The stochastic version of the FT

Later proofs of FT for systems with stochastic dynamics were given by Kurchan, Lebowitz and Spohn, Farago.

R. van Zon and E.G.D. Cohen extended the results for the heat fluctuations.

The consequences of these extensions are extremely important to characterize out of equilibrium systems where thermal fluctuations cannot be neglected.

The predictions of the extended version of FT allow one to extimate tiny amount of heat either absorbed or dissipated by the system.

Noise in out of equilibrium devices

Fluctuations of the steady current through a system in contact between two reservoirs is one of the simplest and most fundamental problems of non equilibrium physics

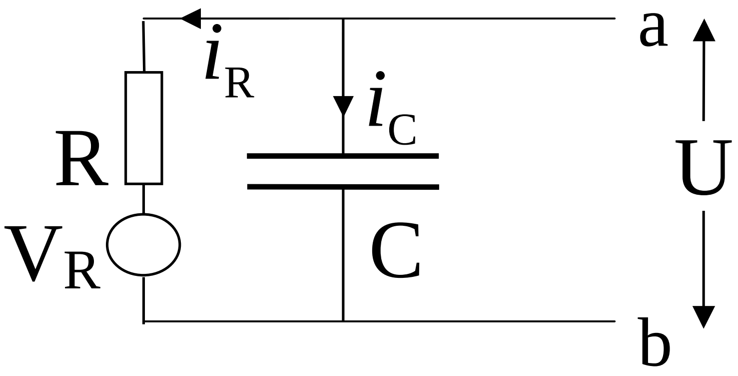
Electrical current fluctuations in a conductor

with N. Garnier, R. van Zon and E.D.G. Cohen

R. van Zon, S. Ciliberto, E. G. D. Cohen, Phys. Rev. Lett. 92, 130601 (2004).
N.Garnier, S. Ciliberto, P.R.E. July 2005

Thermal noise of an electrical resistance in equilibrium

Equivalent circuit



$V_R(t)$ is the noise voltage of the resistance which satisfies

$$\langle V_R(t) \rangle = 0$$

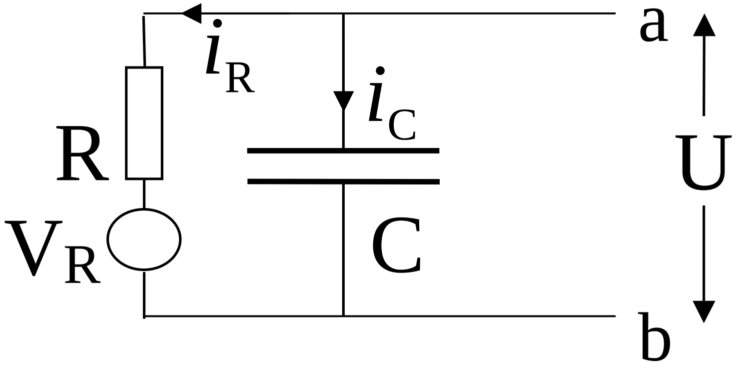
$$\langle V_R(t) V_R(t') \rangle = 2 K_B T R \, \delta(t - t').$$

The circuit impedance is: $Z(f) = \frac{U_\omega}{I_\omega} = \frac{R}{(1 + i\ 2\ \pi\ f\ R\ C)}$

From fluctuation-dissipation-theorem,
the corresponding noise spectrum S_U of U is (Nyquist):

