Even if you're not burning books,
destroying information generates heat.

Information and Thermodynamics: Experimental verification of Landauer's erasure principle with a colloidal particle

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Nature 483, 187-189 (2012)

Outline

- The foundamental laws of Thermodynamics
- Landauer's principle
- How to realise it ?
- Experimental set-up
- Data analyis
- Comparison with numerical results
- Conclusions

Main Laws of Thermodynamics I

Clausius

The **First Law of Thermodynamics** is a version of the **Law of Conservation of Energy**

Clausisus statement of **the First Law**

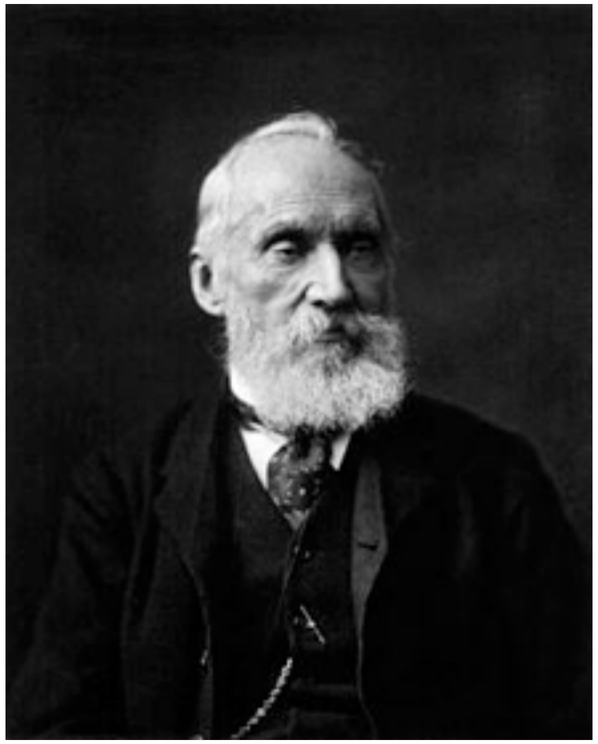
In a thermodynamic process, the increment in the internal energy of a system is equal to the difference between the heat exchanged by the system with the heat bath and the increment of work done on it.

$$\Delta U_{A,B} = W_{A,B} - Q$$

Main Laws of Thermodynamics II

The Second Law is a statement about irreversibility.
It is usually stated in physical terms of impossible processes.

Sadi Carnot was the first to give a formulation of this principle



Lord Kelvin

Lord Kelvin Statement of **the Second Law**

No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work.

Main Laws of Thermodynamics III

The **Second Law of Thermodynamics** is related to the concept of **Entropy**

$$\Delta S = \frac{Q}{T} \qquad\qquad \Delta S_{tot} \geq 0$$

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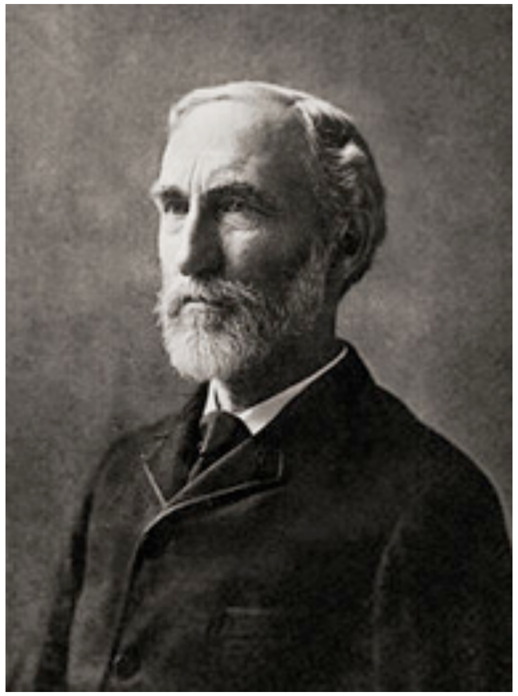

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$$S = -k_B \Sigma_i p_i \log p_i$$

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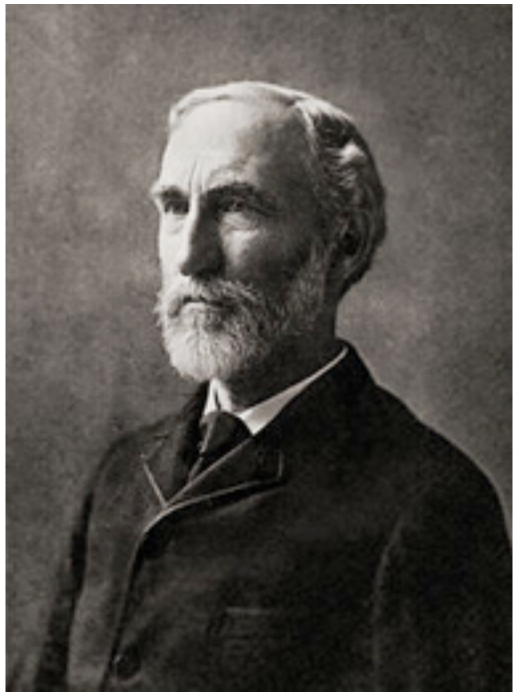

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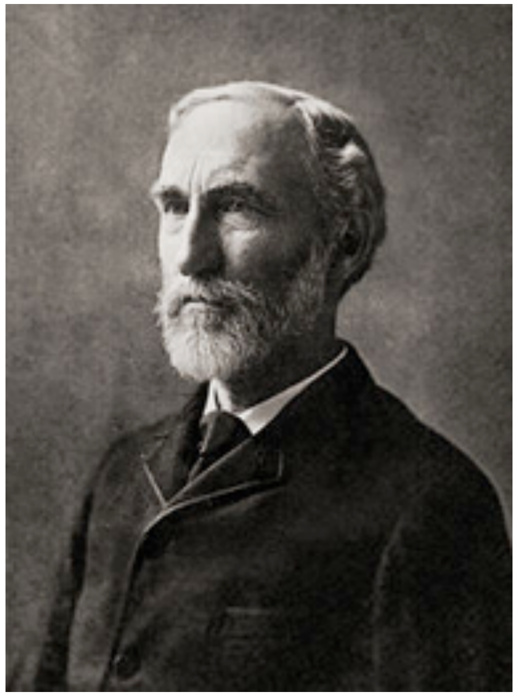

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Question : is Thermodynamic entropy the same that information entropy ?

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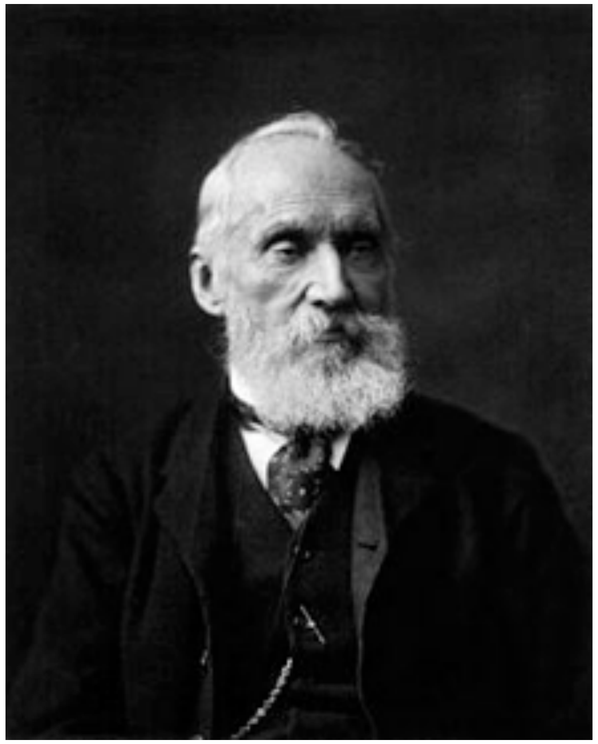

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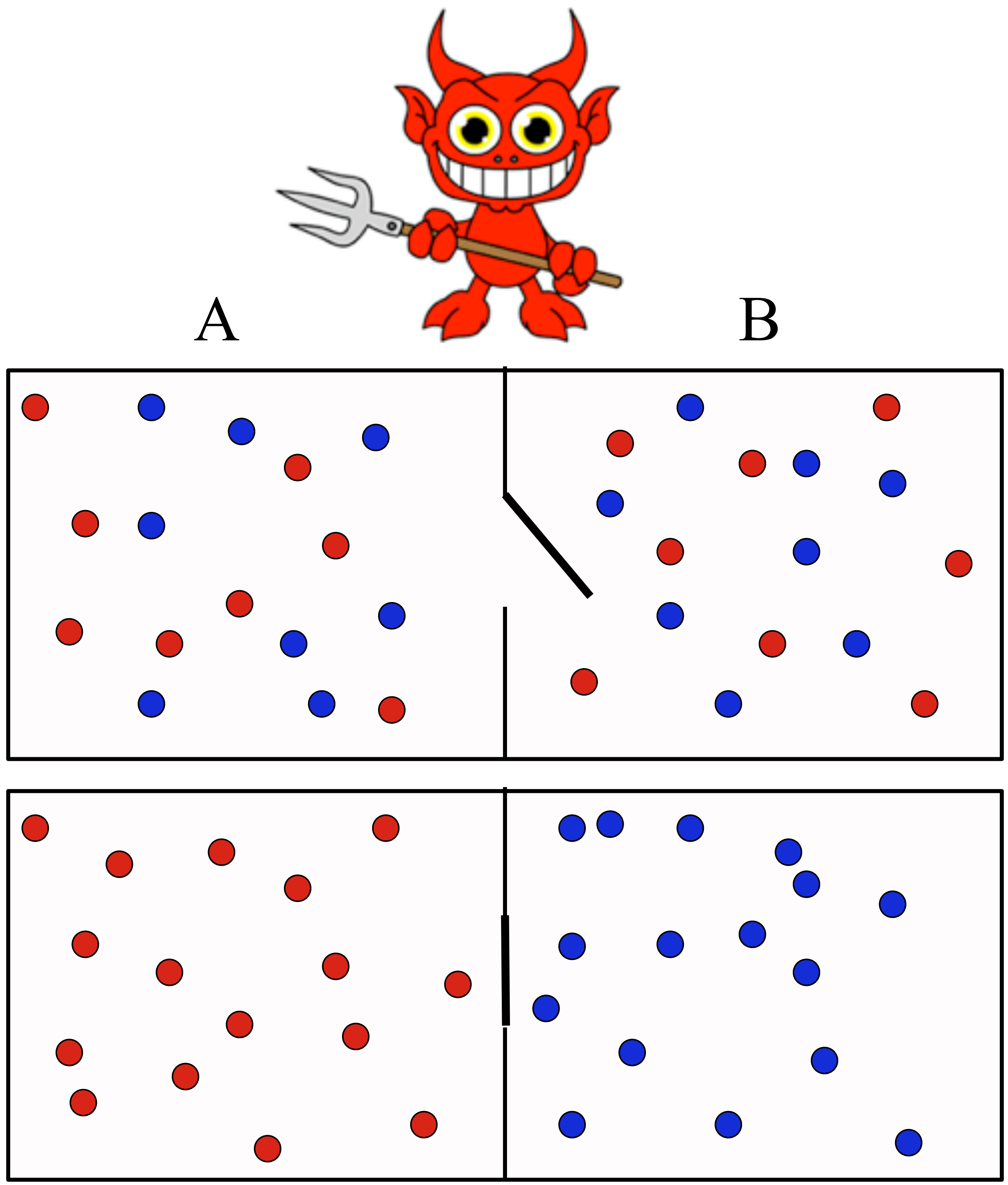
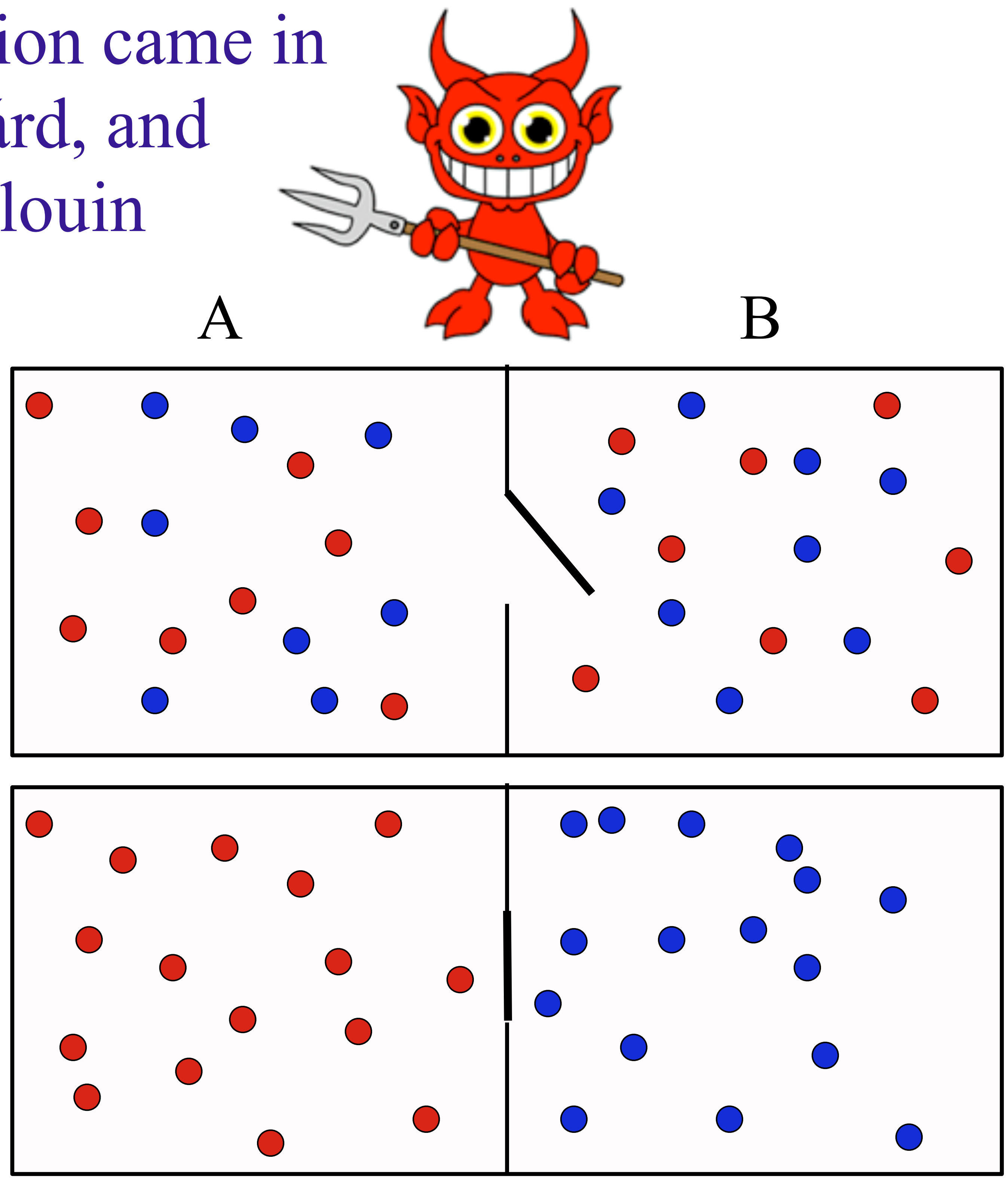
Landauer's Principle and The Maxwell's Demon



- slow molecules
- fast molecules

Landauer's Principle and The Maxwell's Demon

The first explanation came in 1929 by Leó Szilárd, and later by Léon Brillouin



- **slow molecules**
- **fast molecules**

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The Landauer's principle (I)

Any logically irreversible transformation of classical information is necessarily accompanied by the dissipation of at least $k_B T \cdot \ln 2$ of heat per lost bit (about $3 \cdot 10^{-21}$ Joules at room temperature)

Typical examples of logically irreversible transformations are Boolean functions such as AND, NAND, OR and NOR

They map several input states onto the same output state

The erasure of information, the RESET TO ONE operation, is logically irreversible and leads to an entropy production of $k_B \cdot \ln 2$ per erased bit

Landauer’s principle II

Landauer’s principle is a central result which exorcises the Maxwell’s demon

It has been criticised and never tested in a real experiment

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Questions

- Can the Landauer's limit be reached in any experiment ?
- Does any experimentally feasable procedure allow us to reach the limit ?

