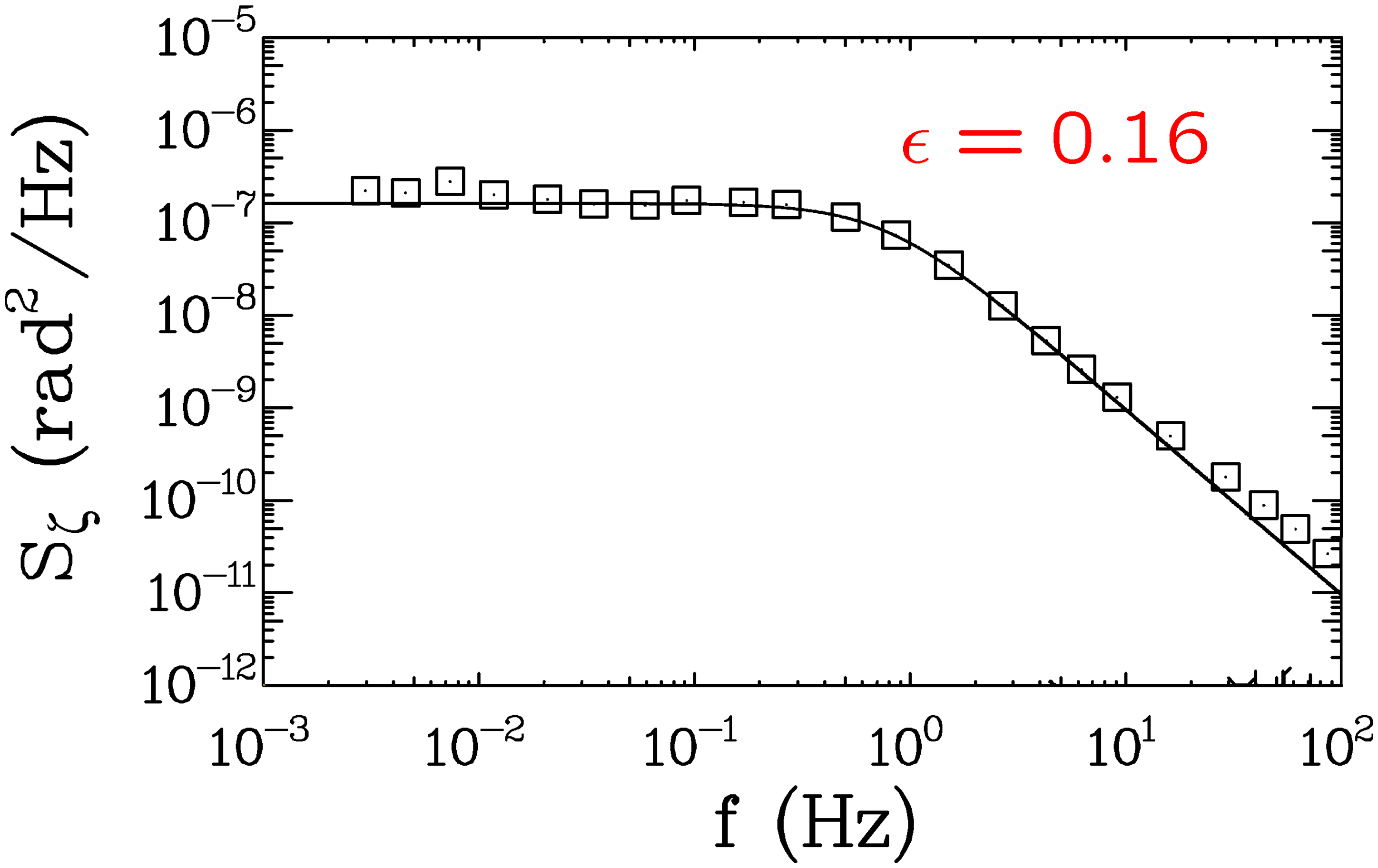
$$<\zeta> = \frac{2\epsilon}{2k+\epsilon+1}$$

For Liquid Crystal 5CB

Computed $V_c = 0.74V$

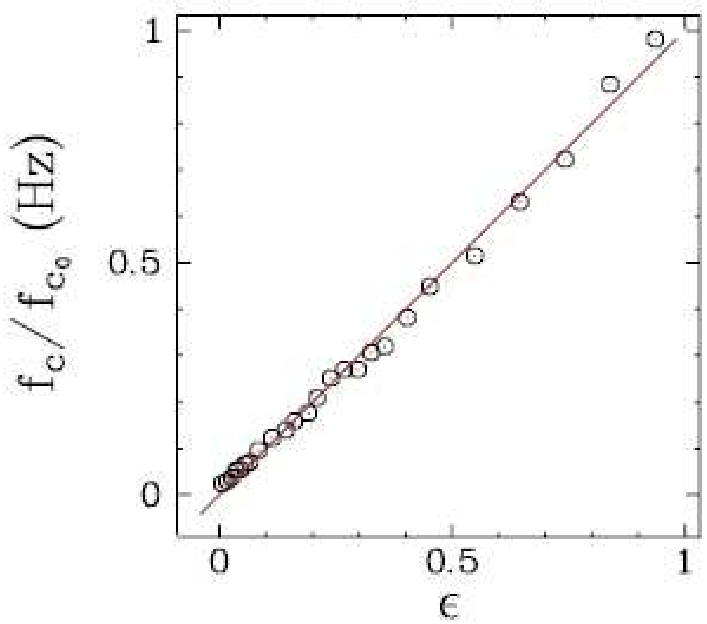
Measured $V_c = 0.73V$ by $<\zeta> \propto \epsilon = 0$

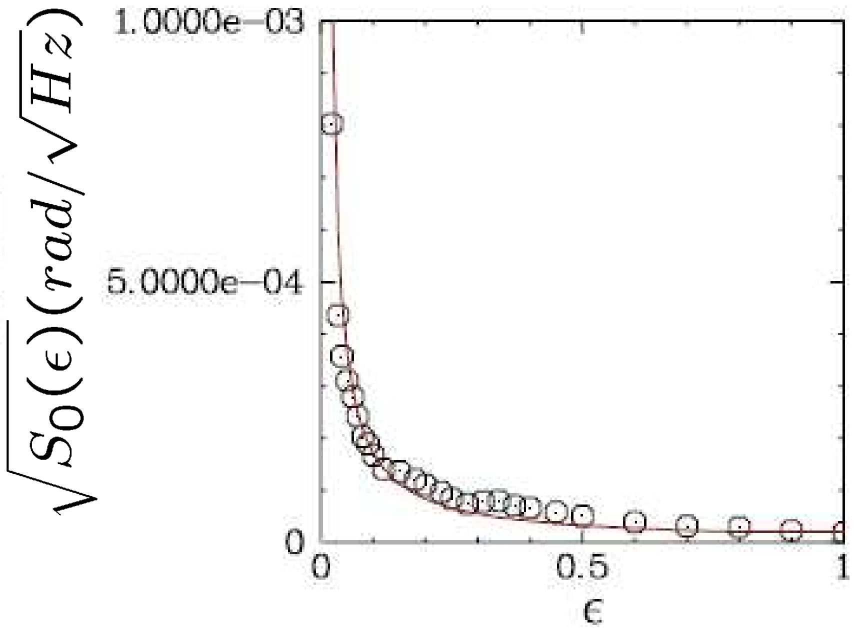
Fluctuation spectra of ζ



$$S_\zeta(f) = \frac{S_o(\epsilon)}{1 + (\frac{f}{f_c(\epsilon)})^2}$$

Relaxation time as a function of ε





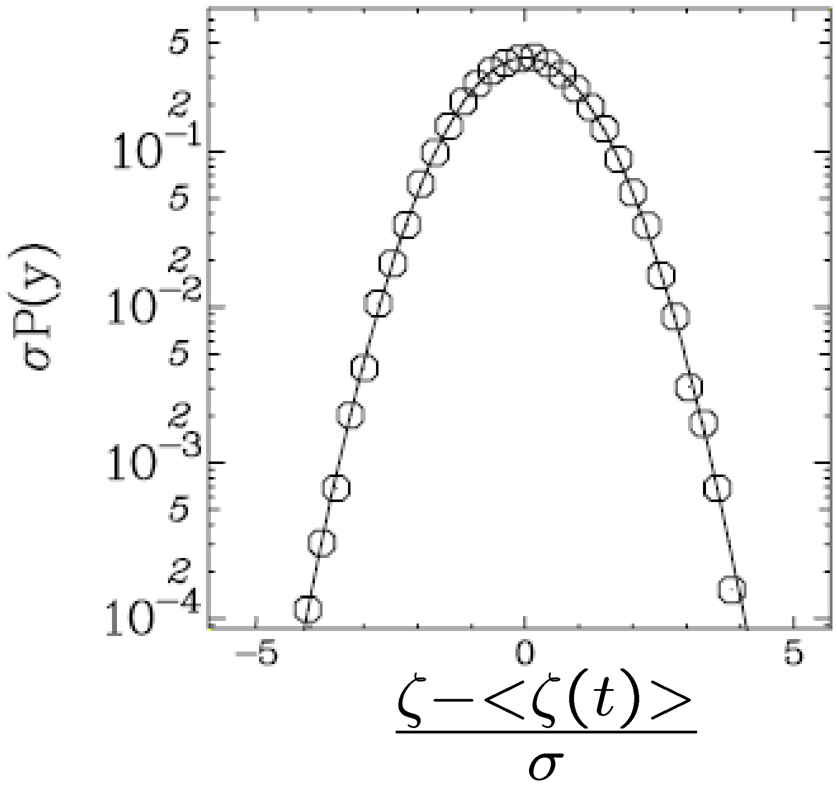
$$f_c = \frac{1}{\tau_{relax}} = \epsilon \, \frac{1}{\tau_o}$$