$$S_U(f) = 4\ K_B\ T\ Real[Z(f)] = \frac{4\ K_B\ T\ R}{1 + (2\pi\ f\ R\ C)^2}$$

A specific example



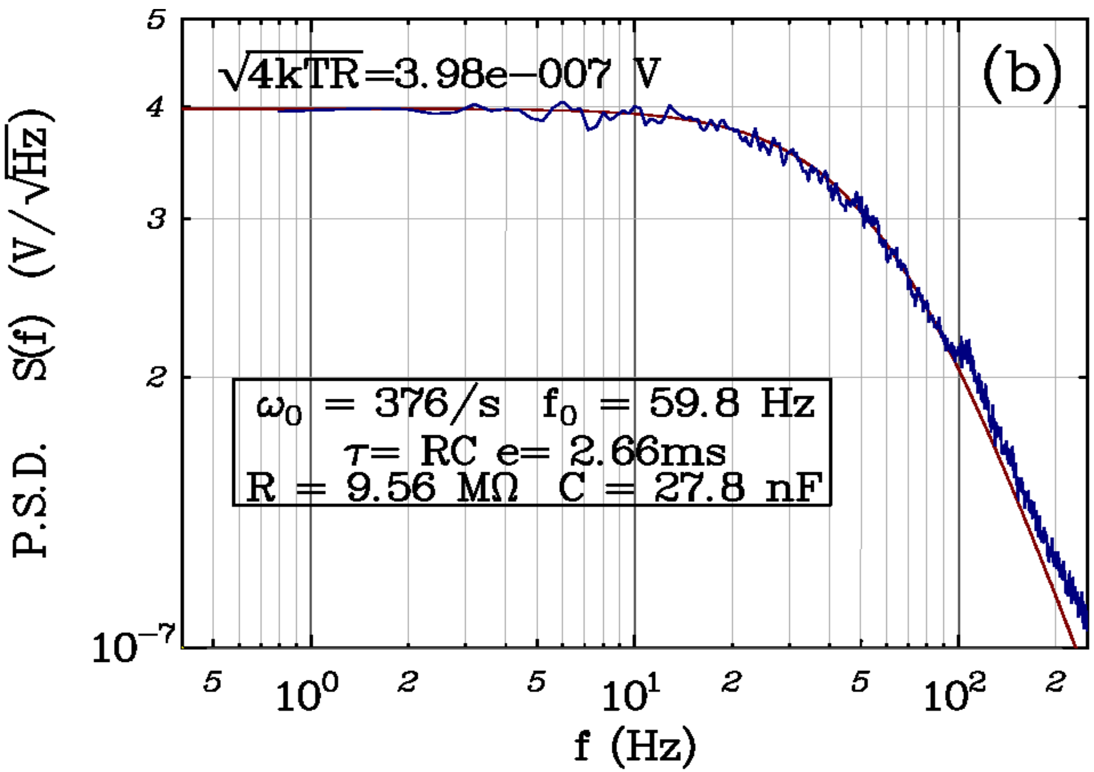
$$R = 9.52\ M\Omega$$

$$C = 200\ pF$$

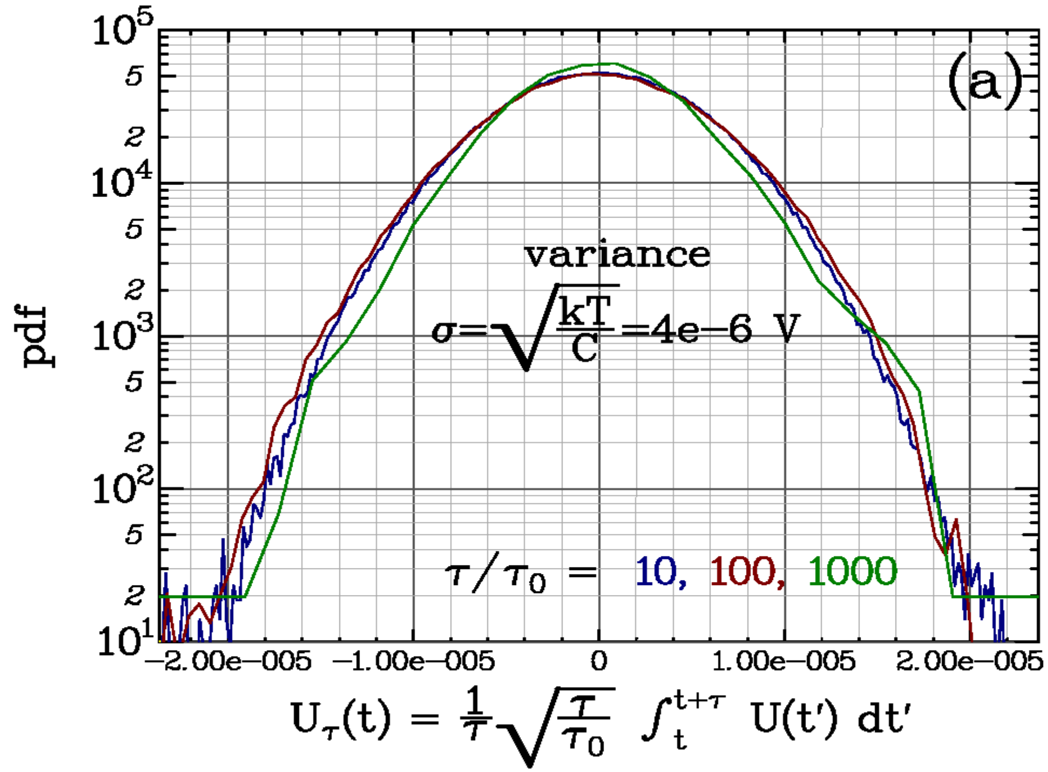
$$\tau_o = R\ C = 3ms$$

$$S_U(f) \simeq 400 \frac{nV}{\sqrt{Hz}} \text{ for } f < 1/(2\pi\ \tau_o)$$

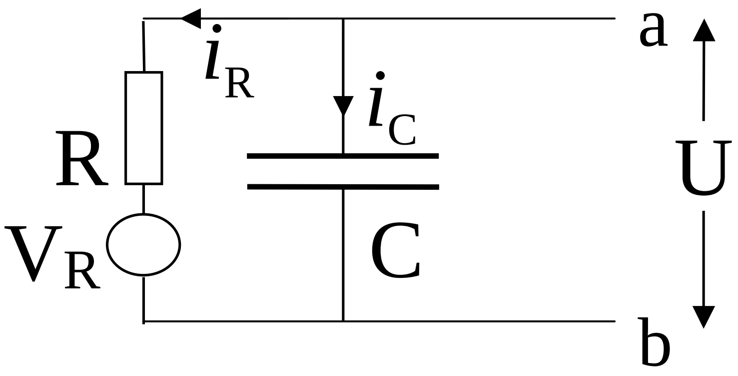
Noise spectrum



Pdf of U



Langevin equation for a resistance in equilibrium