Landauer's principle II

Landauer's principle is a central result which exorcises the Maxwell's demon

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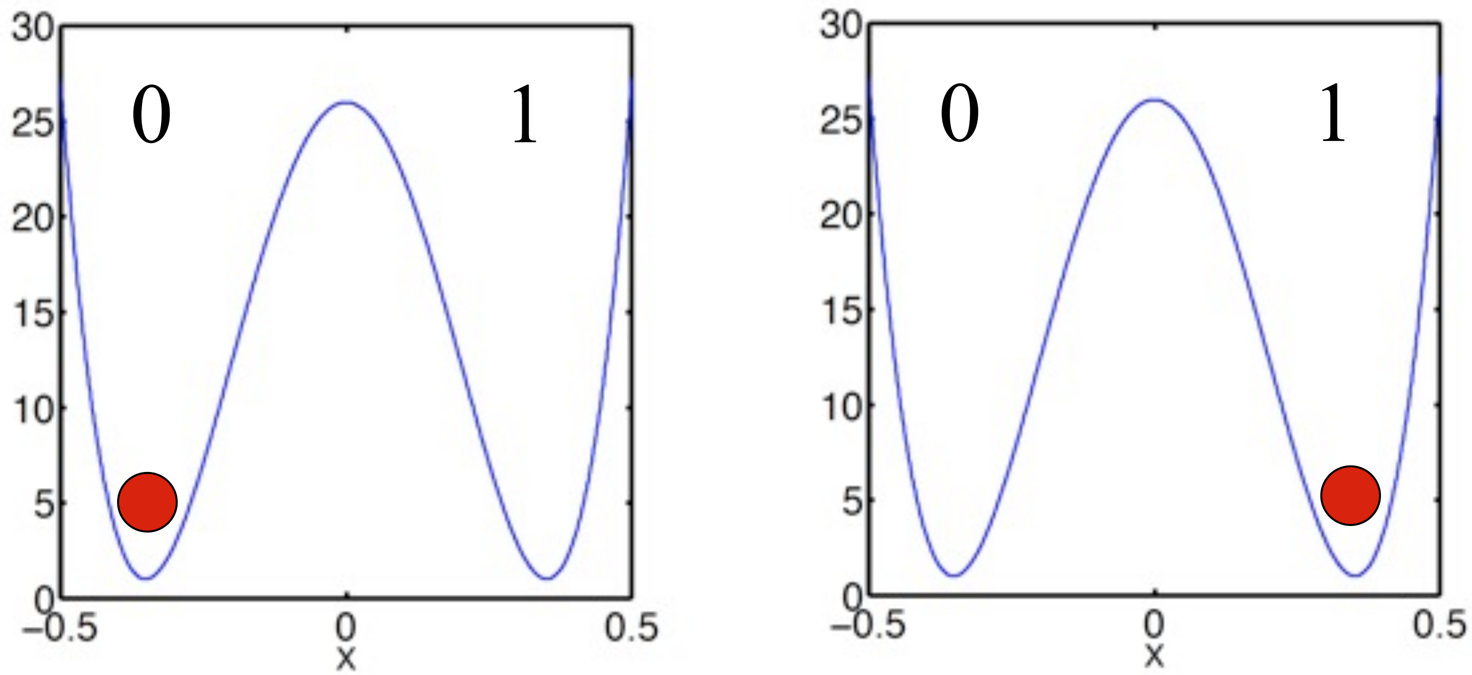
Questions

- Can the Landauer's limit be reached in any experiment ?
- Does any experimentally feasable procedure allow us to reach the limit ?

Following Bennett we use in our experiment the RESET to ONE operation

Bennett, C. H. The thermodynamics of computation, a review. Int. J. Theor. Phys. 21, 905-940 (1982).

The Bennett's erasure procedure



Initial state is 0 or 1 with equal probability 1/2

$S_i = k_B \cdot \ln 2$

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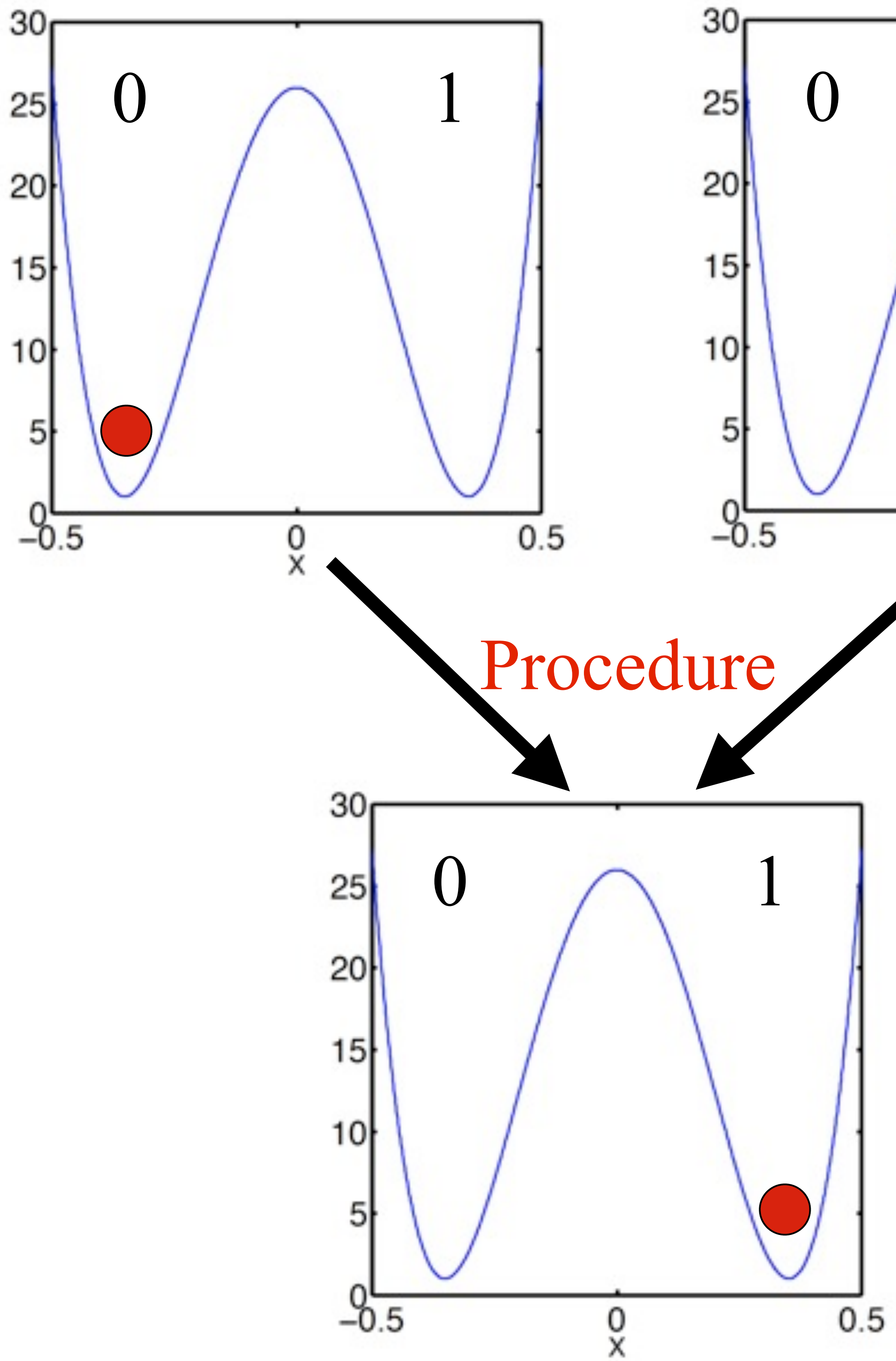

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$S_f = 0$

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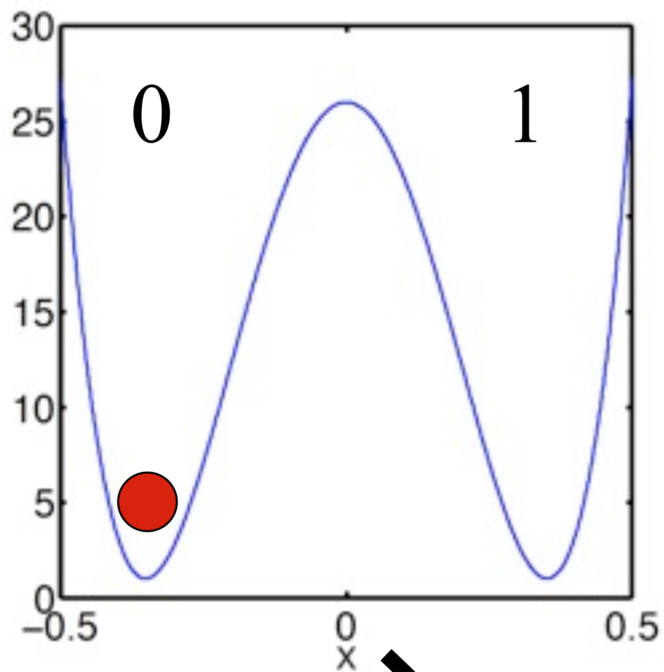

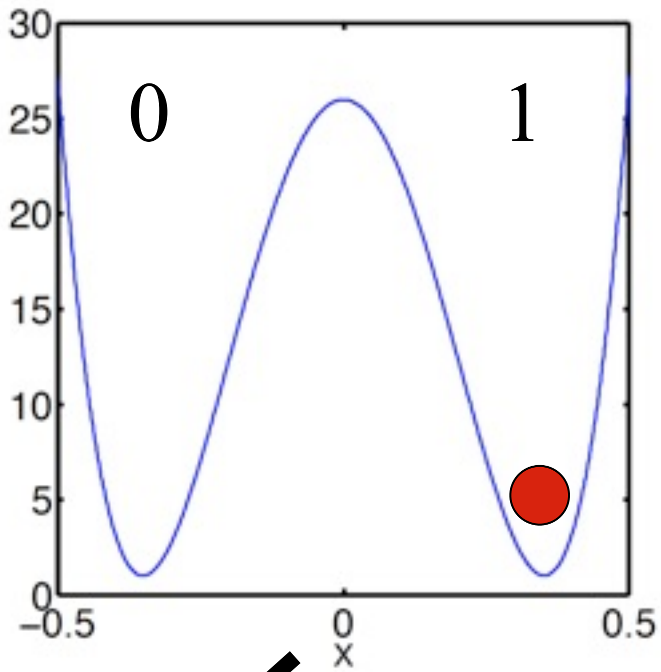

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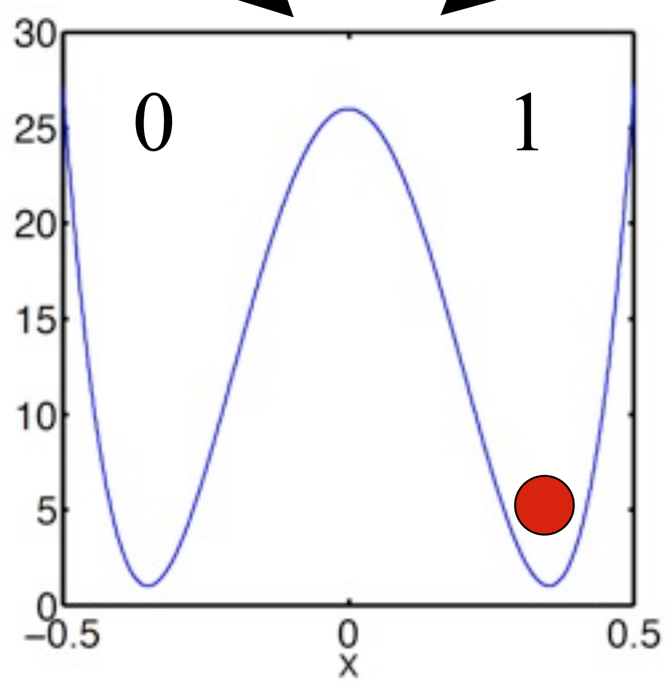
Procedure