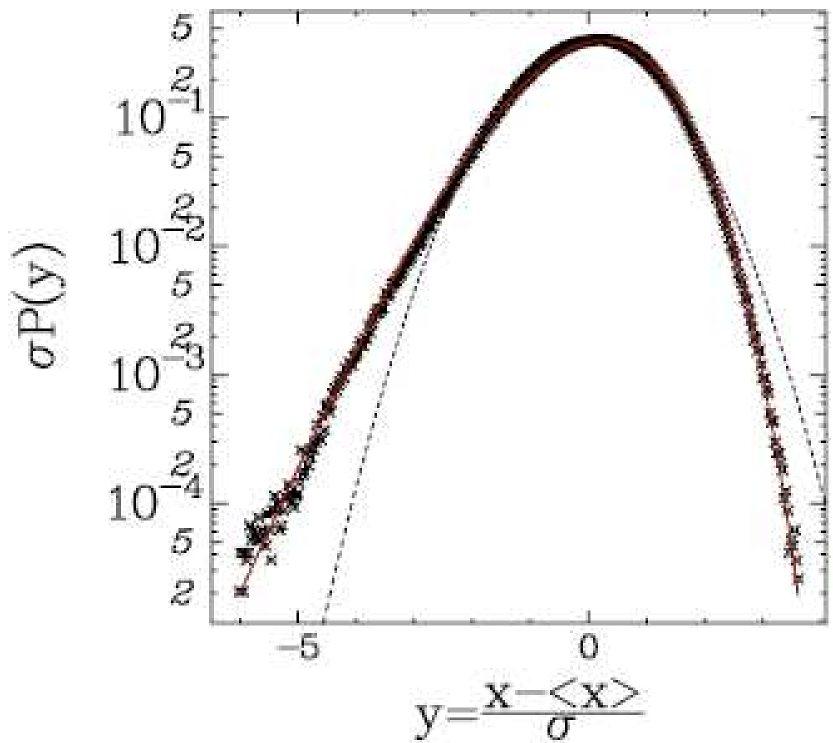
from the fit $U_c = 0.73V$ and $\tau_o = 0.1s$
Computed $\tau_o = 0.09s$.

Variance : $\sigma^2 \propto S_0(\epsilon) f_c(\epsilon) \propto \epsilon^{-1}$

PDF of ζ

Cell width L=6.7 µm, $\tau_o \simeq 0.1s$ and $D = 38\mu m$





$\epsilon = 0.02$

Gaussian fluctuations

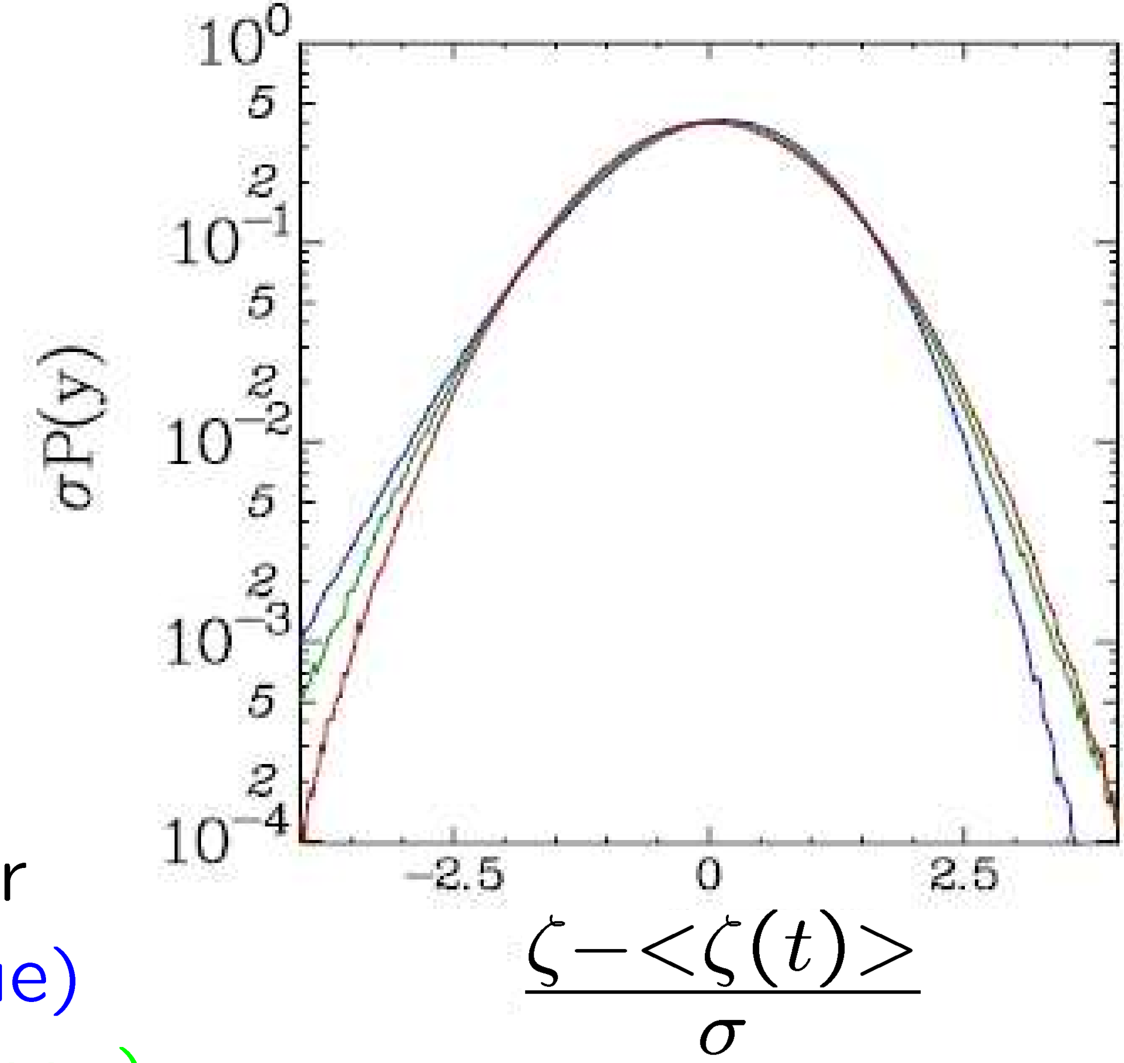
$\xi_r = \frac{L}{\pi\sqrt{\epsilon}} = 15\mu m$

$\epsilon \simeq 0.001$

Non-Gaussian fluctuations

$\xi_r = \frac{L}{\pi\sqrt{\epsilon}} = 60\mu m$

Low frequency influence



High-pass filter

- $1mHz$ (Blue)
- $50mHz$ (Green)
- $2Hz$ (Red)

Questions

- What is the interpretation for the PDF of θ_0 ?

- Why this behavior has not been observed before?

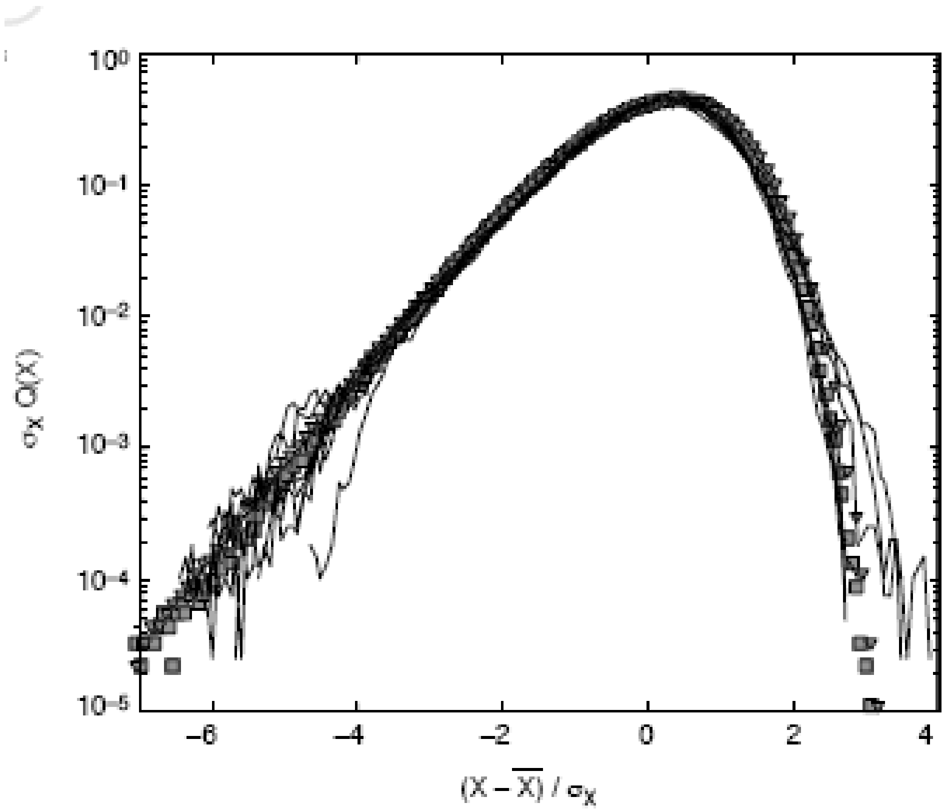
Universality of fluctuation PDF

- In 1998 Bramwell, Holdsworth, Pinton proposed that in spatial extended systems the PDF of a global quantity x may take an universal form:

$$P(x) = K \exp\{-a\ [b(x-s) - \exp(b(x-s))]\}$$

BHP distribution is equivalent to Gumbel for a integer. (Bertin 2005).