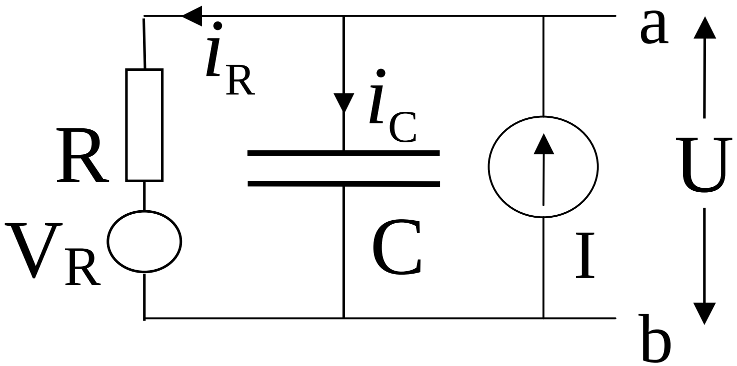
$$U = i_R \, R \; + \; V_R(t)$$

$$\frac{dq_R}{dt} = i_R, \qquad U = \frac{q_C}{C}, \qquad i_R + i_C = 0$$

$$R\frac{dq_R}{dt} = -V_R(t) - \frac{q_R}{C},$$

Langevin equation for a resistance out of equilibrium

A constant current I is injected into the circuit



$$U = i_R\ R + V_R(t)$$

$$\frac{dq_R}{dt} = i_R, \qquad U = \frac{q_C}{C}, \qquad i_R + i_C = I$$