Final state is 1 with probability 1

$S_f = 0$

Thus $\Delta S_{min} = -k_B \cdot \ln 2$

Procedure

Quasi Static : $-T\Delta S=Q$

Energy variation : $\Delta U=0$

First principle : $\Delta U=-Q+W$

$$\Downarrow$$

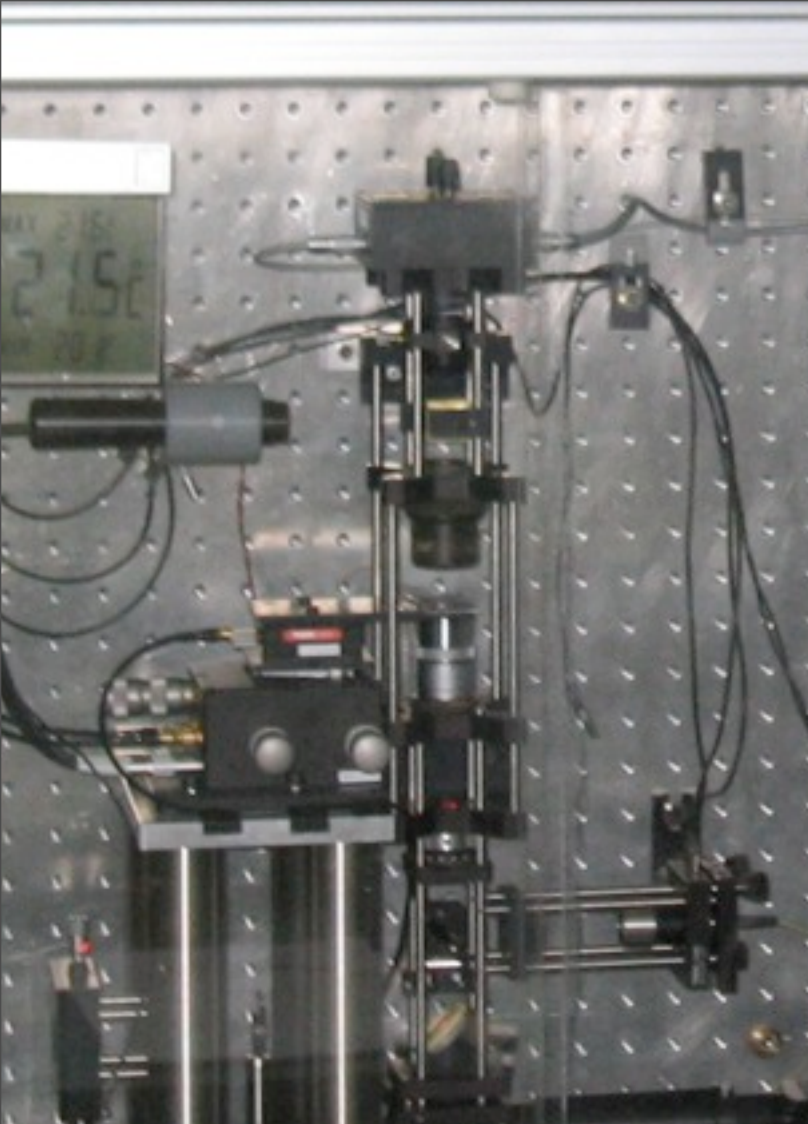
In average : $\langle W \rangle = \langle Q \rangle = -T\,\Delta S \geq k_B T \ln(2)$

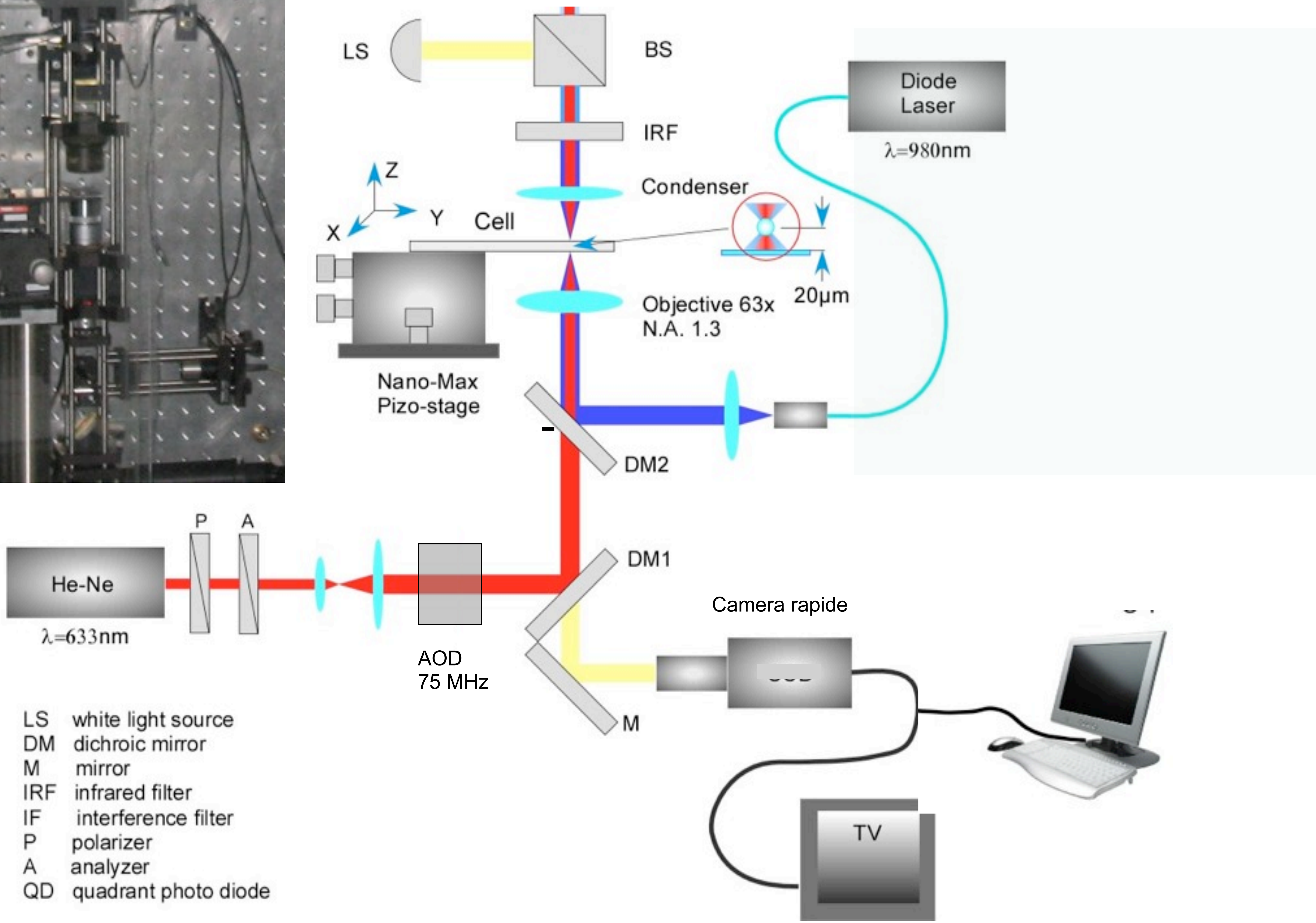
Numerical result :

Memory Erasure in Small Systems,

R. Dillenschneiderand E. Lutz, Phys. Rev. Lett. 102, 210601 (2009)

Experimental set-up Optical trap





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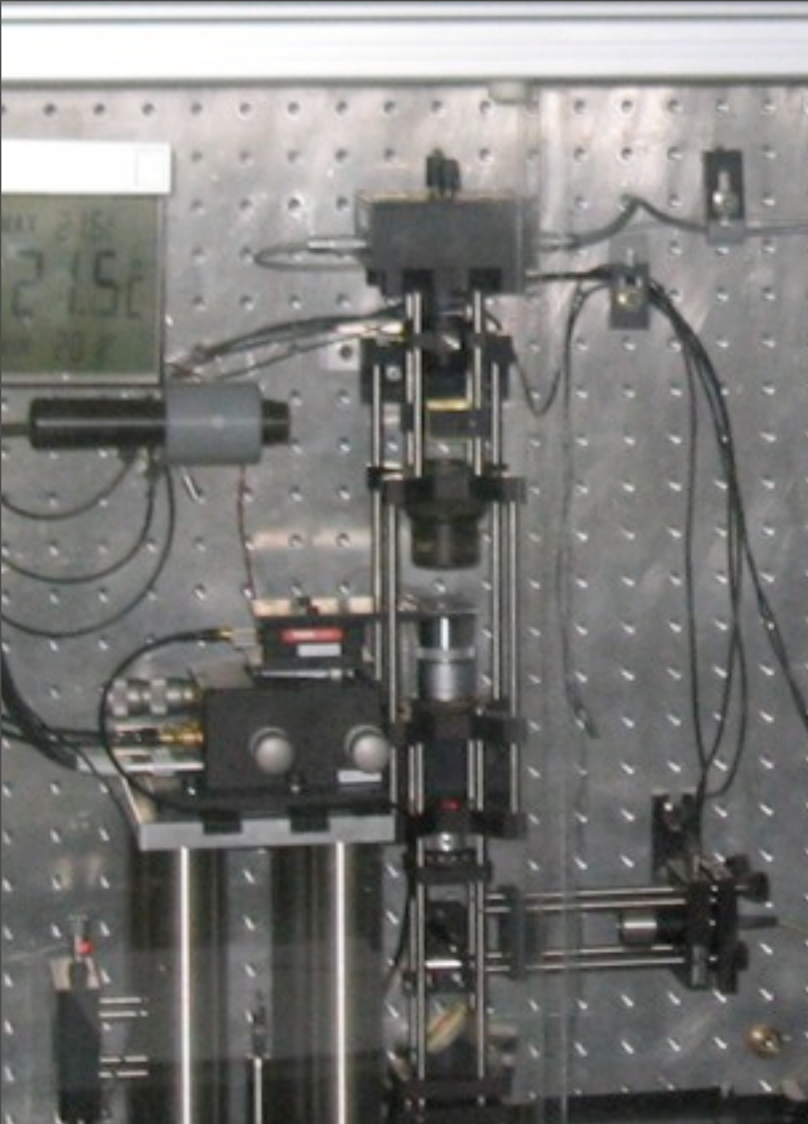
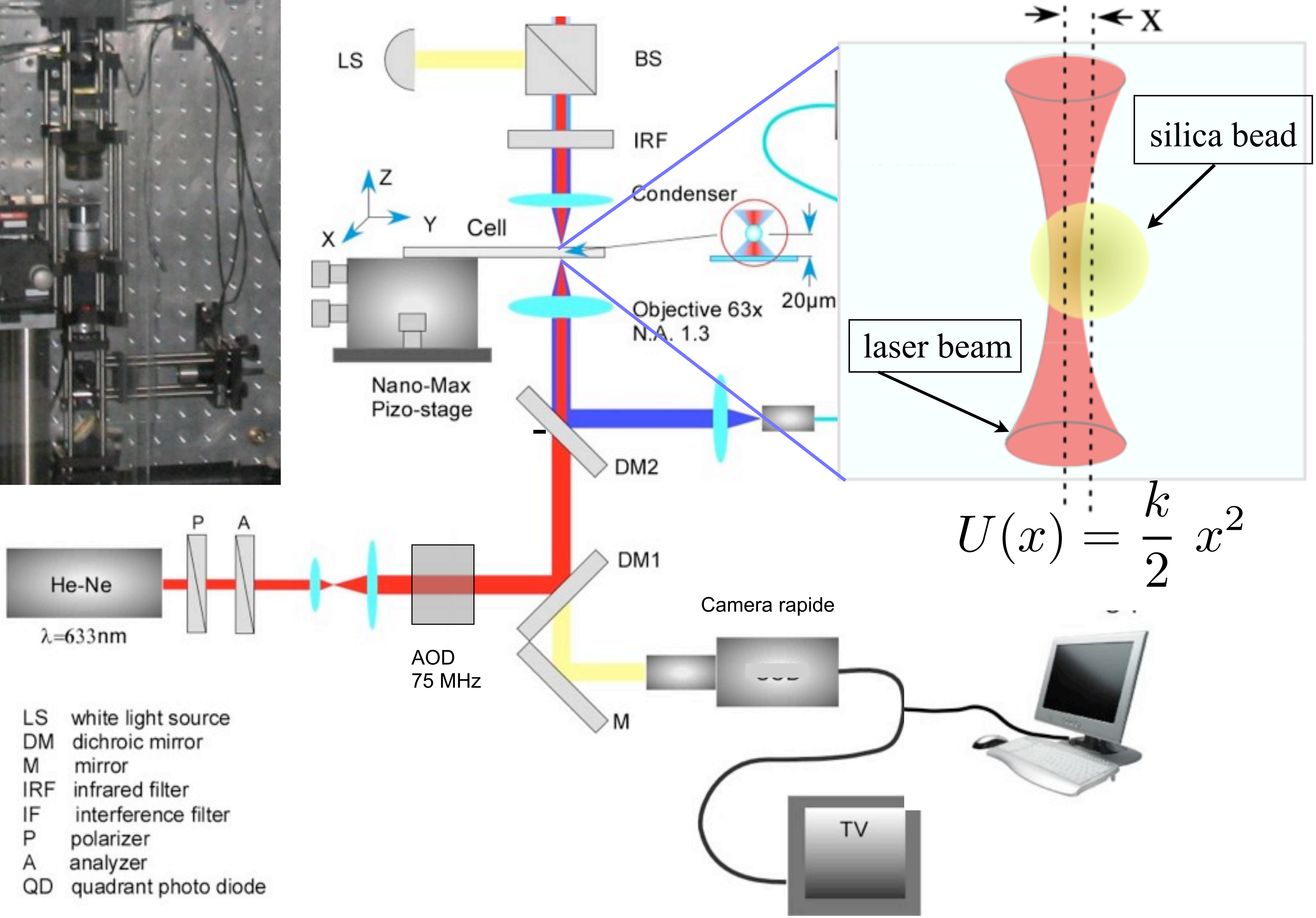
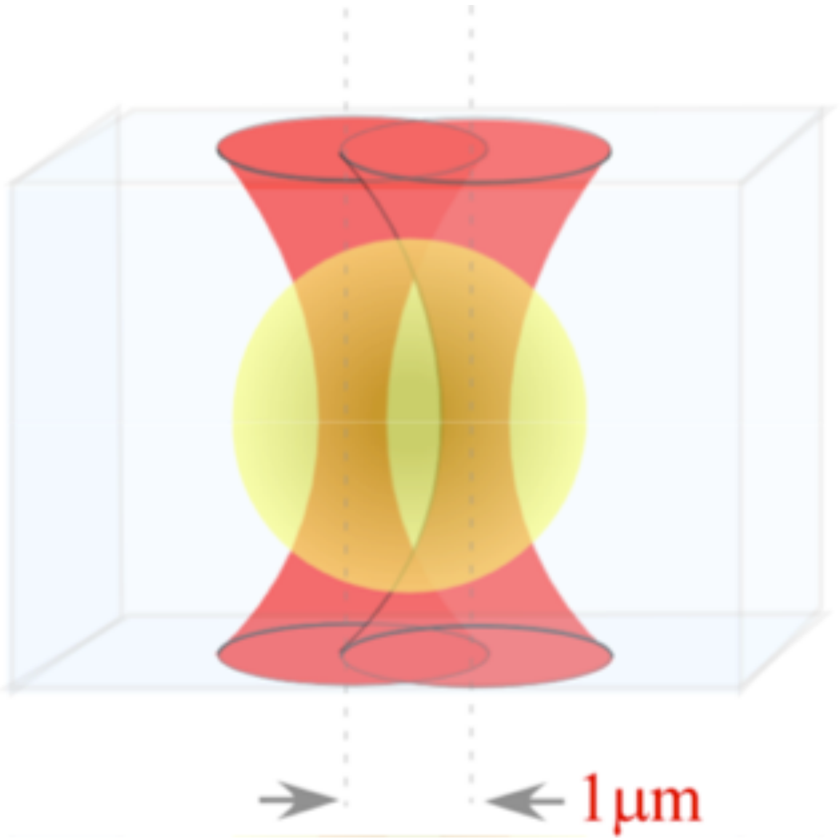
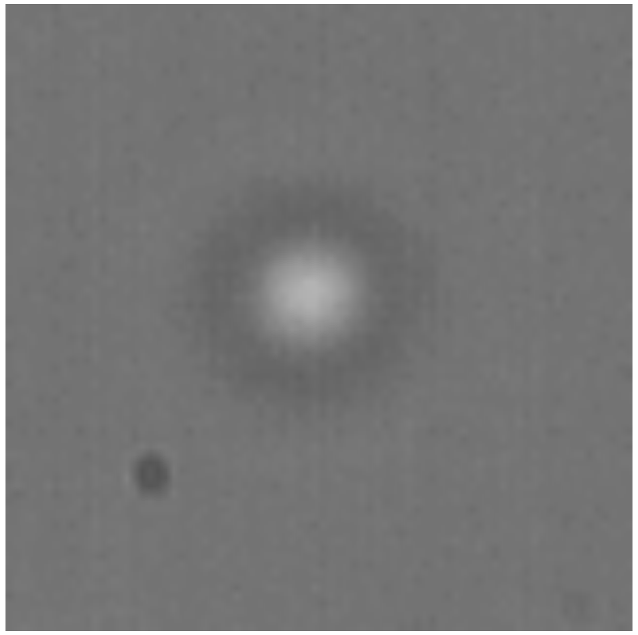