- Correlation lengths are of the order of the system size

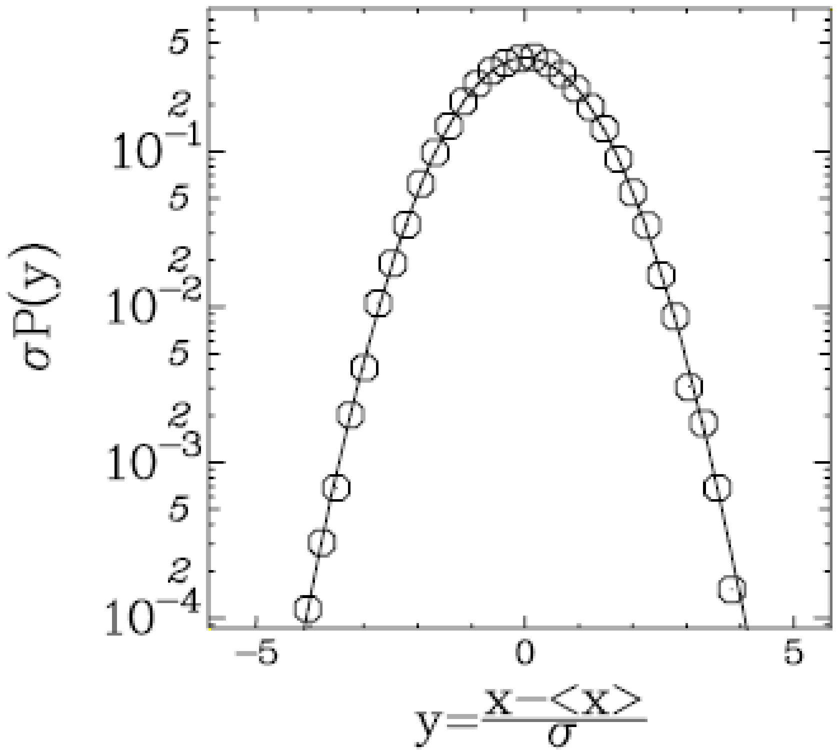


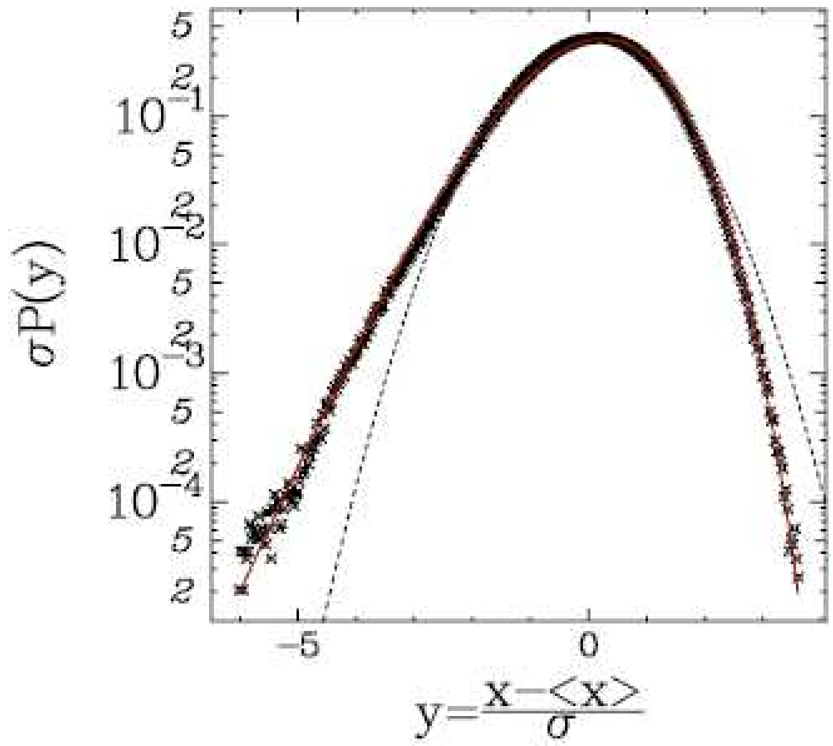
BHP has been observed in the fluctuations of :

- the injected power in an experiment of turbulence
- the magnetizaton in a xy model
- on the height of the Danube river

PDF of ζ

Cell with L=6.7 µm and $\tau_o \simeq 0.1s$





$\epsilon = 0.02$

Gaussian fluctuations

$\epsilon \simeq 0.001$

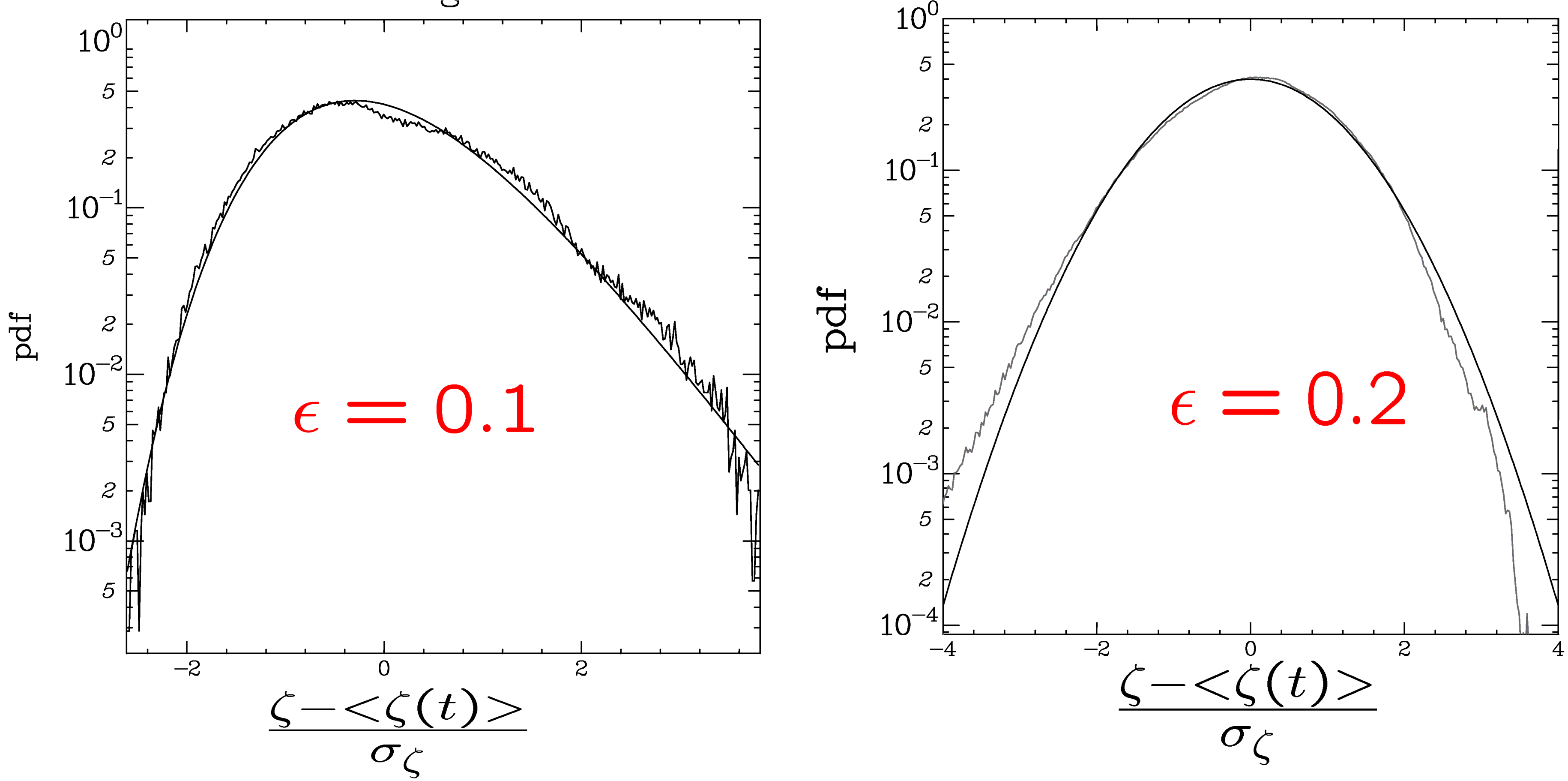
Non-Gaussian fluctuations

Gumbel model ($a \simeq 6$)