$$R\frac{dq_R}{dt} = -V_R(t) - \frac{q_R}{C} + \frac{I\ t}{C}$$

General problem

$$R\frac{dq_R}{dt} = -V_R(t) - \frac{q_R}{C} + \frac{I\ \ t}{C}$$

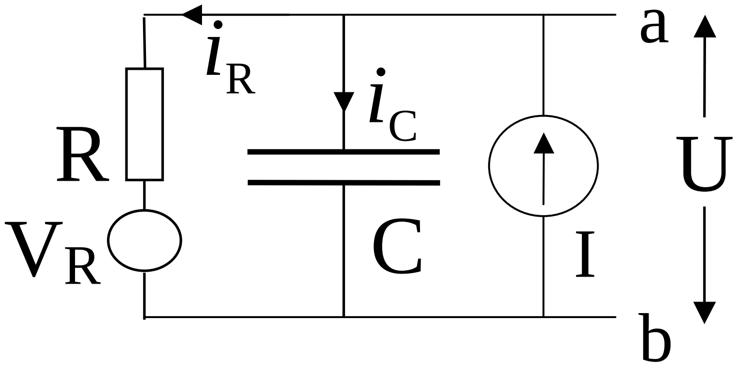
Analogy of the circuit and a Brownian particle

$$m\frac{d^2x}{dt^2} = -\alpha\frac{dx}{dt} + \xi_t - k(x - v^*t).$$

Brownian particle:	ξ_t	α	T	x_t	v_t	k	v^*
RC circuits:	$-V_R$	R	T	q_R	i_R	$1/C$	I

Example: Particle of negligible mass trapped by an opital tweezer moving at constant speed v^*