$$U(x) = \frac{k}{2}\, x^2$$

Brownian particle trapped by two laser beams





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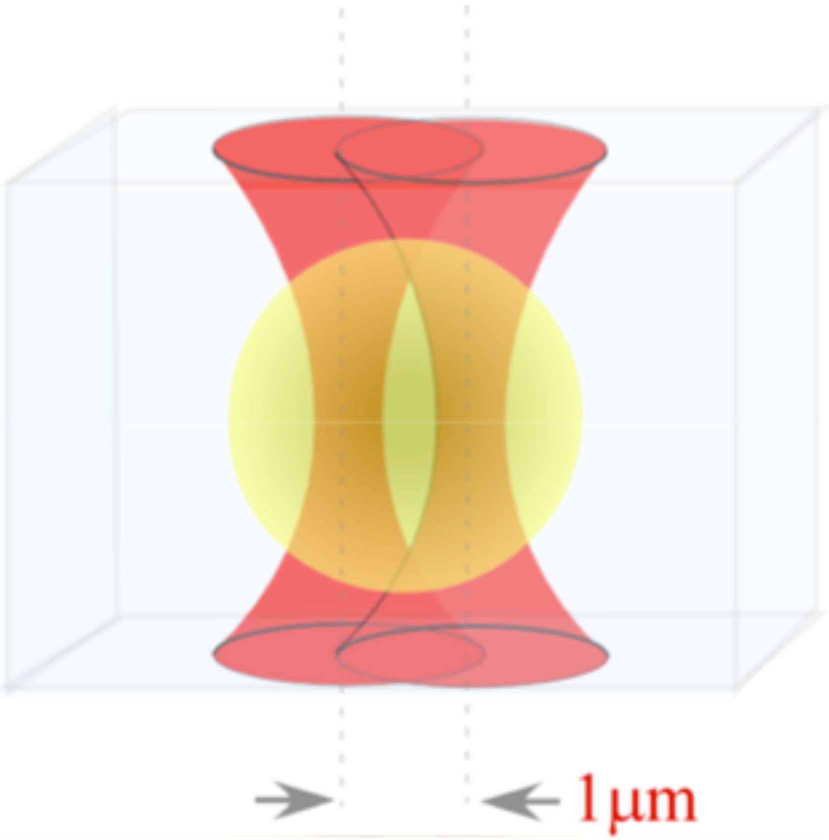

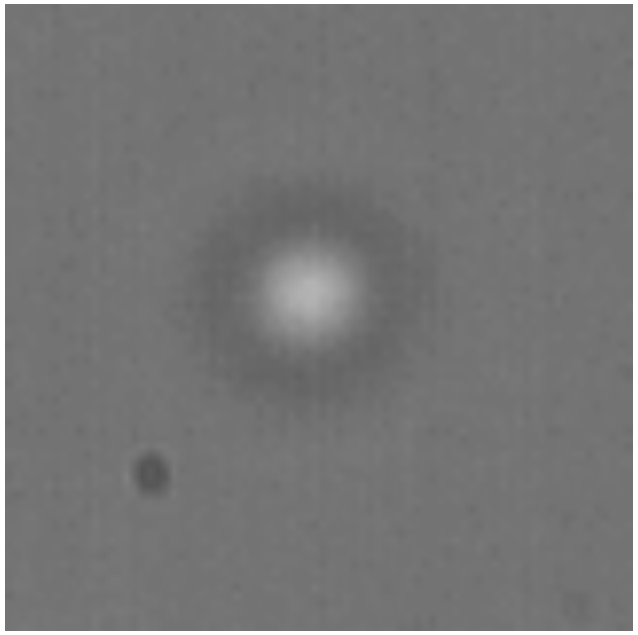

Trapping potential is constructed using the Boltzmann factor

$$P(x) \propto \exp(\frac{U(x)}{k_B T})$$

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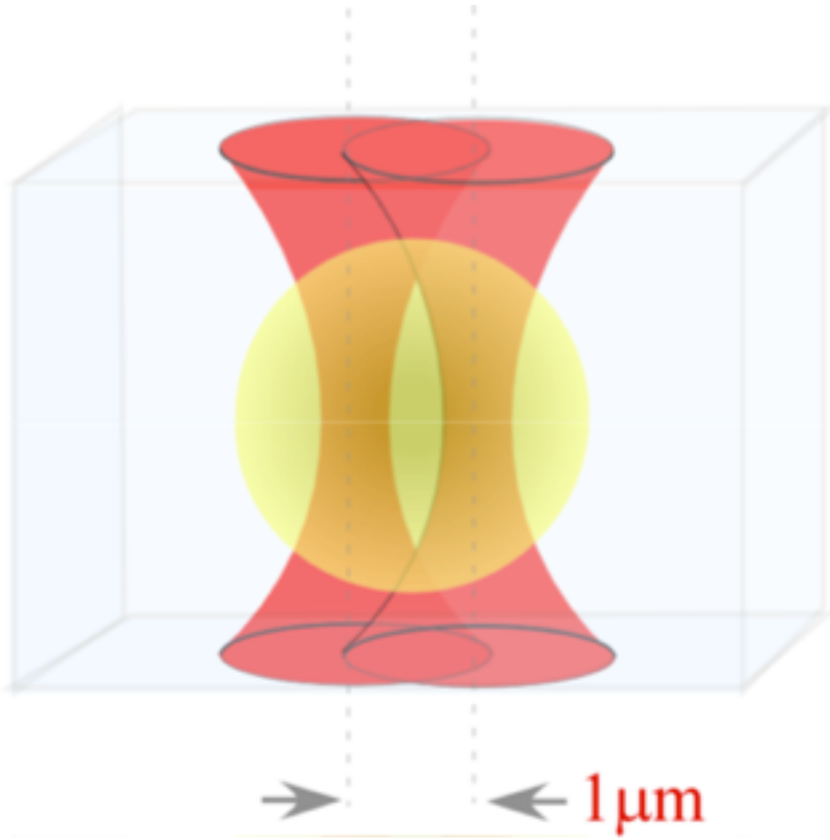

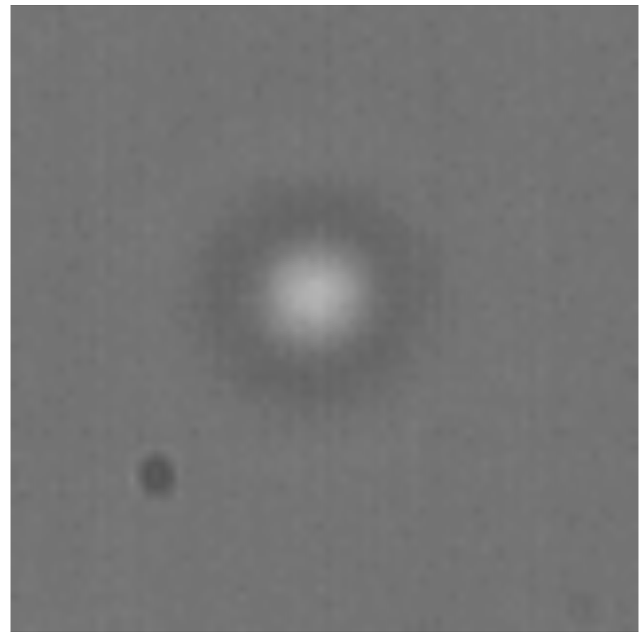

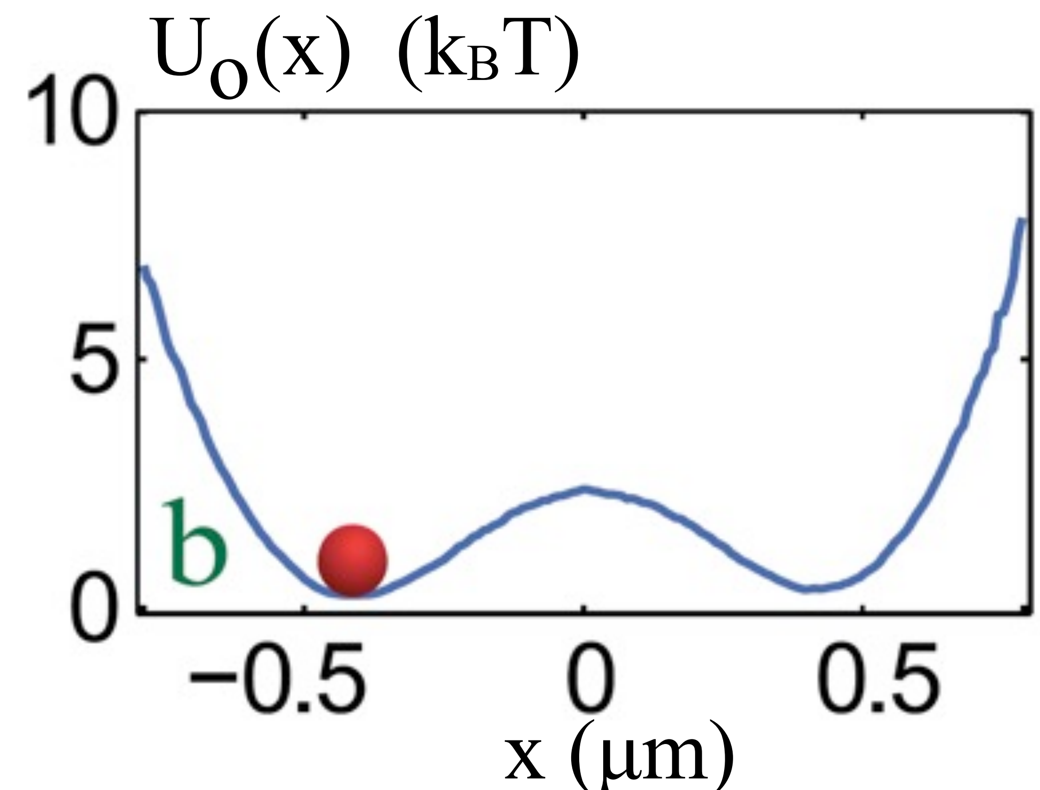

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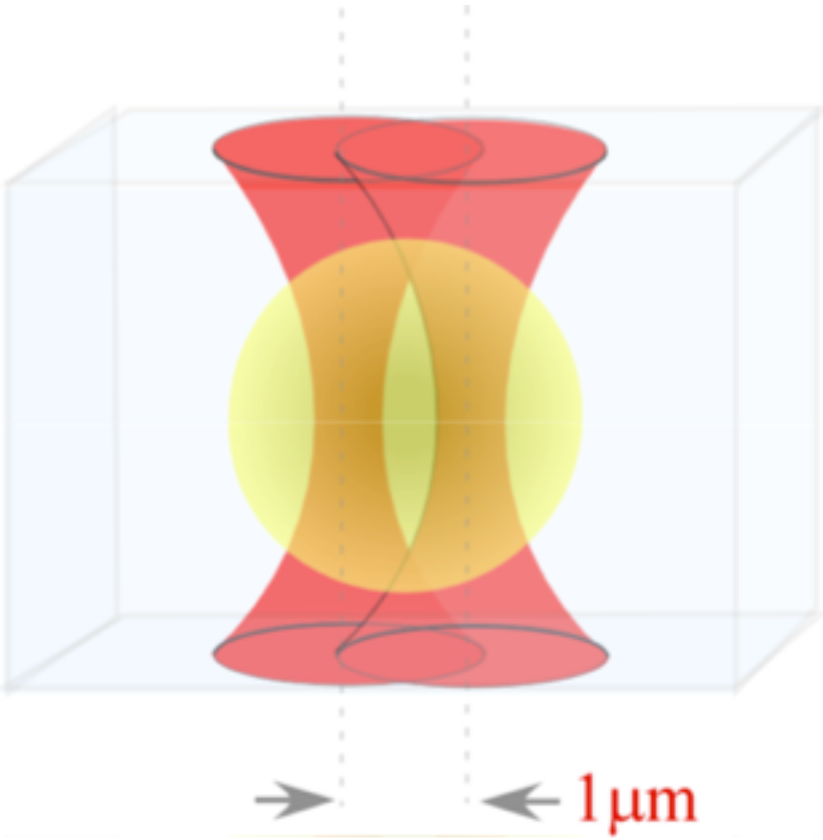