$\xi_r = \frac{L}{\pi\sqrt{\epsilon}} = 60\mu m$

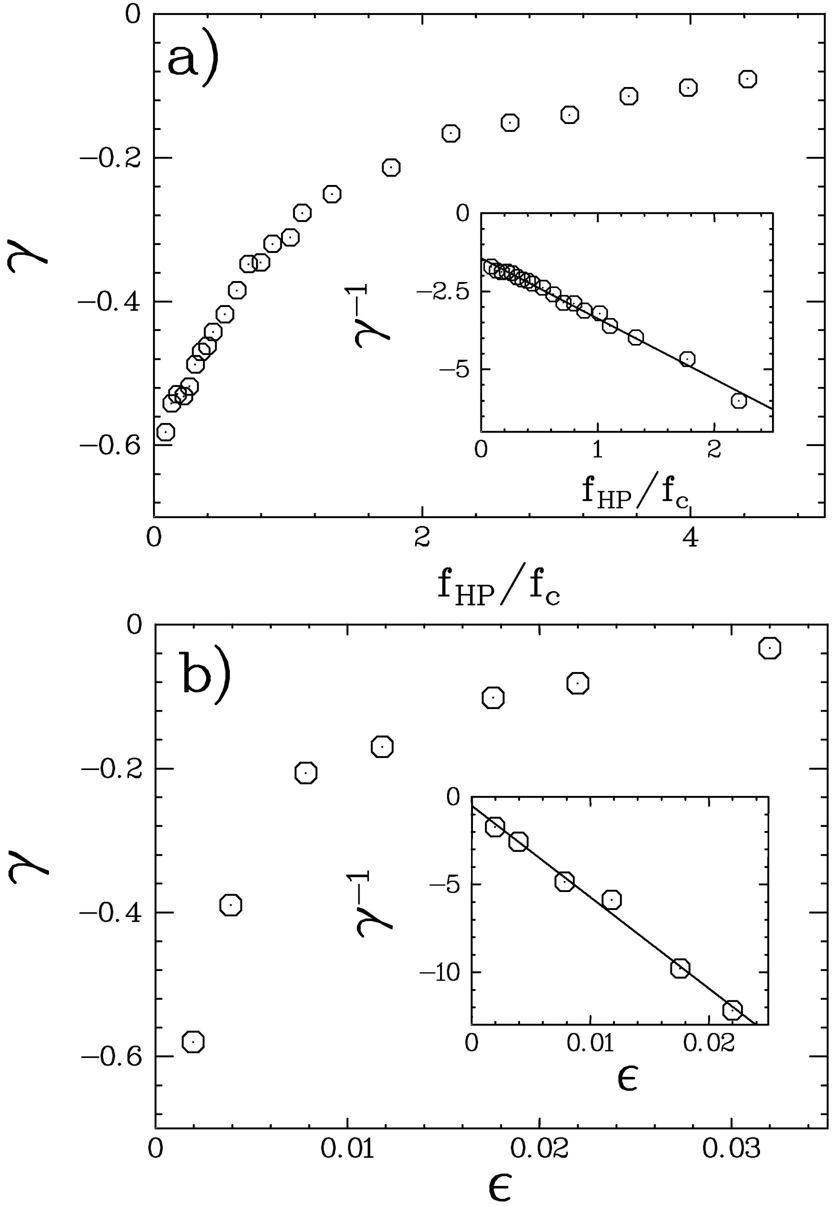
PDF of ζ

Cell with $L = 25\mu m$ $\tau_0 \simeq 2.5s$



- The BHP distributions are observed when the diameter of the probe is smaller then the correlation length
- The BHP is determined by the very slow motion

Dependence of γ as a functtion of ε and fc



$$\gamma = < y^3 > \simeq \frac{1}{\sqrt{a}}$$

Aging at critical point

(Calabrese, Gambassi)

At t=0 the system is rapidly quenched from $\epsilon = \epsilon_1$ to $\epsilon_0 = 0$.

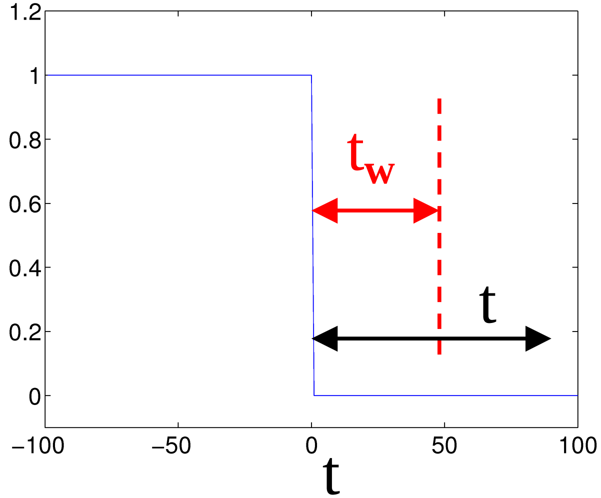
Within the Ginzburg-Landau approximation the dynamics of θ_0 is ruled by:

$$\tau_o \frac{d\theta_0}{dt} = -\theta_0^3$$

whose solution is

$$< \theta_0(t)^2 >= \frac{\tau_o}{2\ (\ t + \tau_m)} \quad \text{with} \quad \tau_m = \frac{\tau_o}{2\epsilon_1}$$

ε



The system remains out of equilibrium forever.

Experimentally the condition $\epsilon = 0$ is never realized

$\chi(t, t_w)$ and $C(t, t_w)$

Fluctuation dissipation theorem after a quench

(mean field)

We consider the fluctuations $\delta\theta(t) = \theta_0(t) - <\theta_0(t)>$

The correlation $C(t, t_w) =< \delta\theta(t)\delta\theta(t_w) >$ for $t_w < t$

$$C(t, t_w) = \frac{(t_w + \tau_m)^4 - \tau_m^4}{2(t_w + \tau_m)^{3/2}(t + \tau_m)^{3/2}}$$

$$R(t, t_w) = \frac{\Delta\theta_0}{\Delta h} = \frac{(t_w + \tau_m)^{3/2}}{(t + \tau_m)^{3/2}}$$ Response function

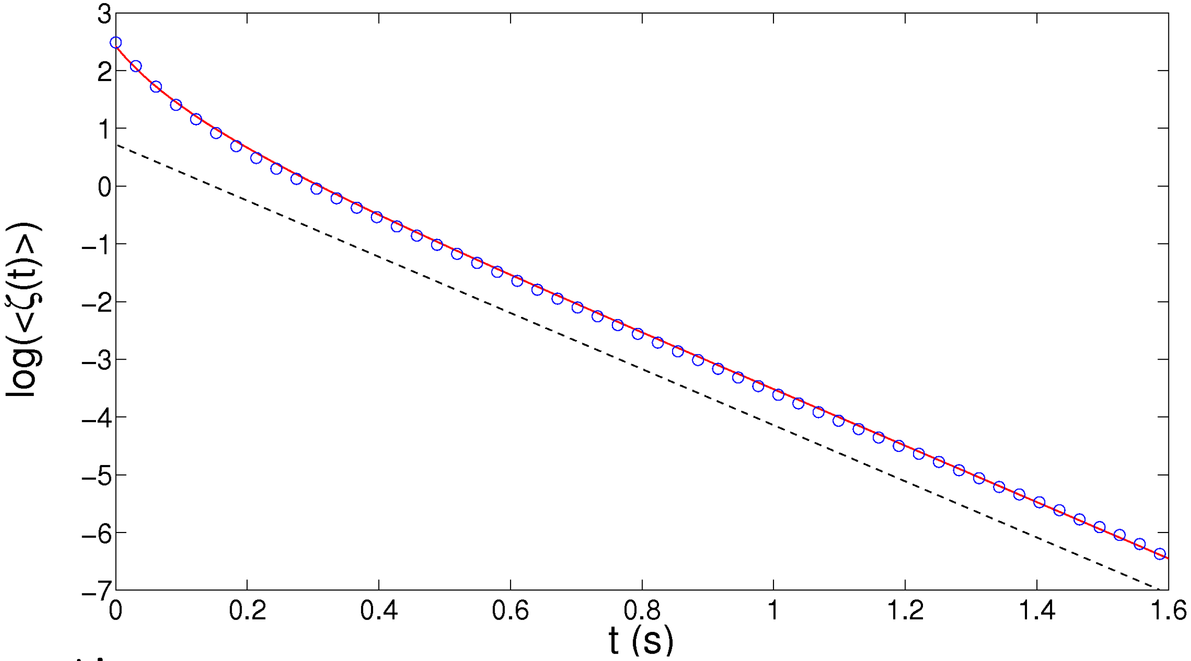
The fluctuation dissipation ratio (FDR) $X(t, t_w) = -k_B T \frac{R(t, t_w)}{\partial_{t_w} C(t, t_w)}$