Out of equilibrium



$$I = i_R + i_C$$

$$i_c = \frac{dq_C}{dt} = C \frac{dU}{dt}$$

$$I = i_R + C\frac{dU}{dt} \quad \text{and} \quad \mathcal{P}_{\text{in}} = \mathcal{P}_{\text{diss}} + \frac{C}{2}\frac{dU^2}{dt}$$

Power injected into the system:

$$\mathcal{P}_{\text{in}} = UI$$

Power is dissipated in the resistive part only

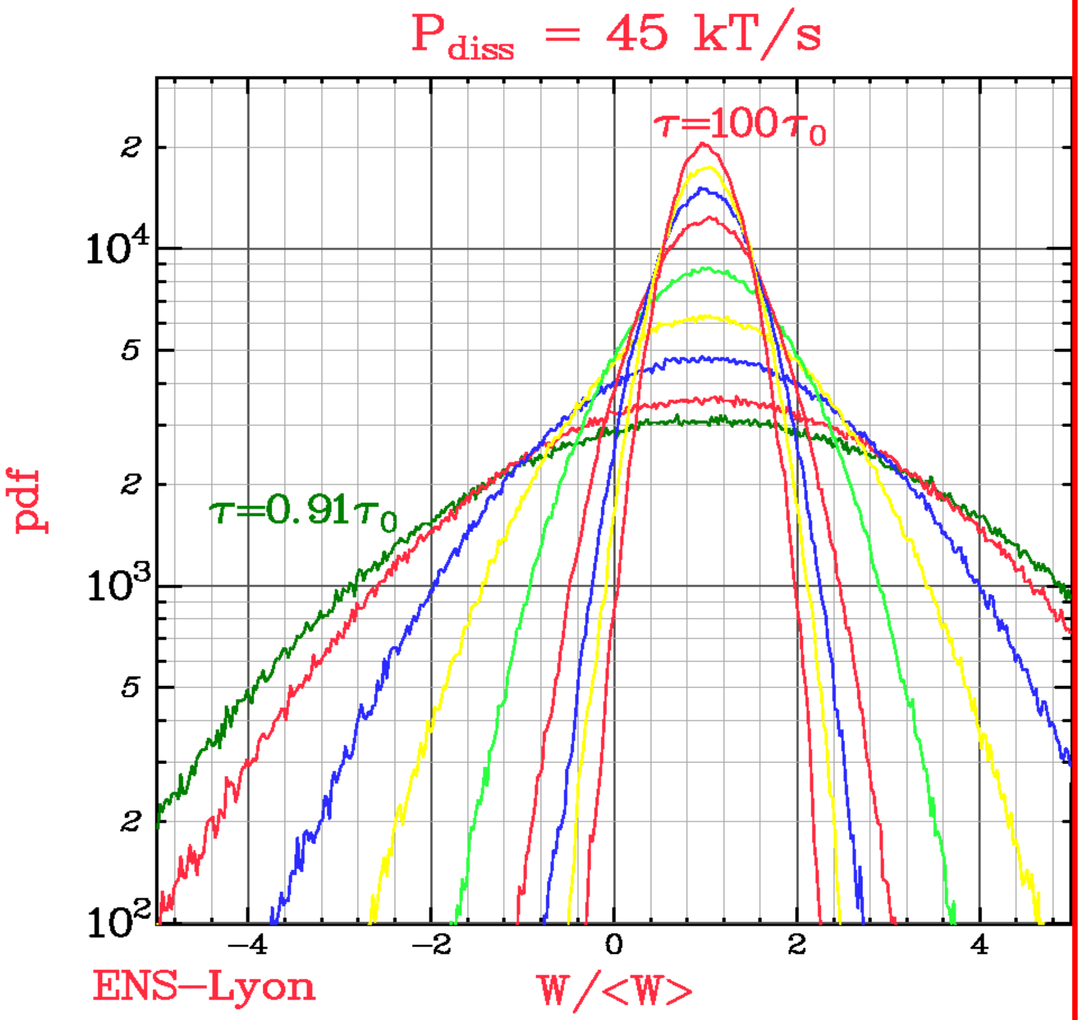
$$\mathcal{P}_{\text{diss}} = U i_{\text{R}}$$