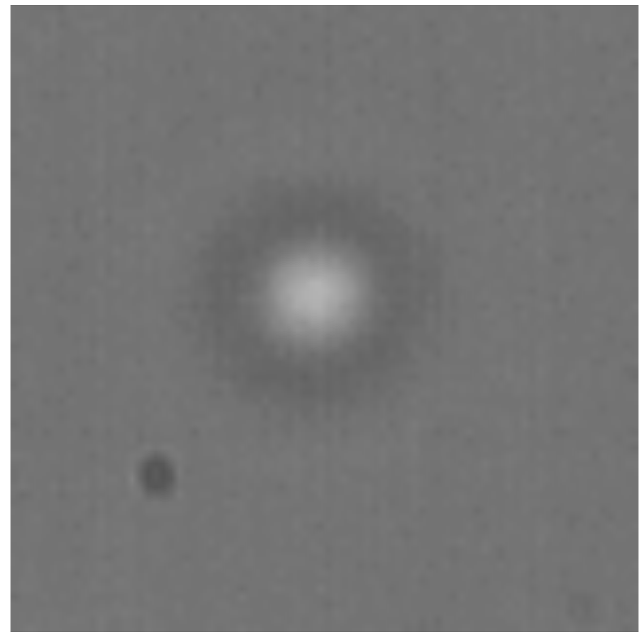

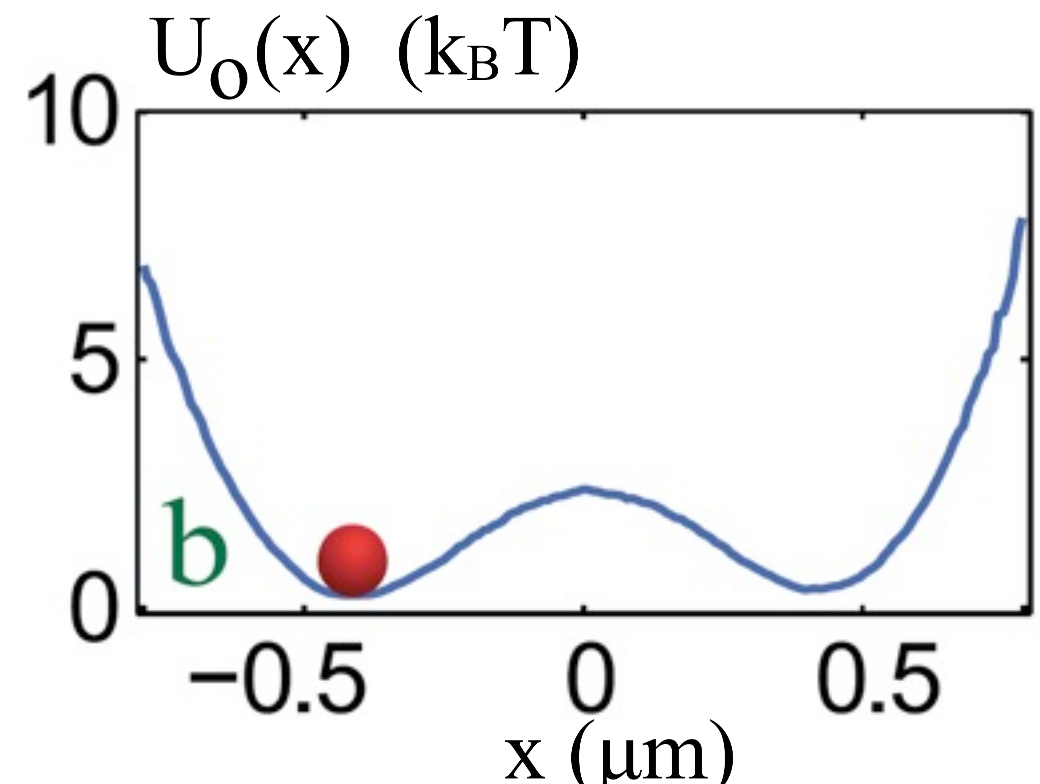

The Kramers time

$$\tau_K = \tau_o \ \exp[\frac{\delta U}{k_B T}]$$

with $\tau_o = 1 \ s$

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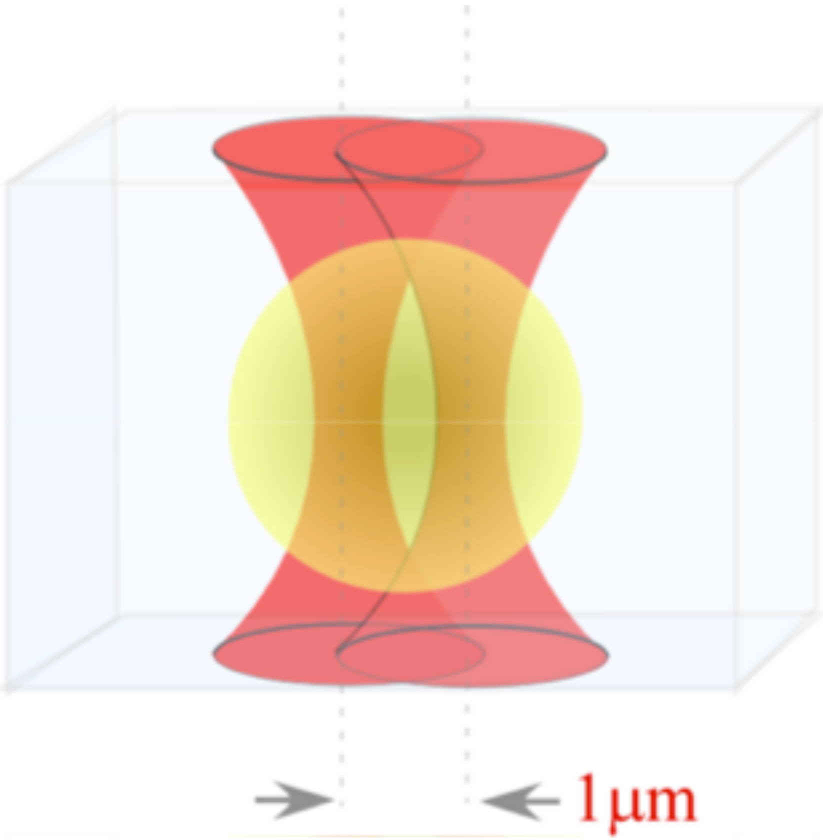

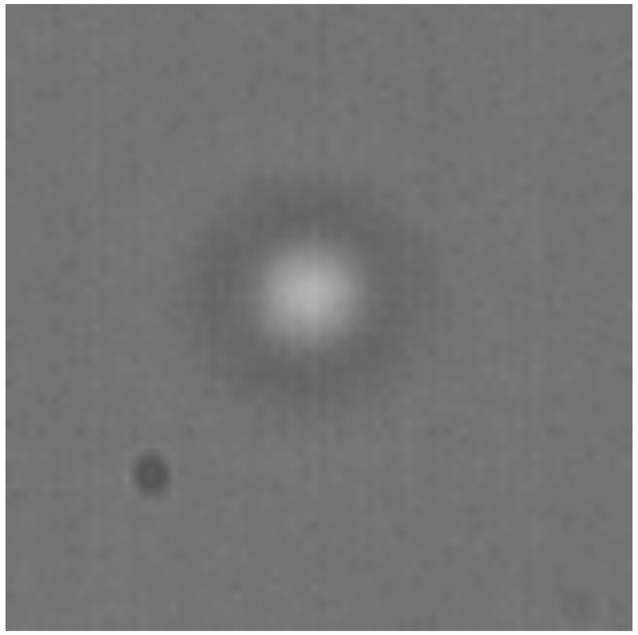

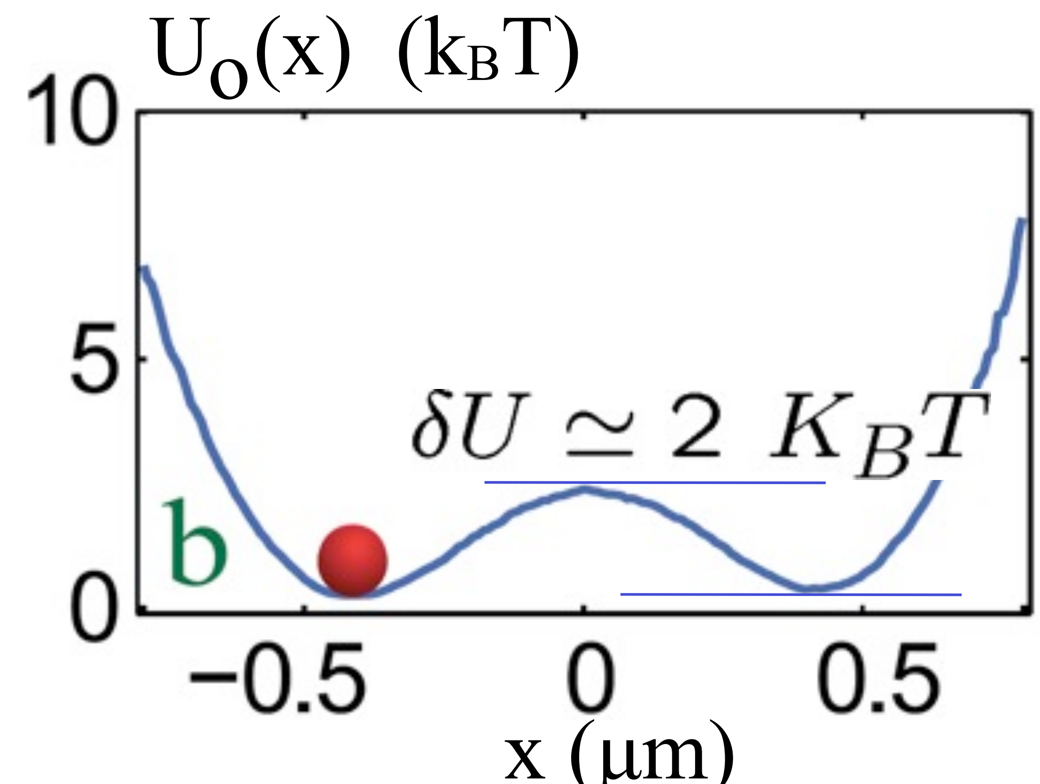

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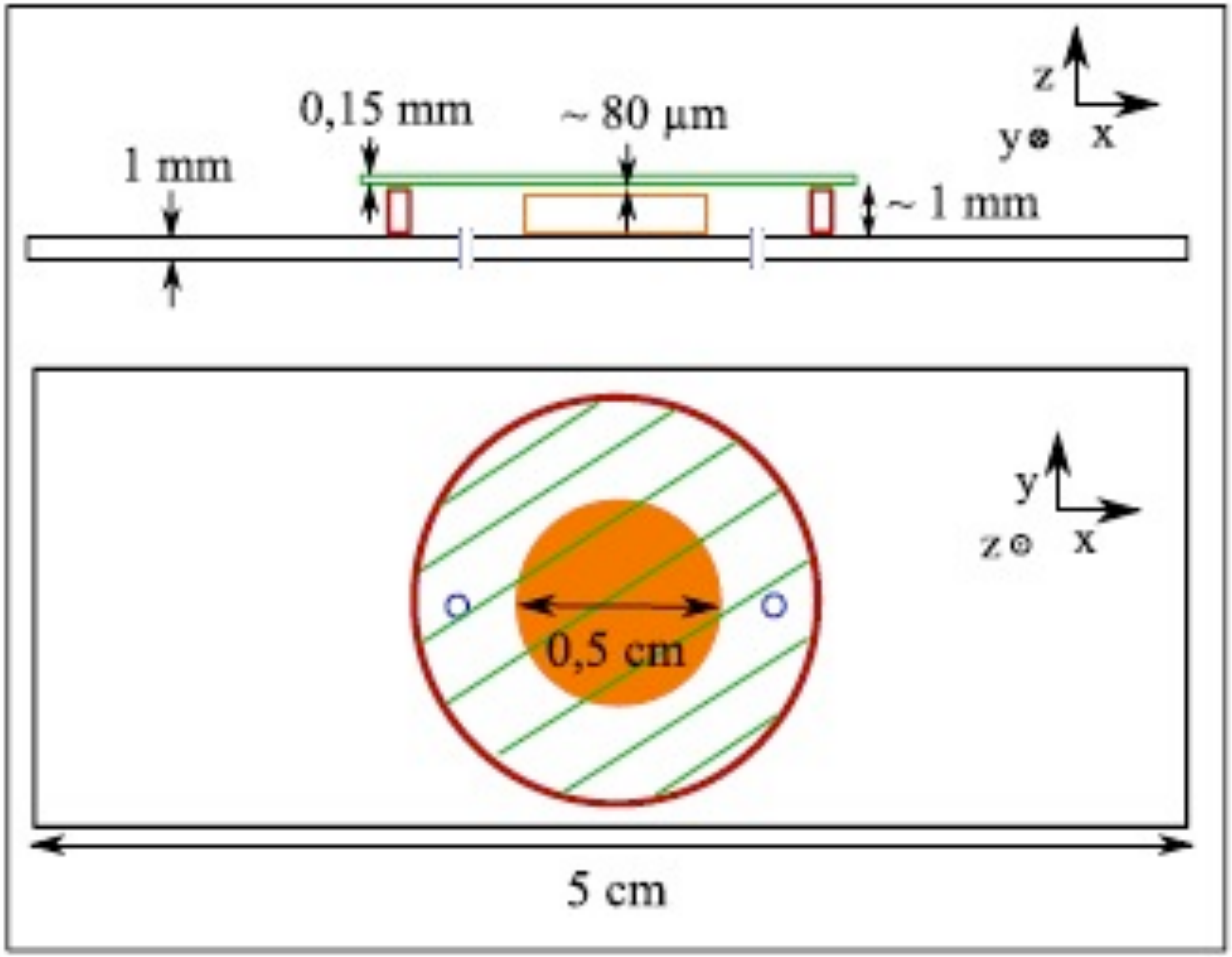
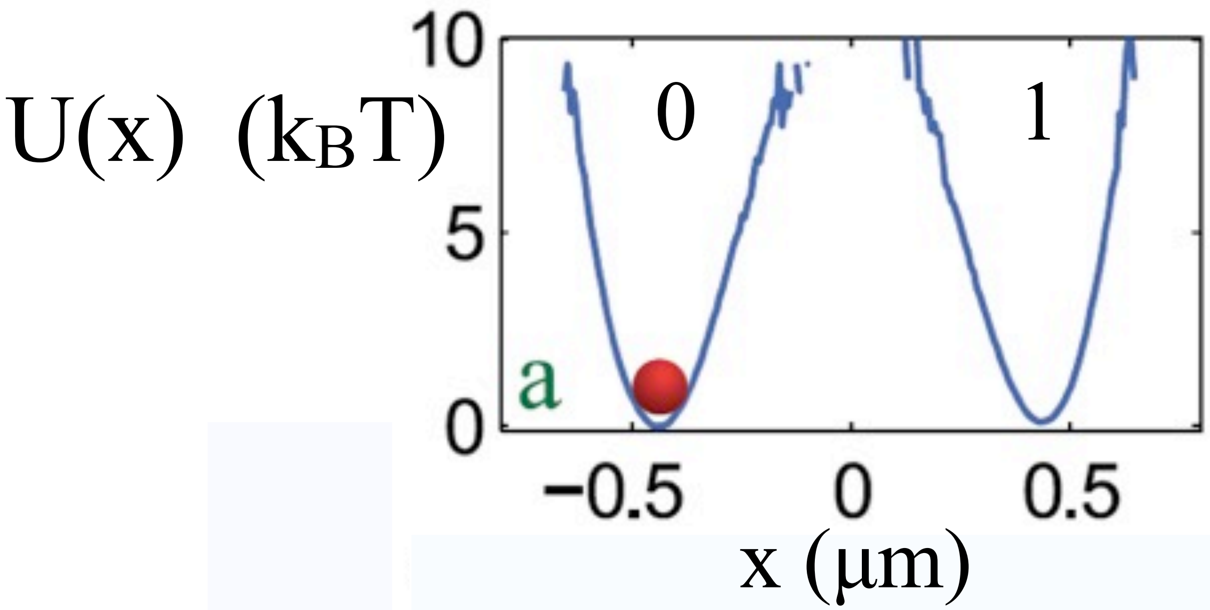
The cell for the bead