In equilibrium $X(t, t_w) = 1$

After the quench at critical point $X(t, t_w) < 0.8$ $\forall t$ and t_w

Time evolution after a quench in LC

from $\epsilon_1 = 0.3$ to $\epsilon_0 = 0.01$



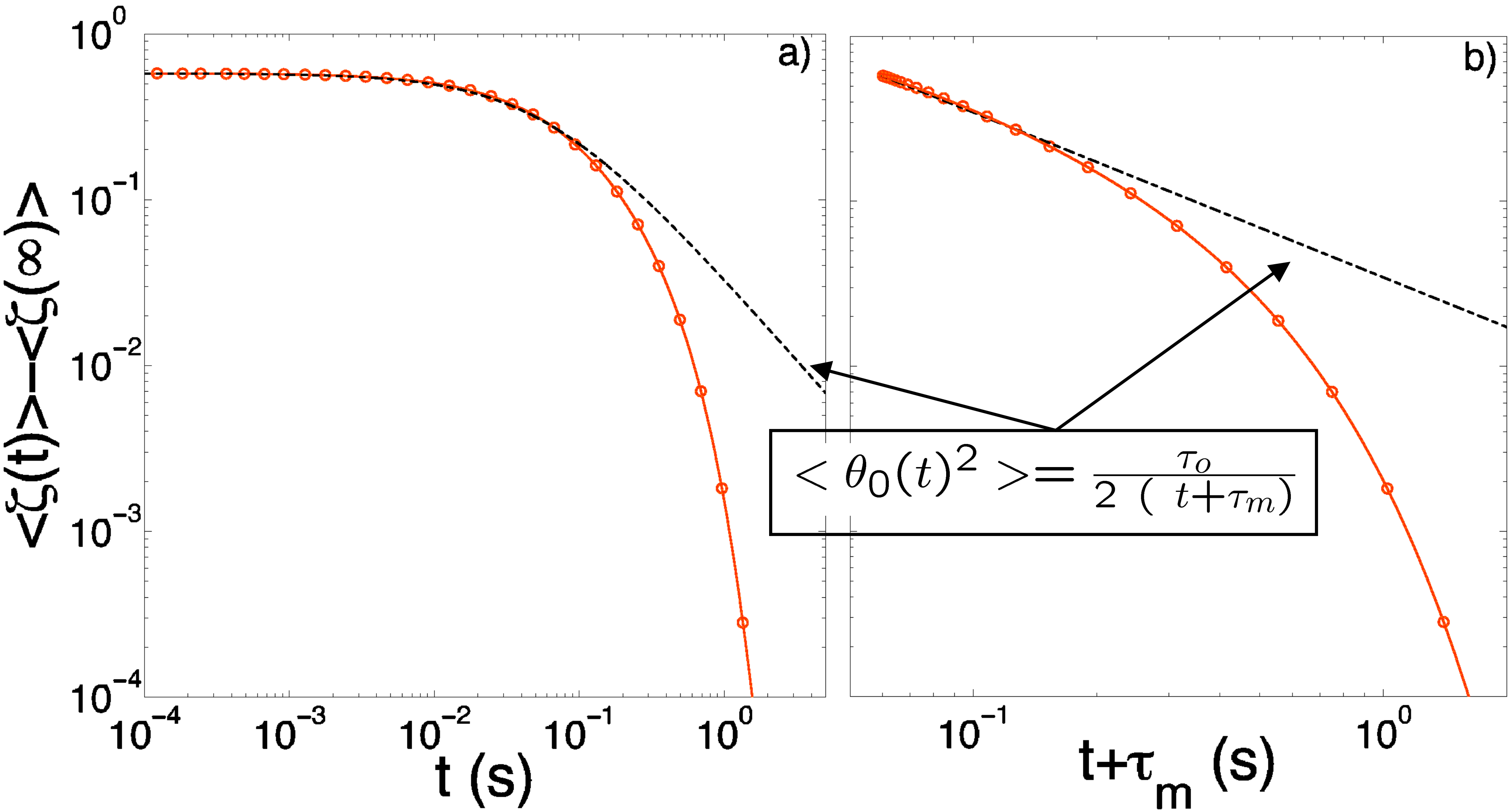
Fit function

$$\langle\ \zeta(t)\rangle = \frac{\langle\zeta(\infty)\rangle}{1+\left(\frac{\langle\zeta(\infty)\rangle}{\langle\zeta(0)\rangle}-1\right)\exp\left(-\frac{2\epsilon_0 t}{\tau_0}\right)},$$

with $<\zeta(\infty)>=\frac{2\epsilon_0}{(2k+\epsilon_0+1)}$ and $<\zeta(0)>=\frac{2\epsilon_1}{(2k+\epsilon_1+1)}$

Time evolution of ζ after a quench

Quench from $\epsilon_1 \simeq 0.3$ to $\epsilon_0 \simeq 0.01$ $\tau = \frac{\tau_0}{2\epsilon} \simeq 0.22s$



Correlations

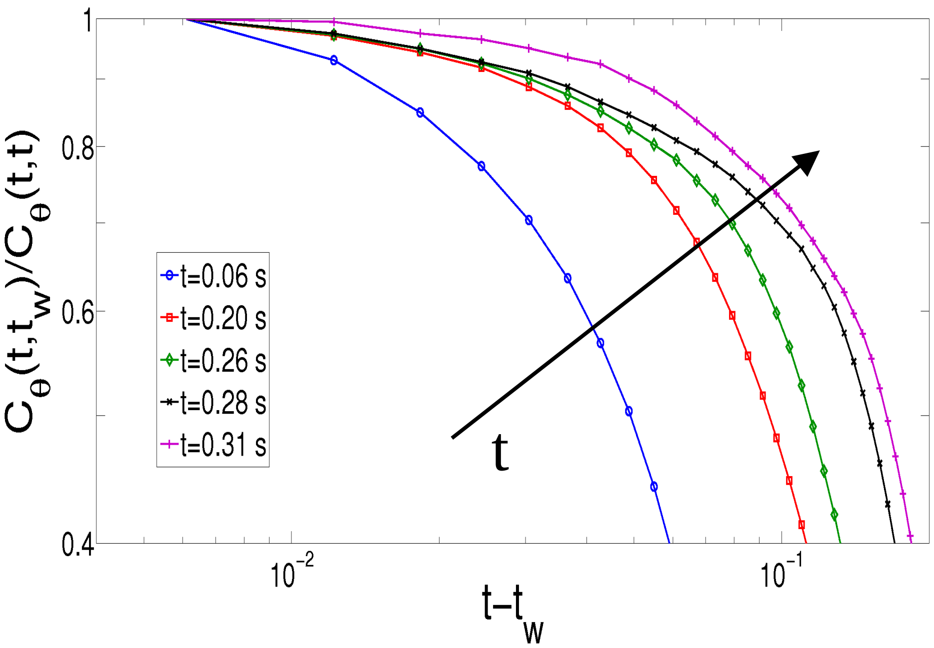
$$\zeta(t) = \theta_0(t)^2 = \psi_0^2(t) + 2\psi_0\delta\theta = < \zeta(t) > + \delta\zeta(t)$$

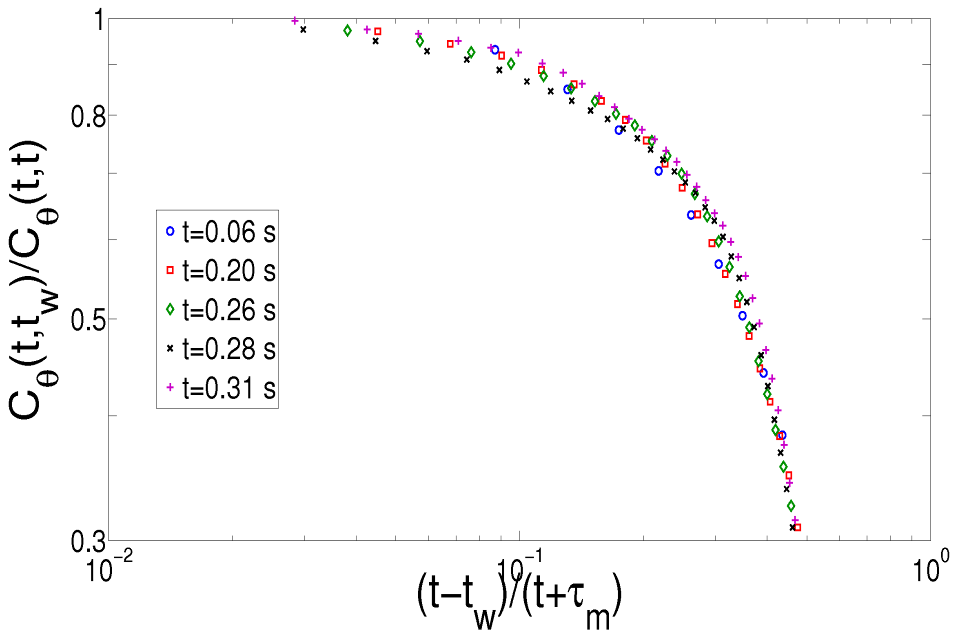
$$\psi_0^2(t) = < \zeta(t) >$$

$$C_\zeta(t, t_w) = < \delta\zeta(t)\delta\zeta(t_w) > = 4\psi_0(t)\psi_0(t_w)\ C_\theta(t, t_w)$$

Master curve by rescaling

$$(t - t_w) \rightarrow (t - t_w)/(t + \tau_m)$$





FDT in the LC experiment: the measure of the response function

$$\chi(t,t_w) = \frac{<\Delta>}{\Gamma_{ext}} = \int_{t_w}^{t} R(t,t')dt' \qquad C_\theta(t,t_w) =< \delta\theta(t)\delta\theta(t_w) >$$

FDR

$$\chi(t,t_w) = \frac{1}{k_B T_{eff}(t,t_w)}(C_\theta(t,t) - C_\theta(t,tw))$$

In equilibrium
$T = T_{eff}$

Which is the appropriate external torque Γ_{ext} for the LC ?