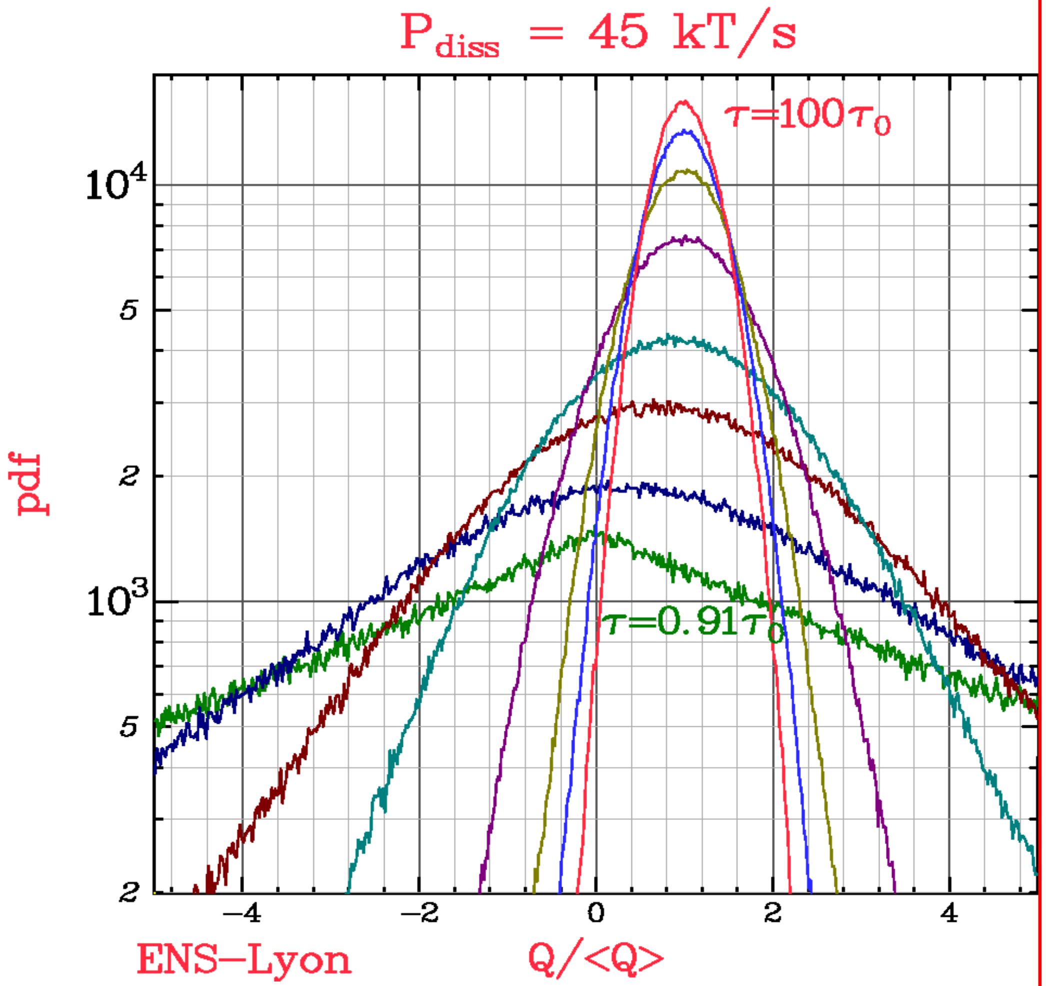
Work and Heat Fluctuations

$I = 1.4\ 10^{-13} A$

$$W_\tau = \int_0^\tau dt\ I\ U$$

$$Q_\tau = \int_0^\tau dt\, i_R\ U$$





Fluctuation function for the work : $f_\tau^W \equiv \frac{K_B T}{\langle W_\tau \rangle} \ln \left[\frac{P(+W_\tau)}{P(-W_\tau)} \right]$

FT for the work

$$f_\tau^W = \frac{\frac{W_\tau}{\langle W_\tau \rangle}}{1 - \varepsilon(\tau)}$$

where $\varepsilon(\tau) = \frac{\tau_o(1 - e^{-\tau/\tau_o})}{\tau}$ and $\tau_o = RC$.

For $\tau \to \infty$ FT fixes the variance σ_W^2 of work fluctuations.
As $P(W)$ is gaussian then from FT

$$\sigma_W^2 = 2 < W_\tau > K_B\, T = I^2 R\, \tau\, K_B\, T$$

Fluctuation function for the heat : $f_\tau^Q \equiv \frac{k_B T}{\langle Q_\tau \rangle} \ln \left[\frac{P(+Q_\tau)}{P(-Q_\tau)} \right]$