The Erasure Procedure

Initial state



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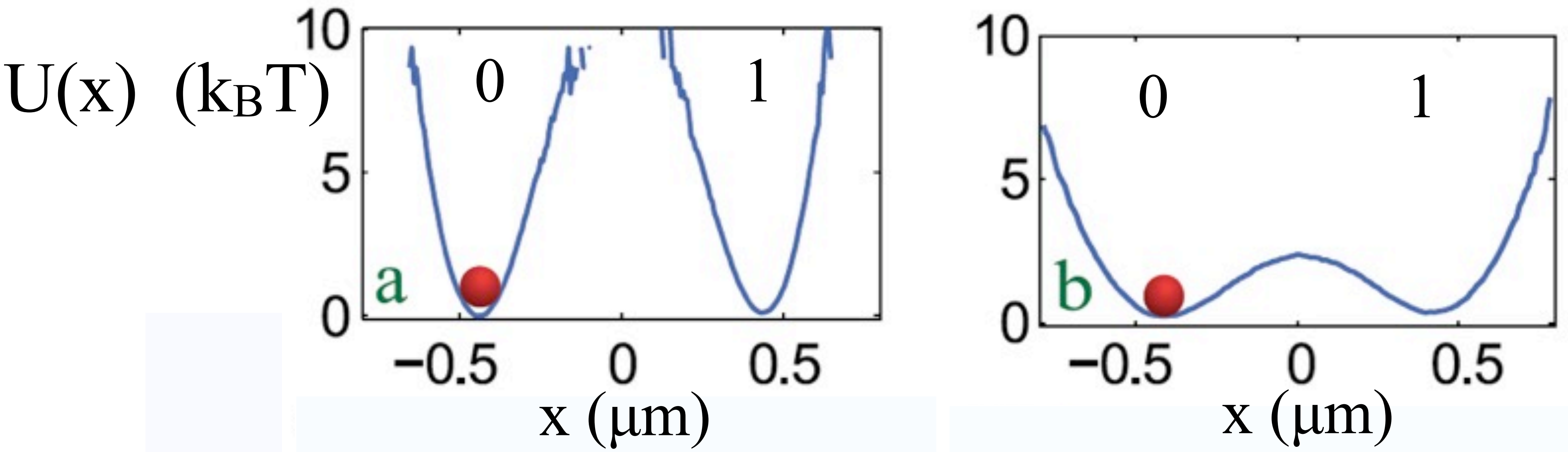
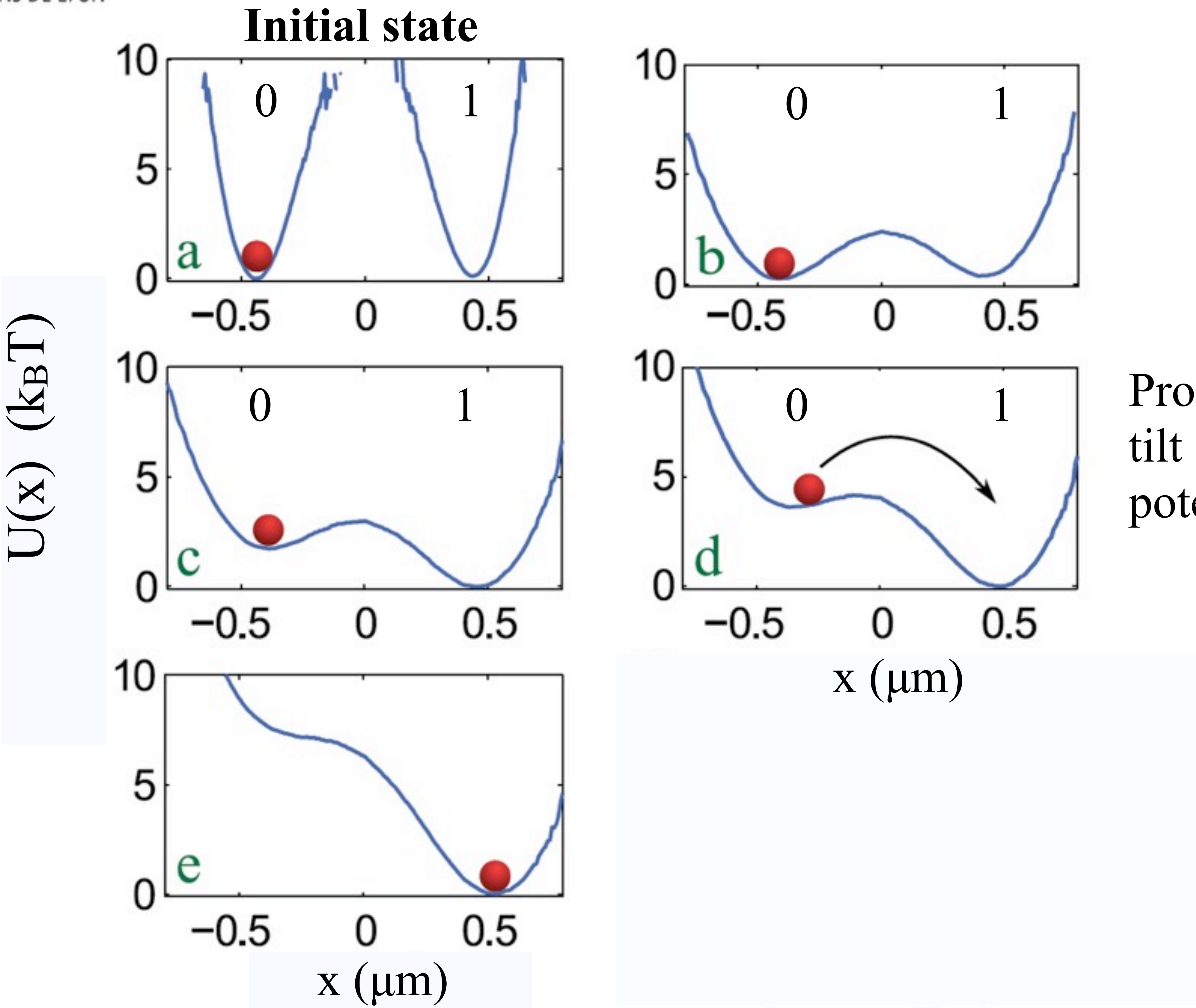