$$\frac{\gamma AL}{2}\frac{d\theta_0}{dt} = B\left[2\epsilon\theta_0 - (\kappa + \epsilon + 1)\,\theta_0^3\right] + \eta$$

$$\begin{cases} B = A\pi^2 K_1/4L \\ \epsilon = \epsilon_0 + \delta\epsilon \\ \theta_0(t) = \psi_0 + \Delta(t) \end{cases}$$

$$\frac{\gamma AL}{2}\frac{d\Delta}{dt} = B\left[2\epsilon_0 - 3\,(\kappa + \epsilon_0 + 1)\,\psi_0^2\right]\Delta + 2B\delta\epsilon\psi_0\left(1 - \frac{\psi_0^2}{2}\right) + \eta$$

Γ_{ext}

FDT in the LC experiment:

$$\delta\theta = \frac{\delta\zeta(t)}{2\psi_0(t)} \qquad \Gamma_{ext} = 4B\ \psi_0(t)\ \delta\epsilon\ \left(1 - \frac{\psi_0(t)^2}{2}\right)$$

Experimental test of these results (JSTAT P01033, 2009) :

1) Out of equilibrium, using the Transient Fluctuation Theorem
2) FDT in equilibrium

In equilibrium $\psi_0(t) = \psi_0(t_w) = \psi_0$

$$\chi(\tau) = \frac{\langle \Delta(\tau) \rangle}{\Gamma_{ext}} = \frac{\chi_{\zeta,\delta\epsilon}}{4B\ \psi_0^2\ \left(1 - \frac{\psi_0^2}{2}\right)}$$

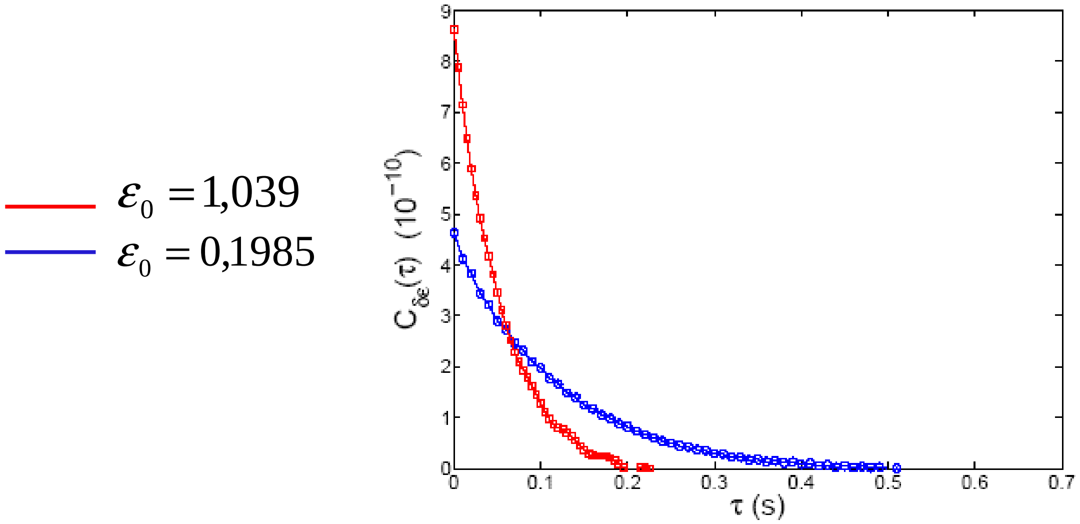
$$C_\zeta(t, t_w) = 4\psi_0^2\ C_\theta(t, t_w)$$

and FDT

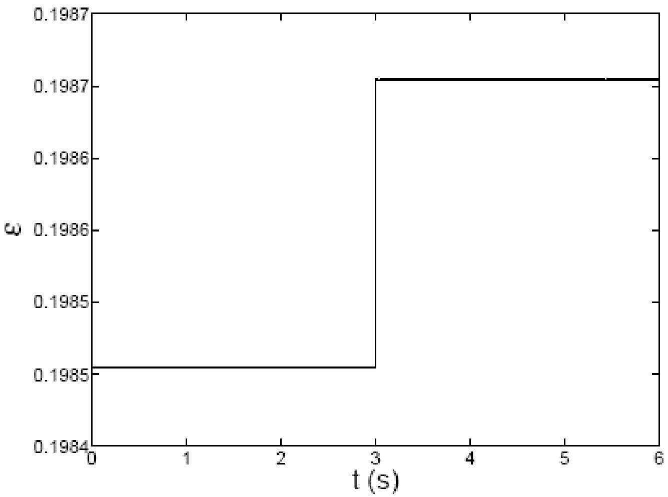
$$\frac{\chi_{\zeta,\delta\epsilon}}{B\left(1 - \frac{<\zeta>}{2}\right)} = \frac{1}{k_B T}(C_\zeta(t,t) - C_\zeta(t, t_w))$$