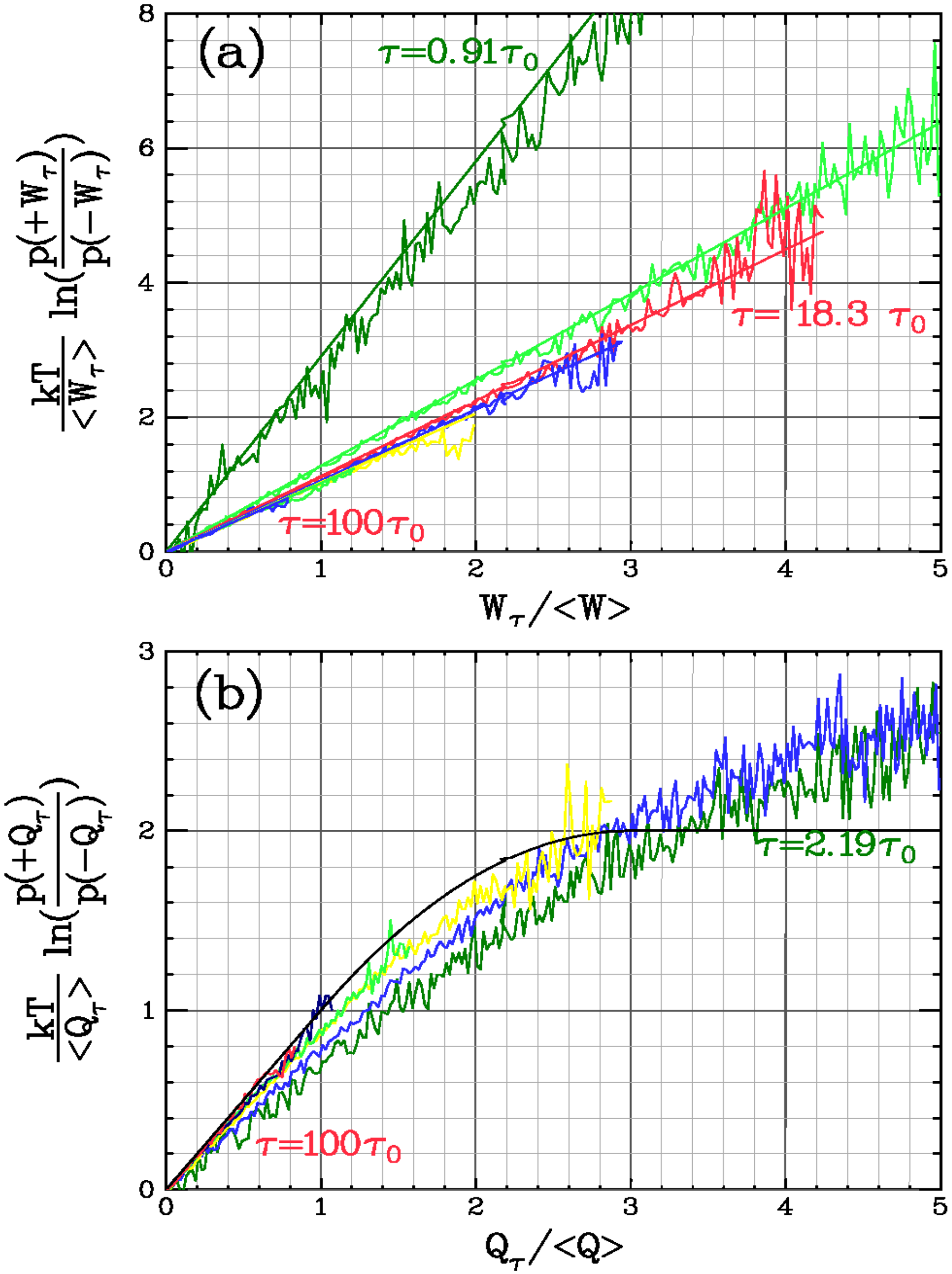
FT for the heat

For $\tau \to \infty$:

$$f_\tau^Q(p_Q) = \begin{cases} p_Q + O(\tau^{-1}) & \text{if } 0 < p_Q < 1 \\ p_Q - \frac{1}{4}(p_Q - 1)^2 + O(\tau^{-1}) & \text{if } 1 < p_Q < 3 \\ 2 + O\left(\sqrt{(p_Q - 3)/\tau}\right) & \text{if } p_Q > 3. \end{cases}$$