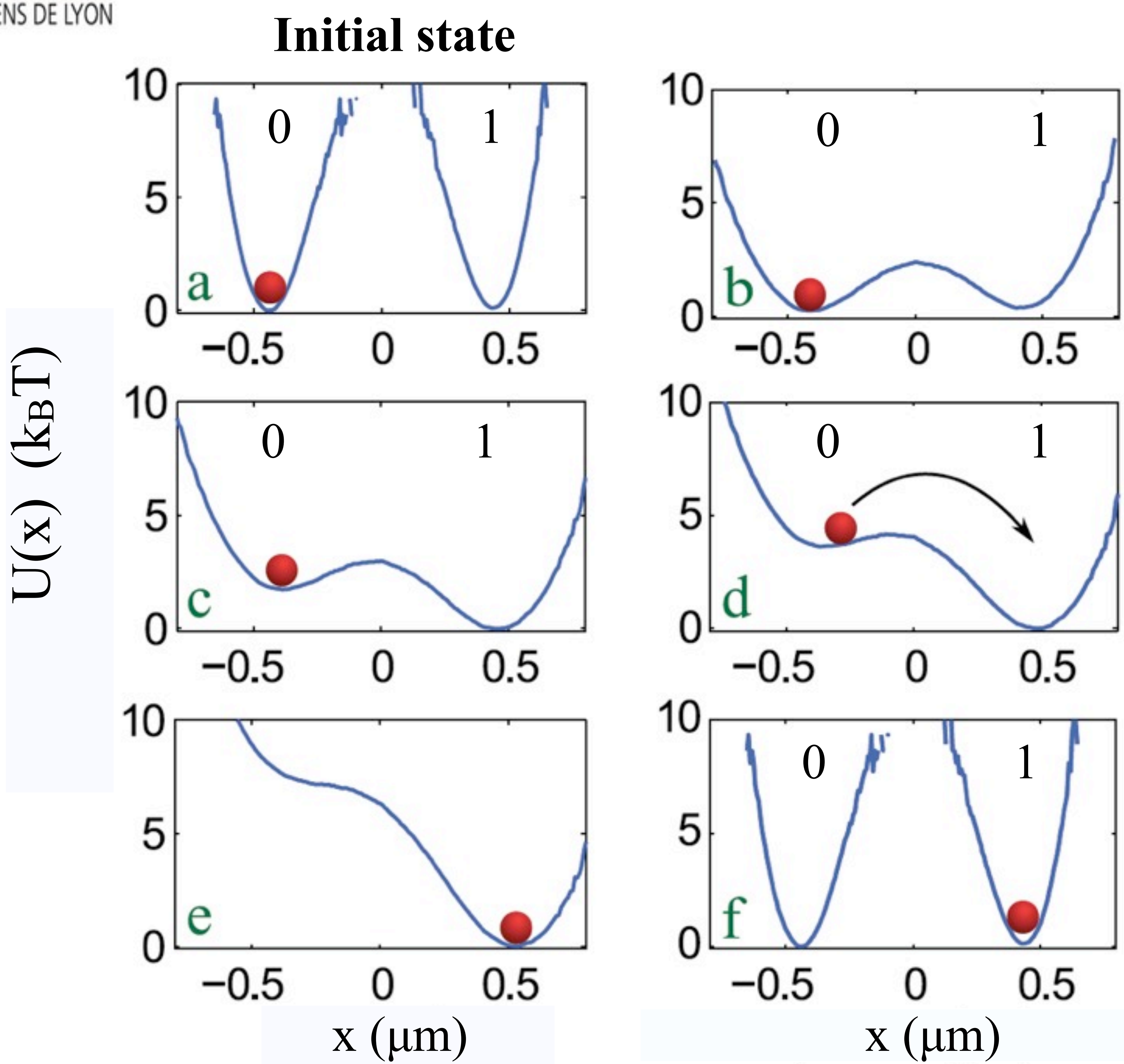
reduction
of the barrier

The Erasure Procedure



Progressive tilt of the potential

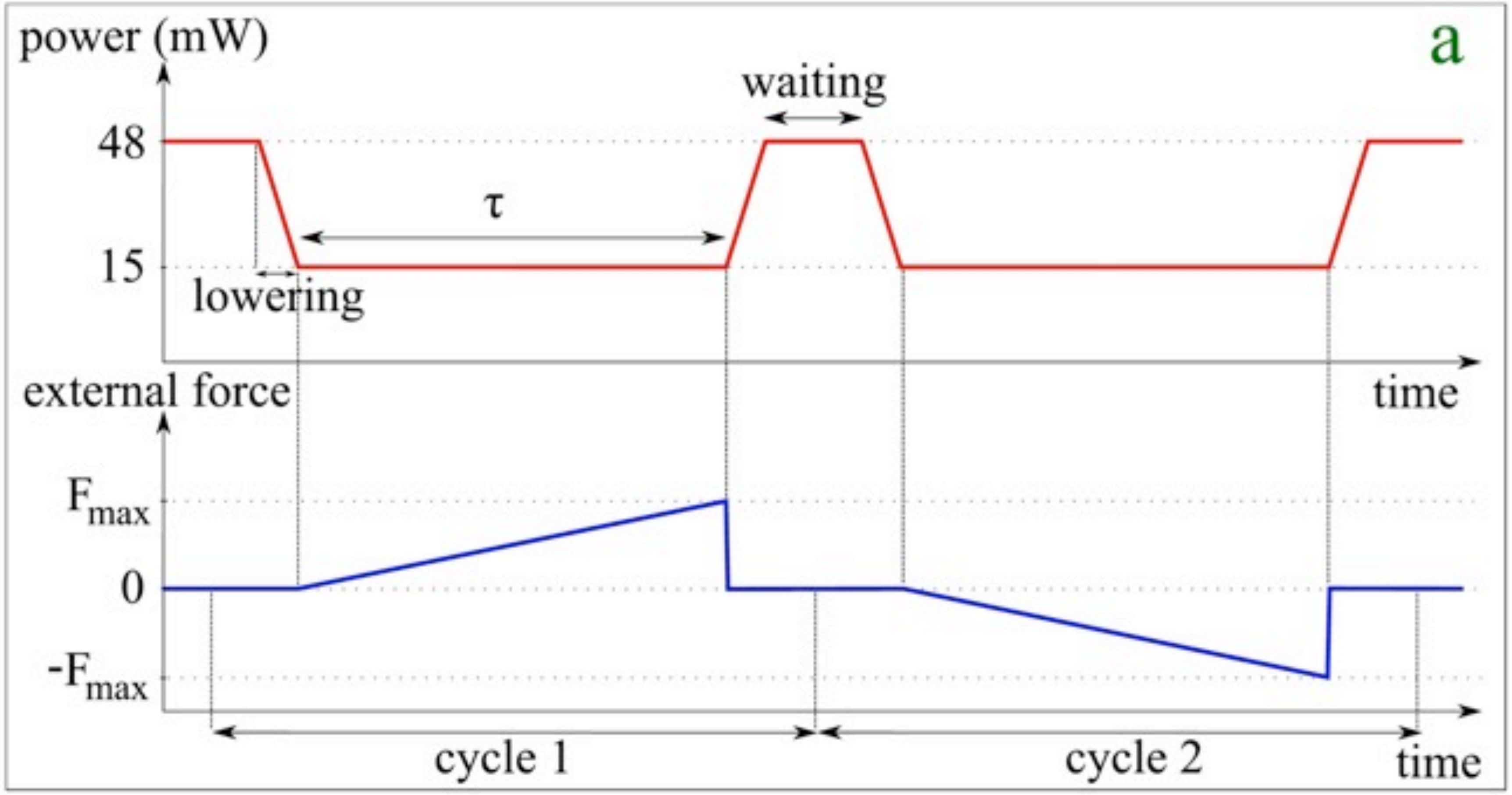
The Erasure Procedure



Increasing of the barrier

Final state

Potential external control as a function of time



The laser intensity controls the barrier height

The potential tilt is produceed by a linearly increasing external force F, applied on the time τ.

$\tau_{cycle} = \tau + 2$ s

The force F is created by displacing the cell with respect to the laser, thus

$$F = -\nu \ v \qquad \text{with} \qquad \nu = 6\pi R\mu$$

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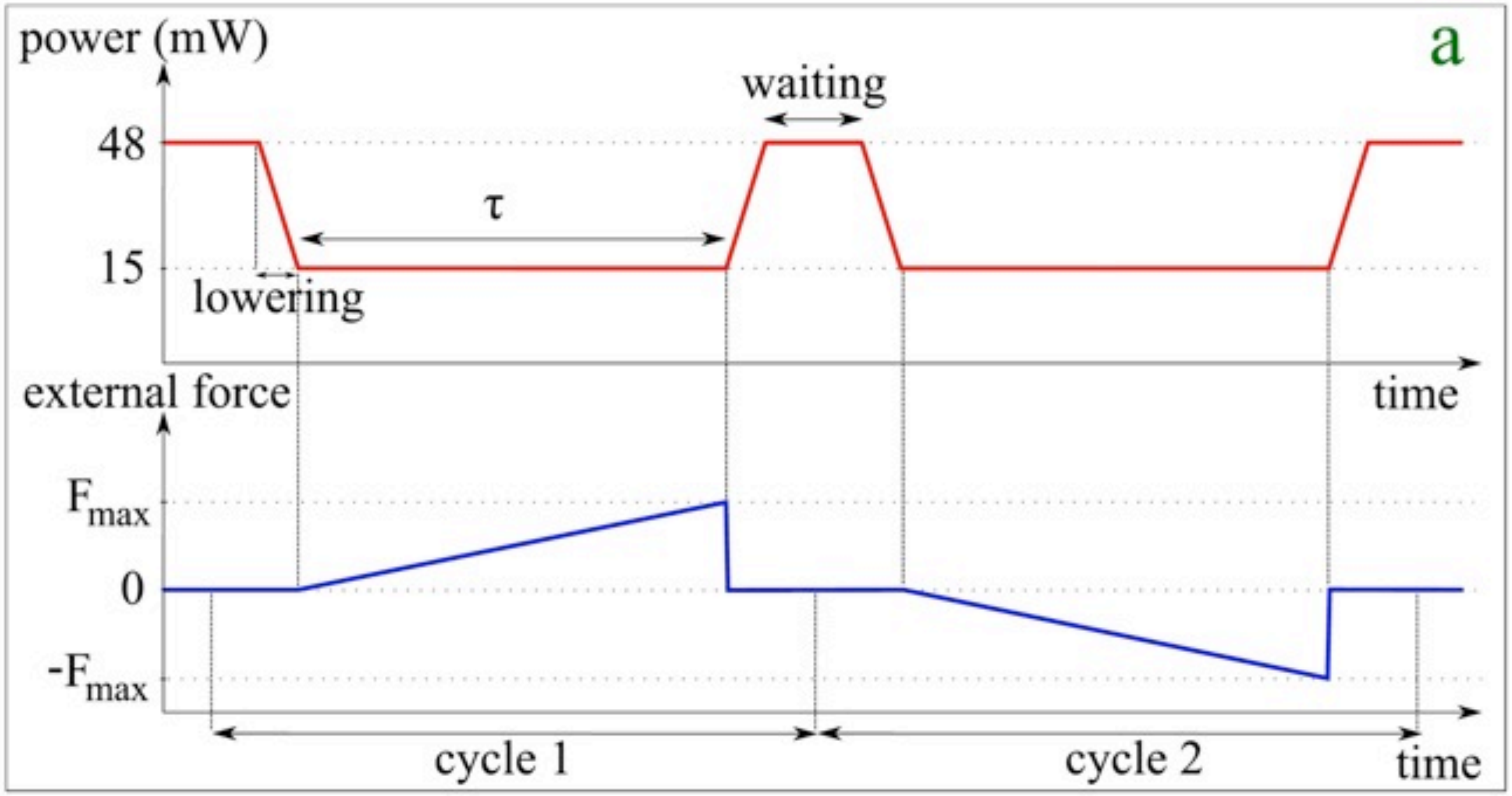

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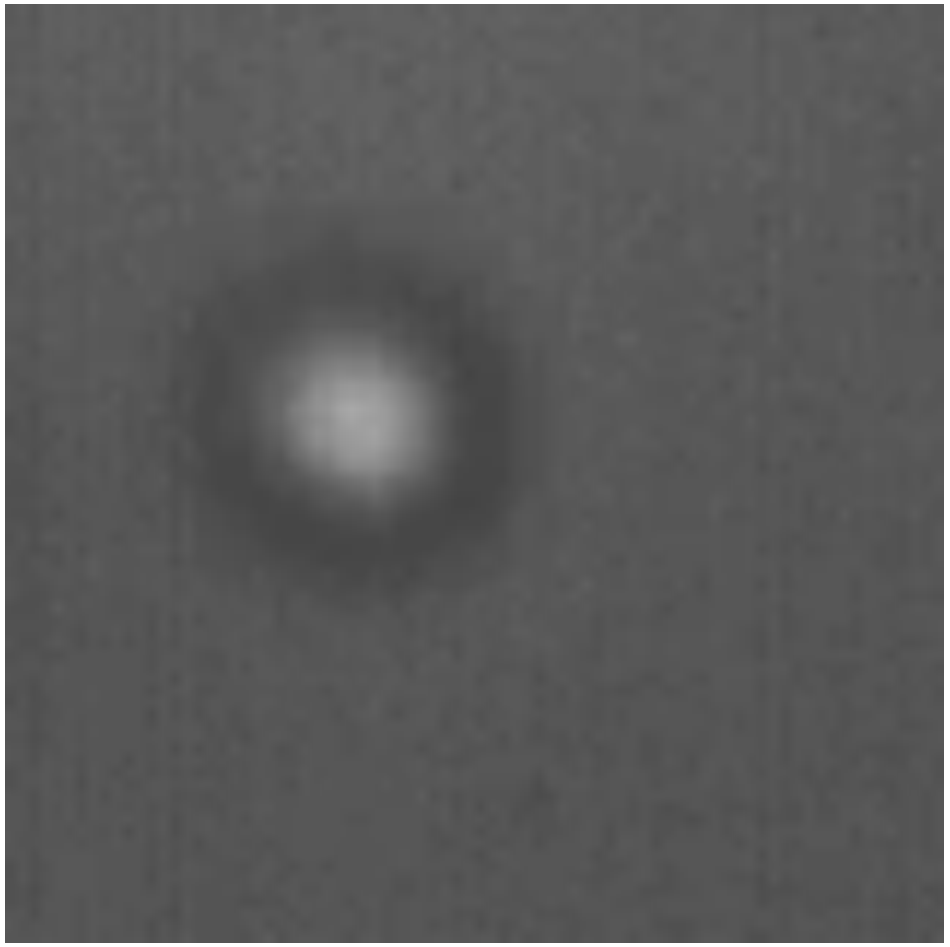
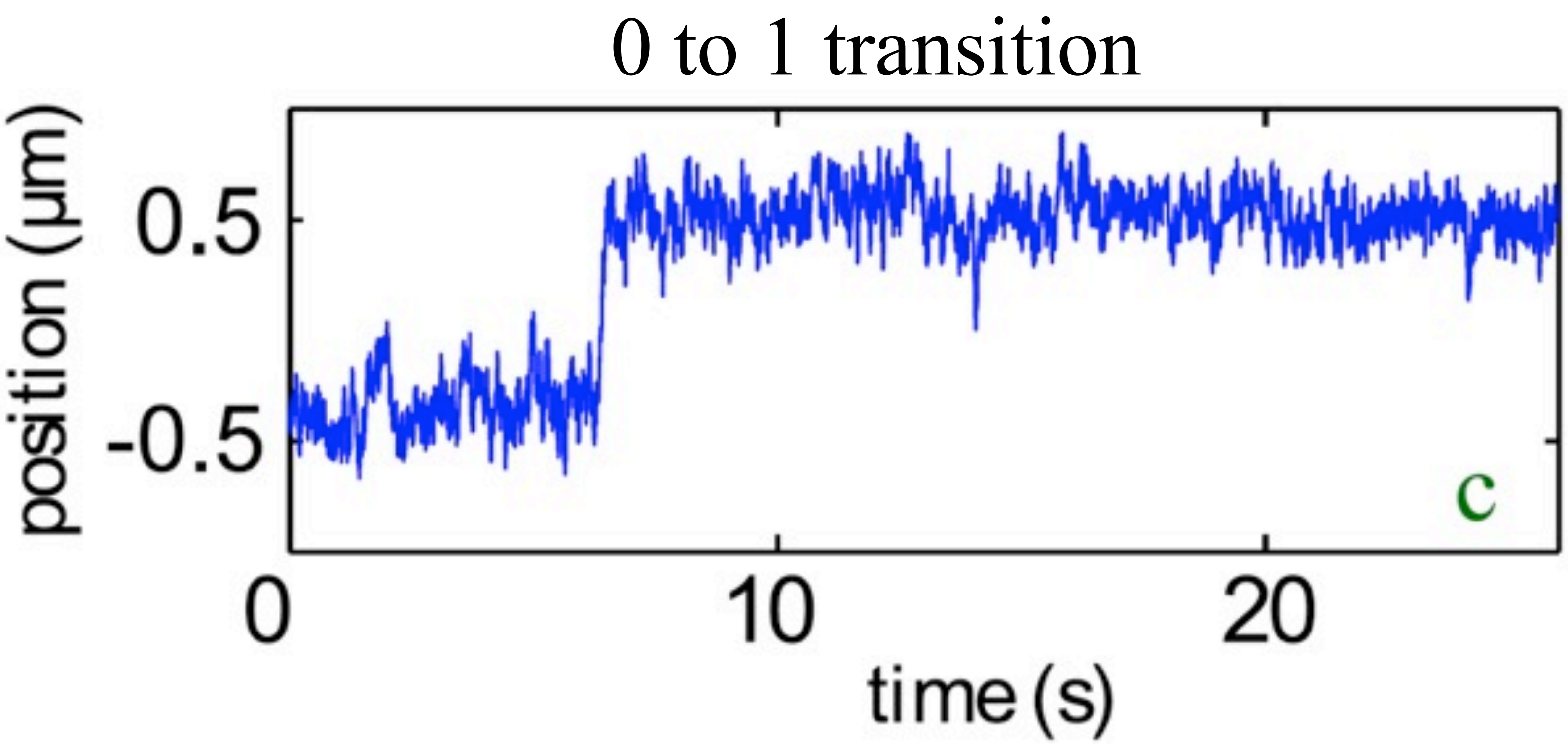
Two control parameters: τ the time of application of F