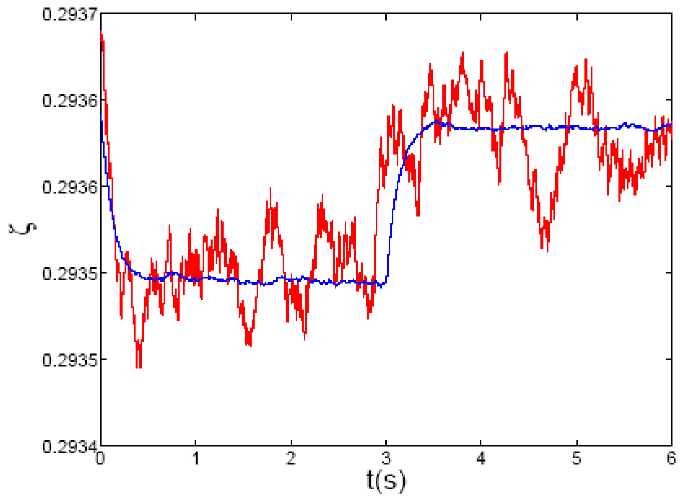
with $B = \mathcal{A}\pi^2 K_1 / 4L$

FDT in the LC experiment in equilibrium



- <u>response function</u>

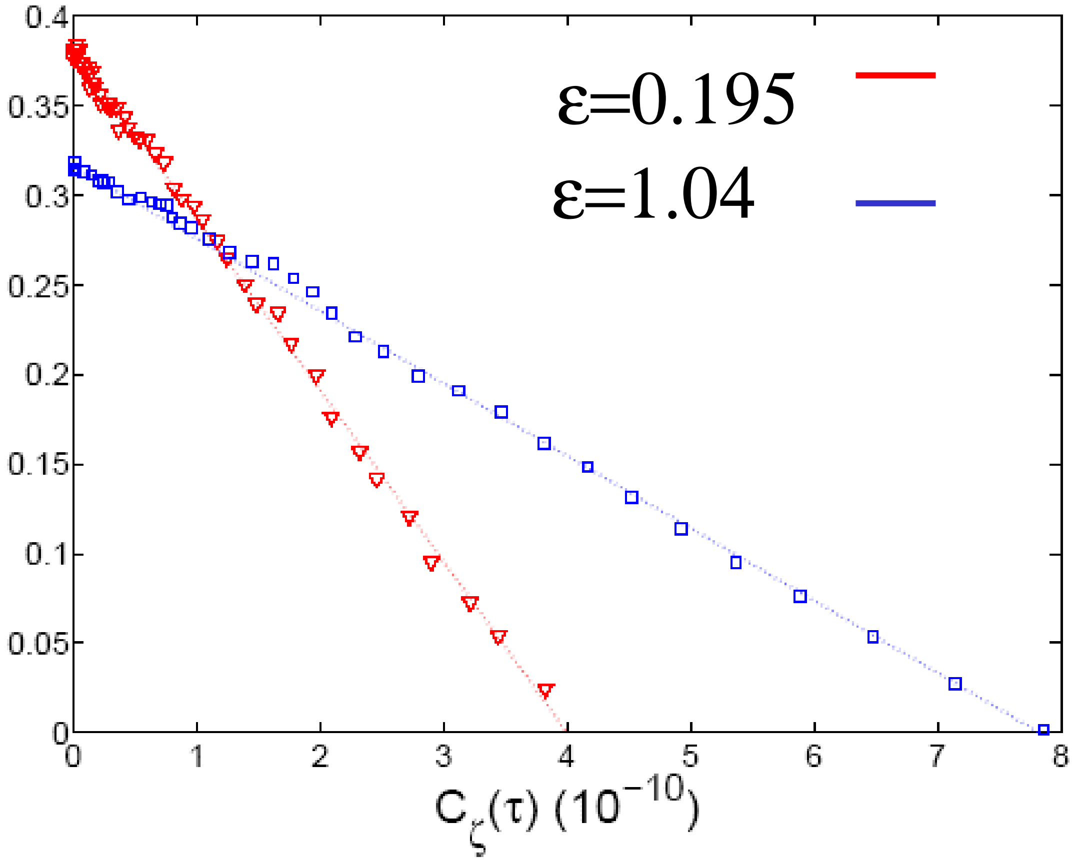




FDT in the LC experiment in equilibrium

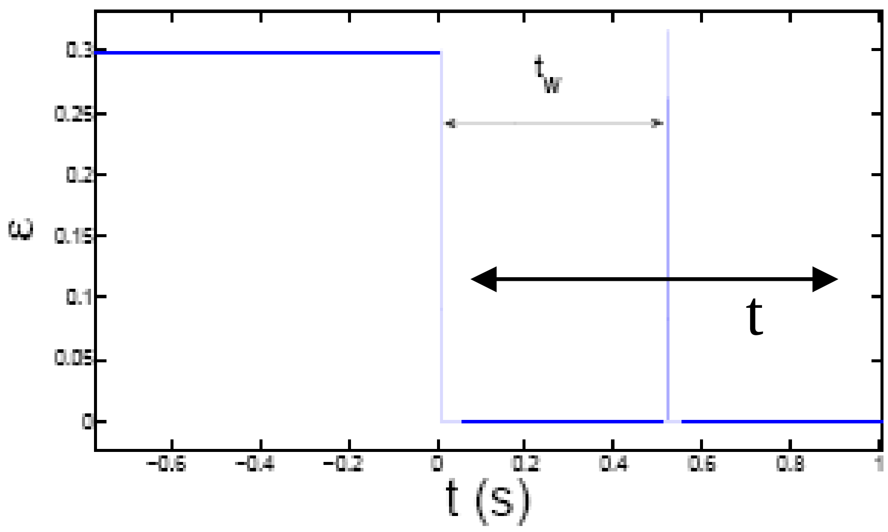
$$\frac{\chi_{\zeta,\delta\epsilon}}{B\left(1-\frac{<\zeta>}{2}\right)} = \frac{1}{k_B T}(C_\zeta(t,t) - C_\zeta(t,t_w)) \quad \text{with } B = \mathcal{A}\pi^2 K_1/4L$$

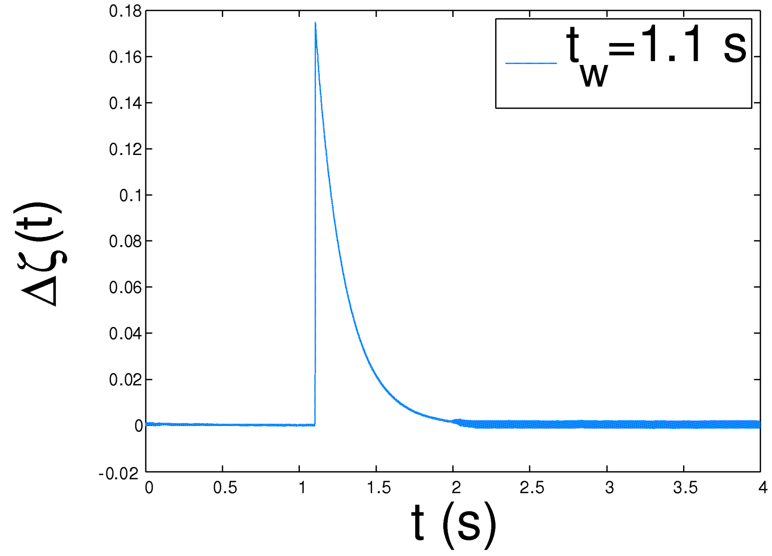
$\chi_{\zeta,\delta\epsilon}(t,t_w)$



at $\epsilon = 0.195$, $\frac{B\left(1-\frac{<\zeta>}{2}\right)}{k_B T} = 8.4\ 10^8$, $\mathcal{A} = 2.4mm^2$ and $D_0 \simeq 1.8mm$

Response function during aging





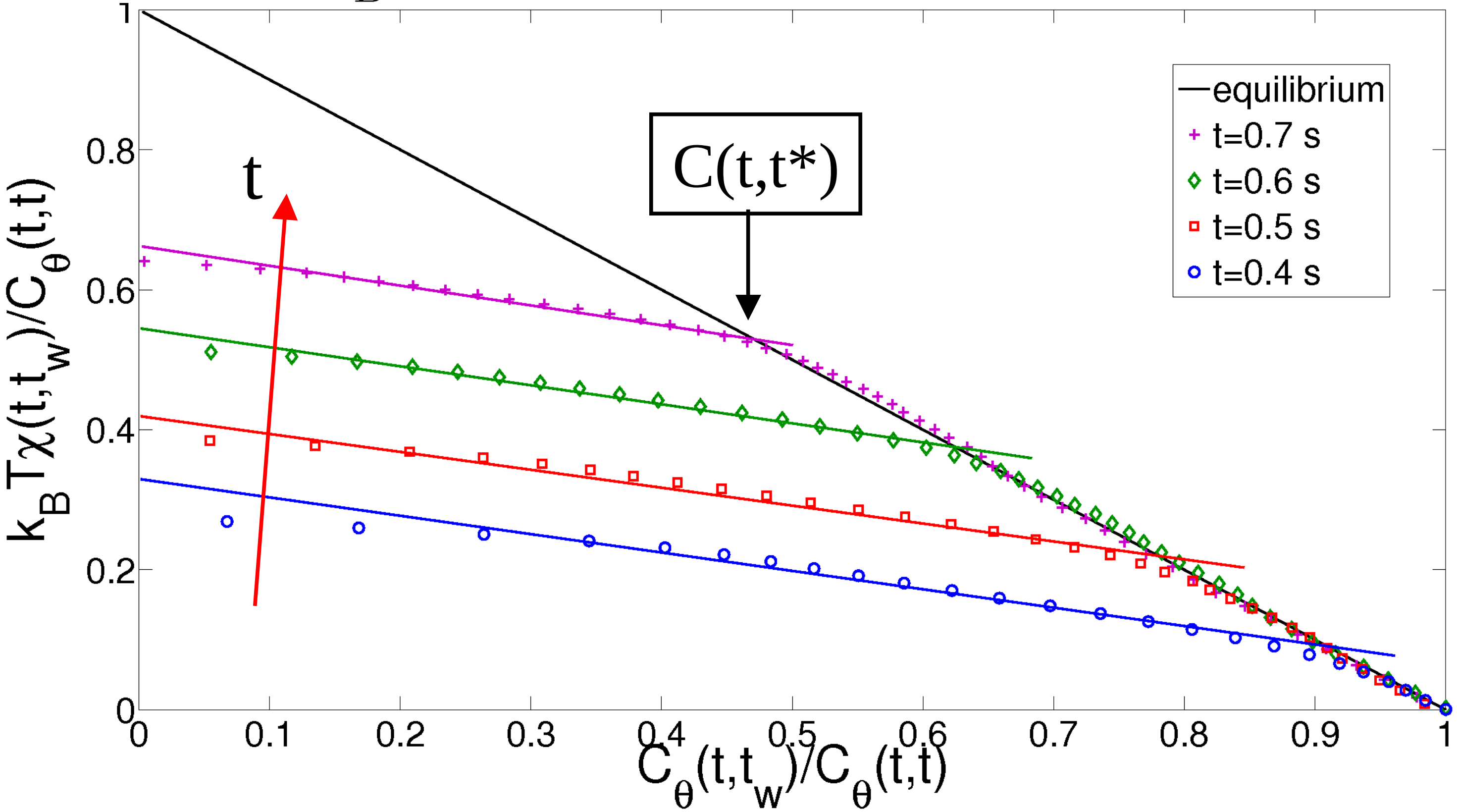
$$\Delta = \theta_0(t) - \psi_0(t) \quad \text{and} \qquad \Delta = \frac{\zeta(t) - \psi_0(t)^2}{2\psi_0(t)} = \frac{\zeta(t) - <\zeta(t)>}{2\Psi_0(t)} = \frac{\Delta\zeta}{2\psi_0(t)}$$

$$R(t,t_w) = \frac{<\Delta(t)>}{\Gamma_{ext}(t_w)} = \frac{R_{\zeta,\delta\epsilon}(t,t_w)}{4B\ \psi_0(t_w)\ \psi_0(t)\ \left(1 - \frac{\psi_0(t_w)^2}{2}\right)}$$

$$\chi(t,t_w) = \frac{<\Delta(t)>}{\Gamma_{ext}} = \int_{t_w}^{t} R(t,t')dt'$$

FDT out of equilibrium: fixed t as a function of tw

$$\chi(t,t_w) = \frac{X(t,t_w)}{k_B T}\left(C_\theta(t,t) - C_\theta(t,tw)\right) \qquad \text{with} \quad T_{eff} = \frac{T}{X(t,t_w)}$$



for $C(t,t_w) > C(t,t^*)$, $X = 1$ and $T_{eff} = T$

for $C(t,t_w) < C(t,t^*)$, $X \simeq 0.33$ and $T_{eff} \simeq 3T$

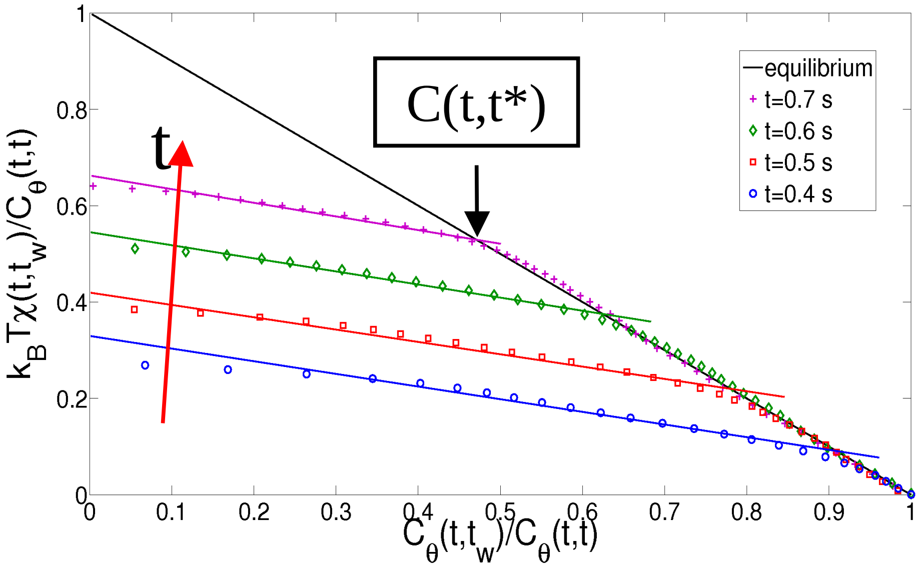
L. Cugliandolo, J. Kurchan, and L. Peliti, Phys. Rev. E 55, 3898 (1997).