where $p_Q \equiv Q_\tau / \langle Q_\tau \rangle$

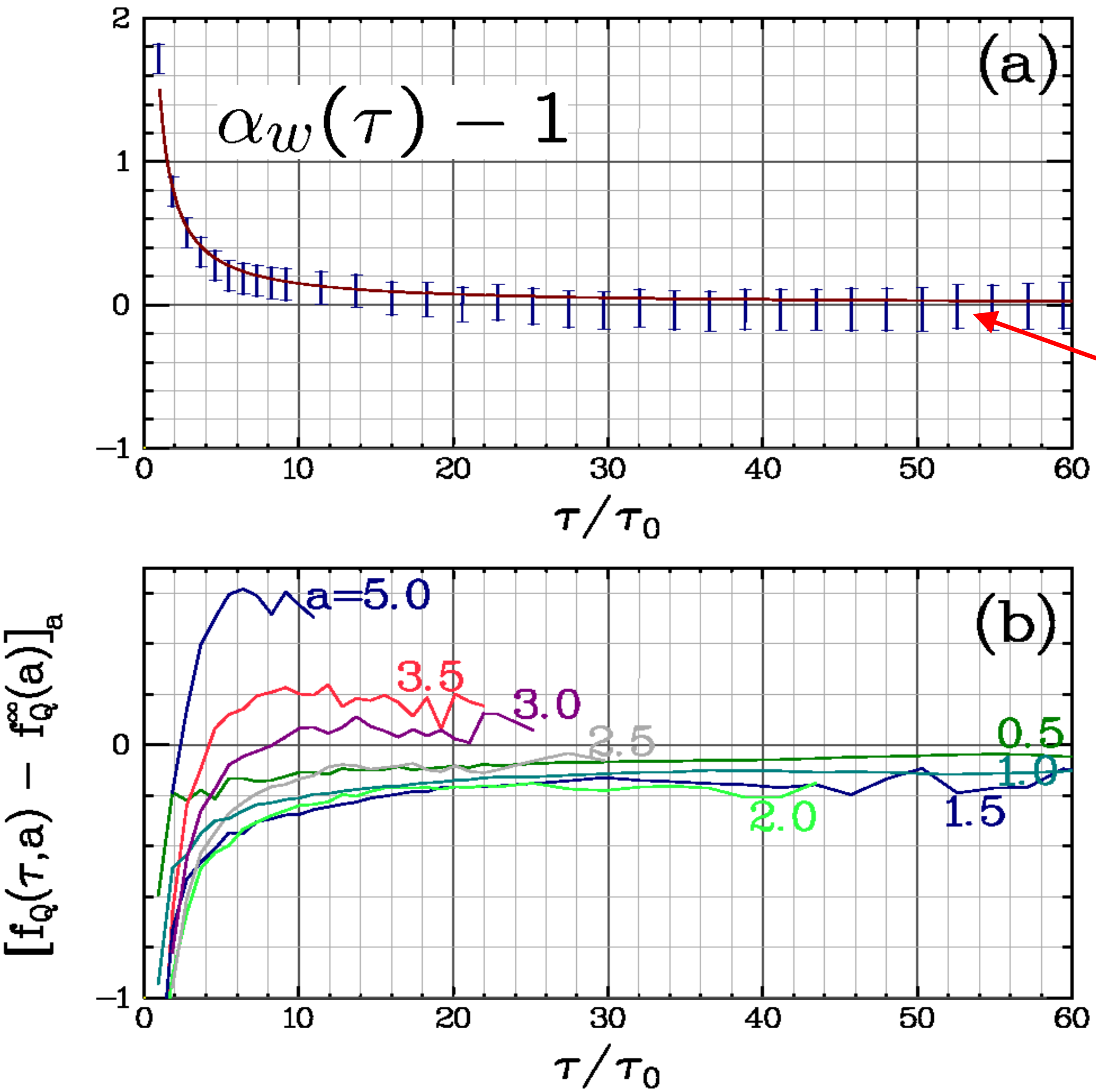
Fluctuation functions



$$f_\tau^W = \frac{\frac{W_\tau}{\langle W_\tau \rangle}}{1-\varepsilon(\tau)},$$

$$f_\infty^Q(p_Q) = \begin{cases} p_Q & \text{if } 0 < p_Q < 1 \\ p_Q - \frac{1}{4}(p_Q - 1)^2 & \text{if } 1 < p_Q < 3 \\ 2 & \text{if } p_Q > 3. \end{cases}$$

Convergence at P_{in}=45 K_BT



$$f_\tau^W = \alpha_w(\tau)\ \frac{W_\tau}{\langle W_\tau \rangle}$$

$$\frac{1}{1-\varepsilon(\tau)} - 1$$

Conclusion on electrical circuit

- We have studied the work and heat fluctuations in an electric circuit.
- The convectional FT holds for work but not for heat which presents a much more complex behavior.
- The analogy of this circuit with a Brownian motion of a particle trapped in a moving potential shows the generality of these results
- FT can be used in order to extract the average power from the measure of the fluctuations

Perspectives

- This is a starting point for more complex systems (non-linear resistance, aging resistance, more complex heat dissipation).