D. Hérisson and M. Ocio, Phys. Rev. Lett. 88, 257202,(2002)

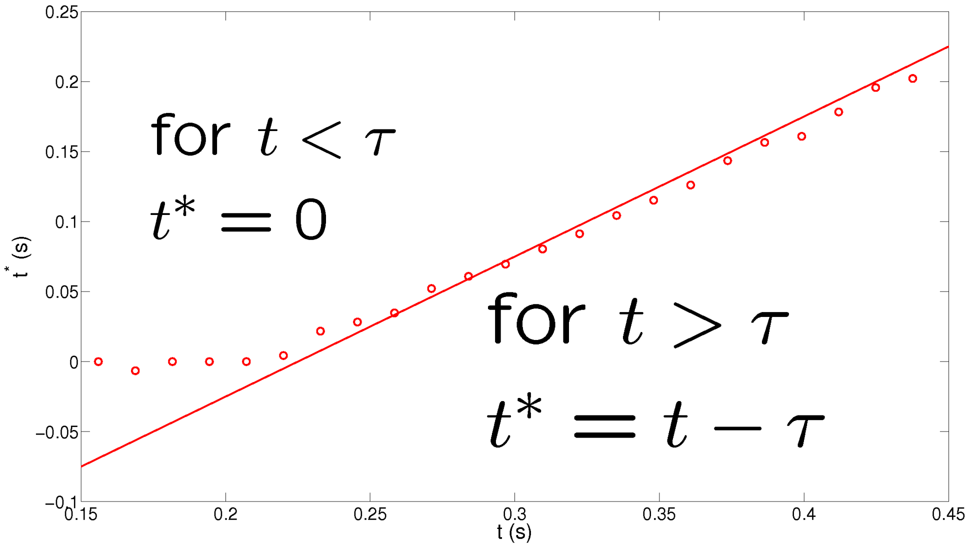
FDT out of equilibrium: fixed t as a function of tw

for $C(t, t_w) > C(t, t^*)$, $X = 1$ and $T_{eff} = T$

for $C(t, t_w) < C(t, t^*)$, $X \simeq 0.33$ and $T_{eff} \simeq 3T$



evolution of t* as a function of t



$\frac{t^*}{t} = 1 - \frac{\tau}{t}$ for $t > \tau$ defines the lenght of the equilibrium interval with respect to the total time.

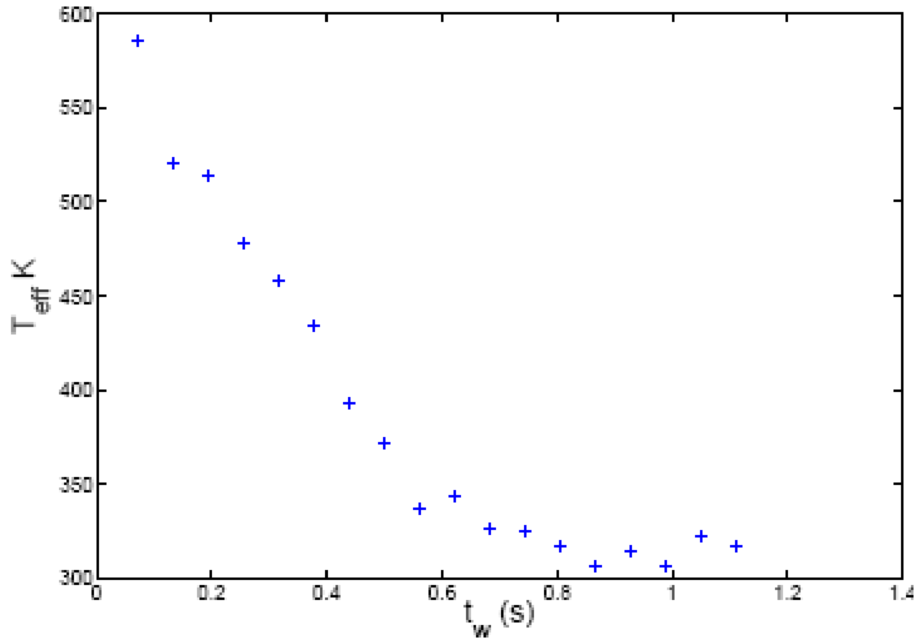
At $\epsilon_0 = 0, \Rightarrow \ \tau = \infty$: the equilibrium interval does not exist.

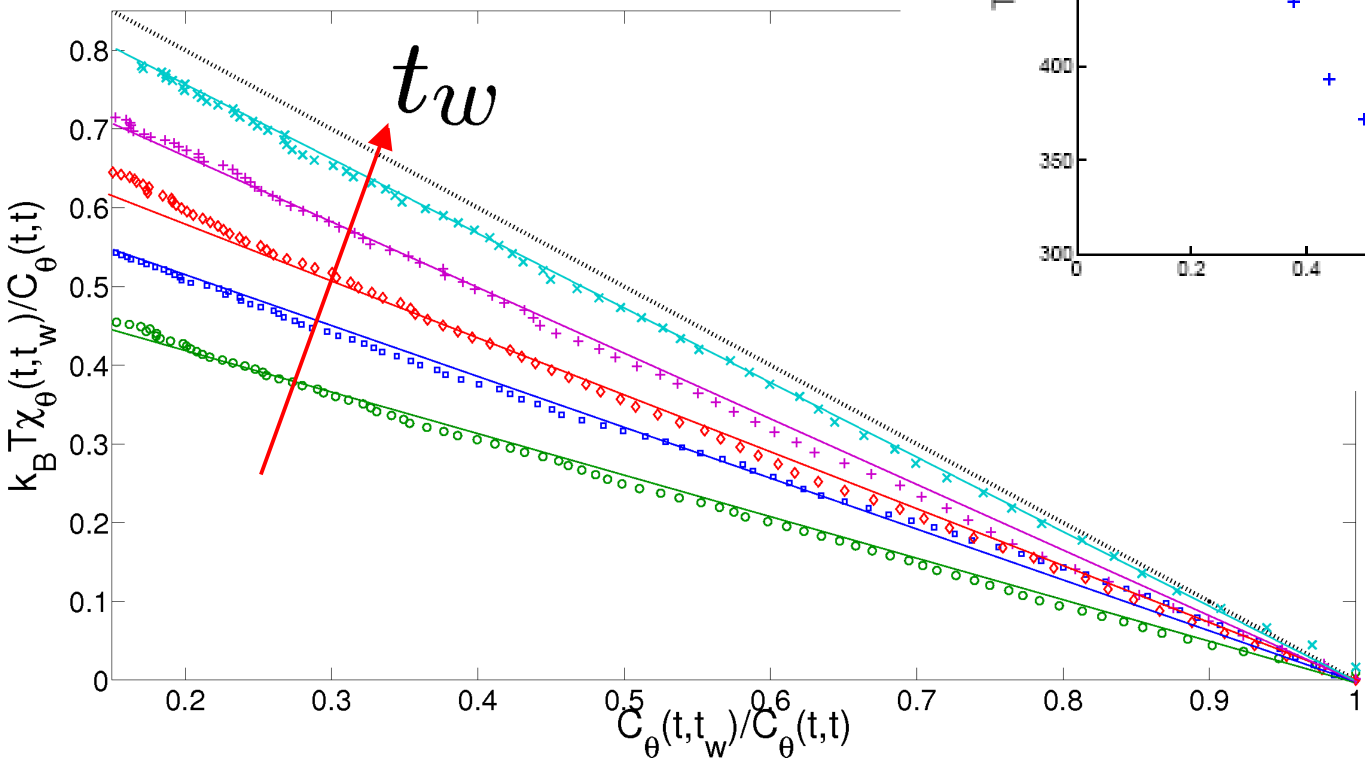
FDT out of equilibrium: fixed tw as a function of t

$$\chi(t, t_w) = \frac{X(t, t_w)}{k_B T}(C_\theta(t, t) - C_\theta(t, tw))$$

with $T_{eff} = \frac{T}{X(t, t_w)}$

evolution of Teff versus tw





Conclusions

Using a liquid crystal driven by an electric field at the Fréedericksz transition we observe that :

- The probability distribution function of the order parameter fluctuations are well described by a generalized Gumbel, when the correlation length is compared to the probe size.
- After a quench close to the critical point the system presents power law decay. A rescaling similar to the one used in aging materials, produces a master curve of correlations
- FDT is violated during the decay. The observed violation depends on the procedure used to define t and tw.
- For the ‘’good procedure’’ an asympothique temperature can be defined, which is not the one computed from mean field.