F_{max} the maximum applied force

Bead trajectories





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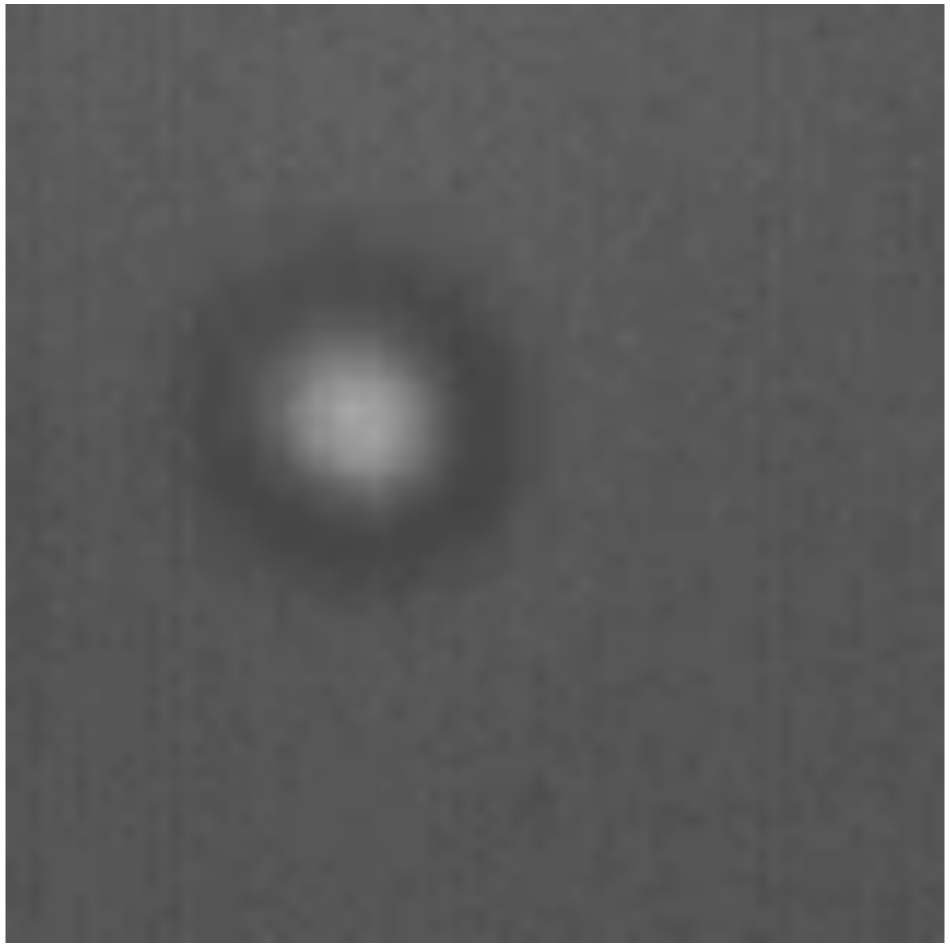

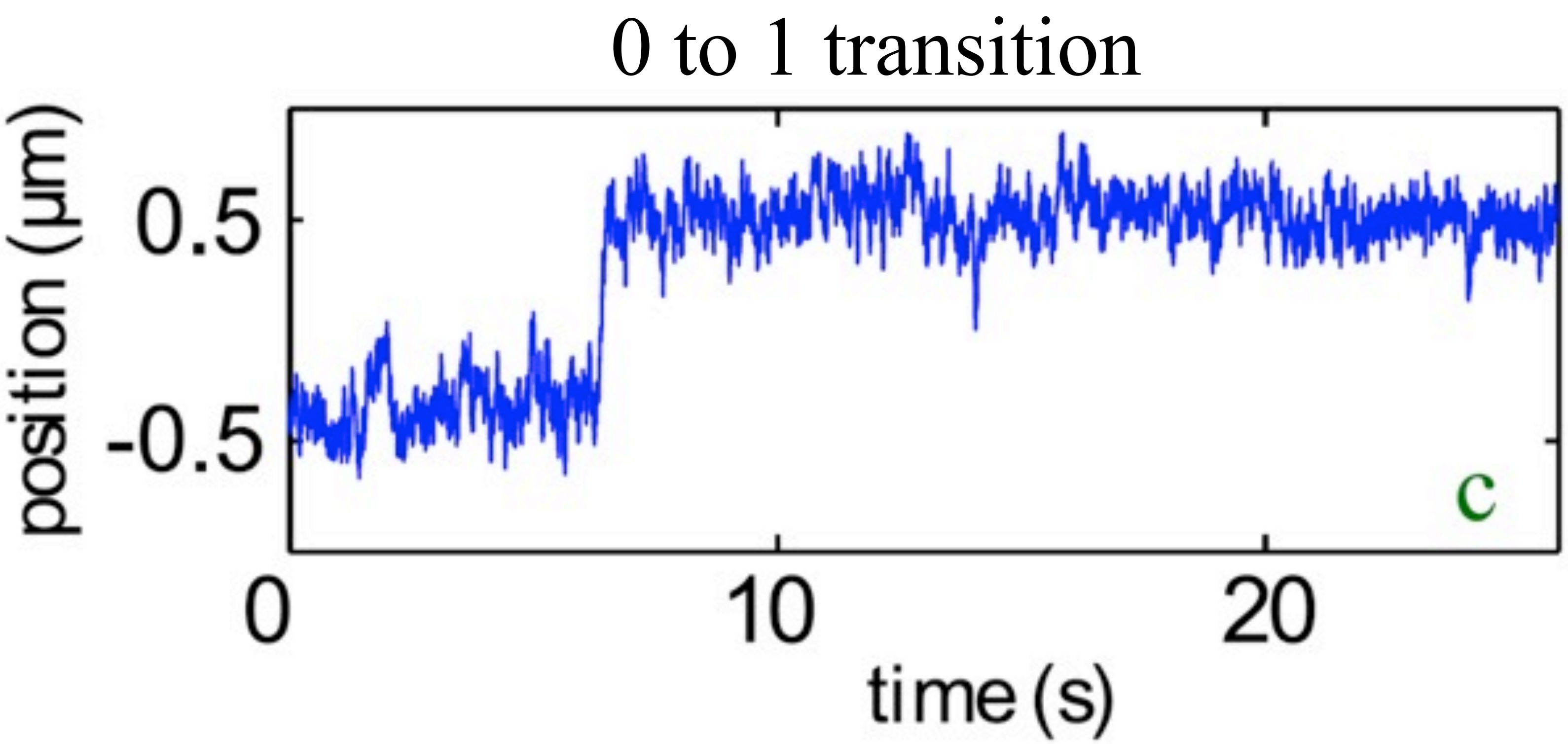

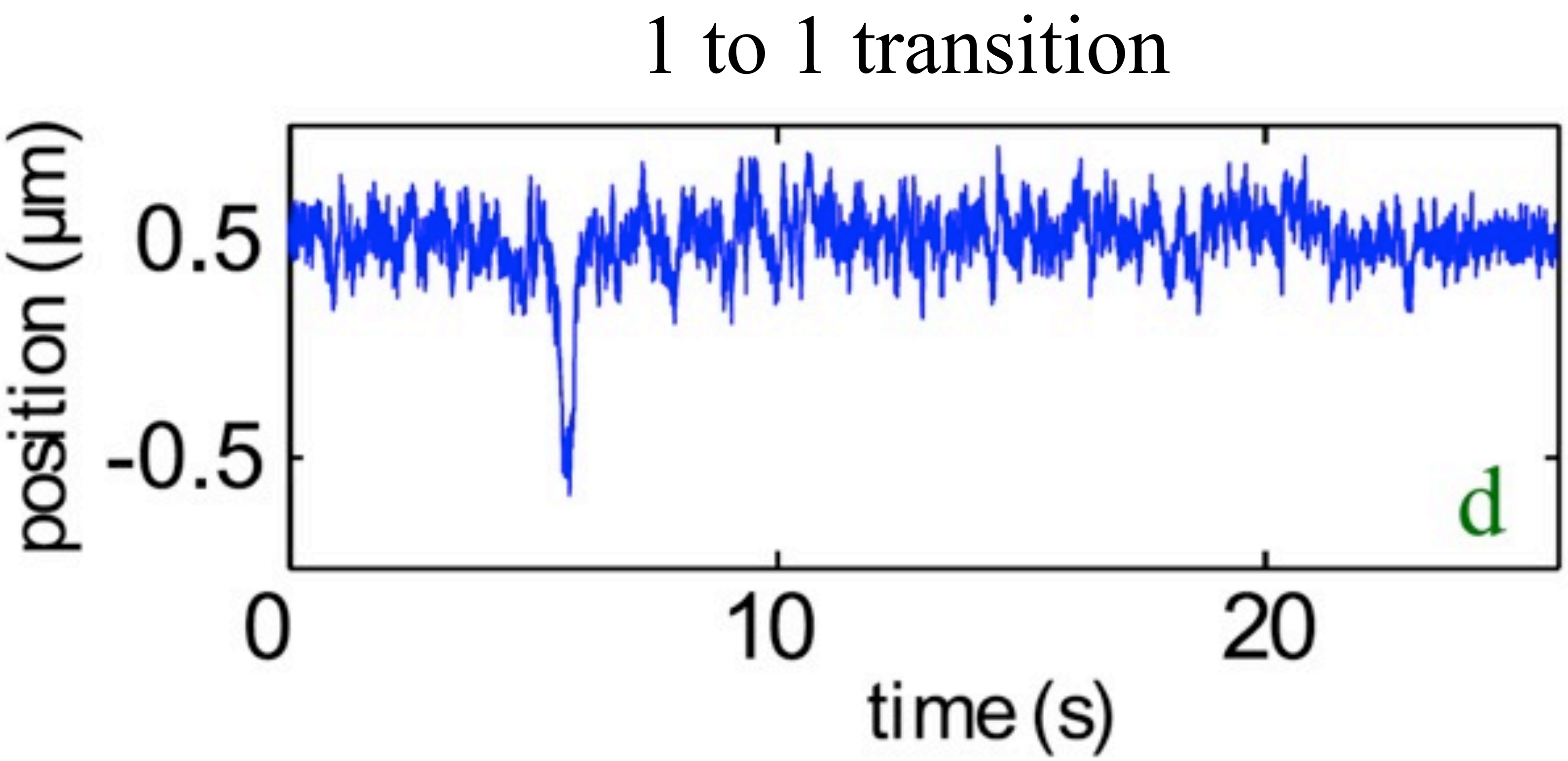

The work on the erasure cycle

$$\nu \dot{x} = -\frac{\partial U_o(x,t)}{\partial x} + F(t) + \eta$$

multiplying by ẋ and integrating for a time τ we get :

$$\Delta U_\tau = W_\tau - Q_\tau$$

Stochastic thermodynamics

$$\Delta U_\tau = -\int_0^\tau \frac{\partial U_o}{\partial x}\, \dot{x}\, dt$$

$$W_\tau = \int_0^\tau F\, \dot{x}\, dt$$

$$Q_\tau = \int_0^\tau \nu \dot{x}^2\, dt - \int_0^\tau \eta \dot{x}\, dt$$

Sekimoto K, Progress of Theoretical Phys. supplement (130), 17 (1998).

The work on the erasure cycle

The two erasure cycles have been considered

0 → 1, 1 → 1 and 0 → 0, 1 → 0

$$W_F = -\int_0^{\tau_{cycle}} \nu \; v(t)\dot{x} \; dt = \int_0^{\tau_{cycle}} F_{max}\frac{t}{\tau}\dot{x} \; dt$$

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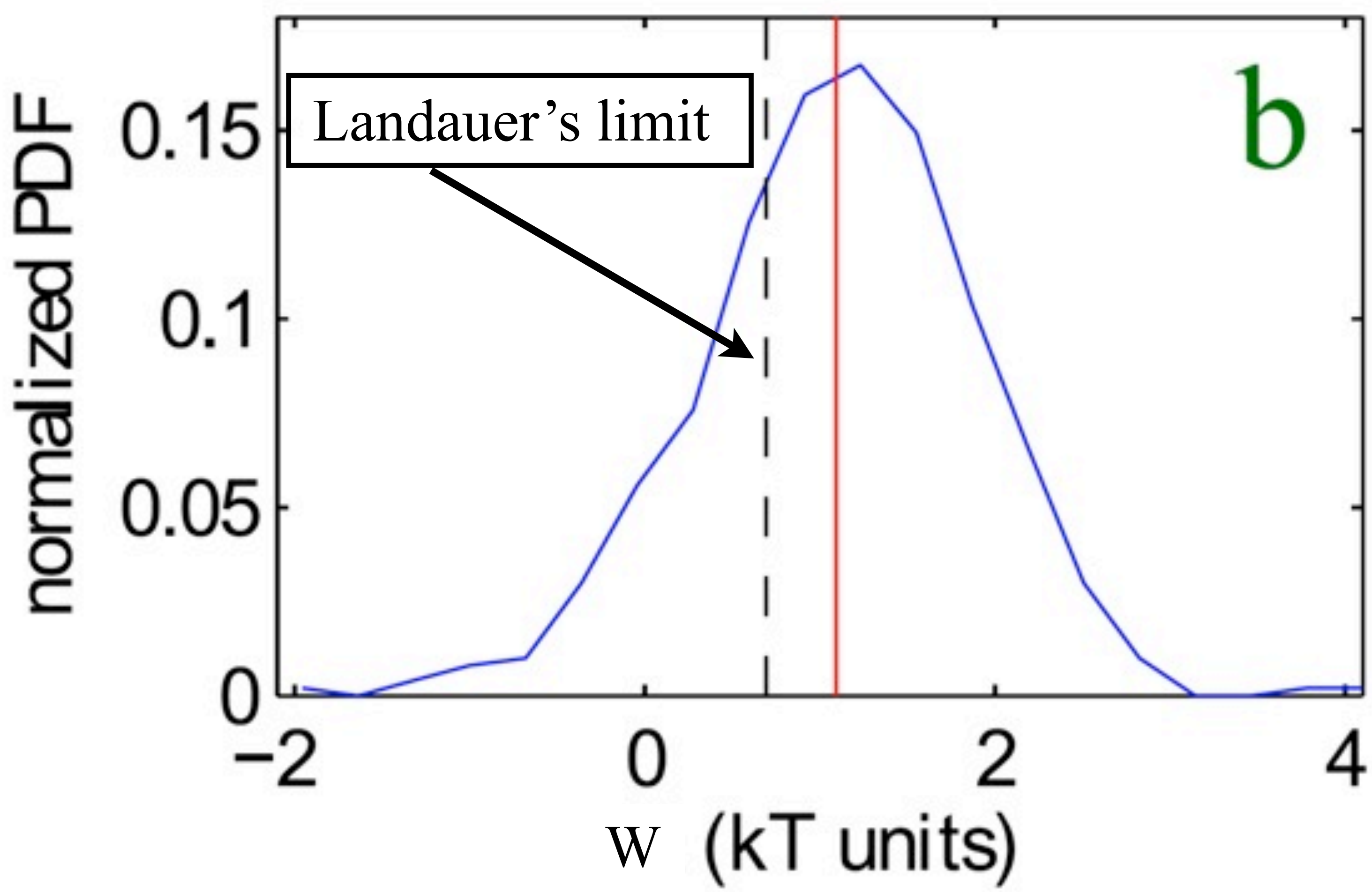

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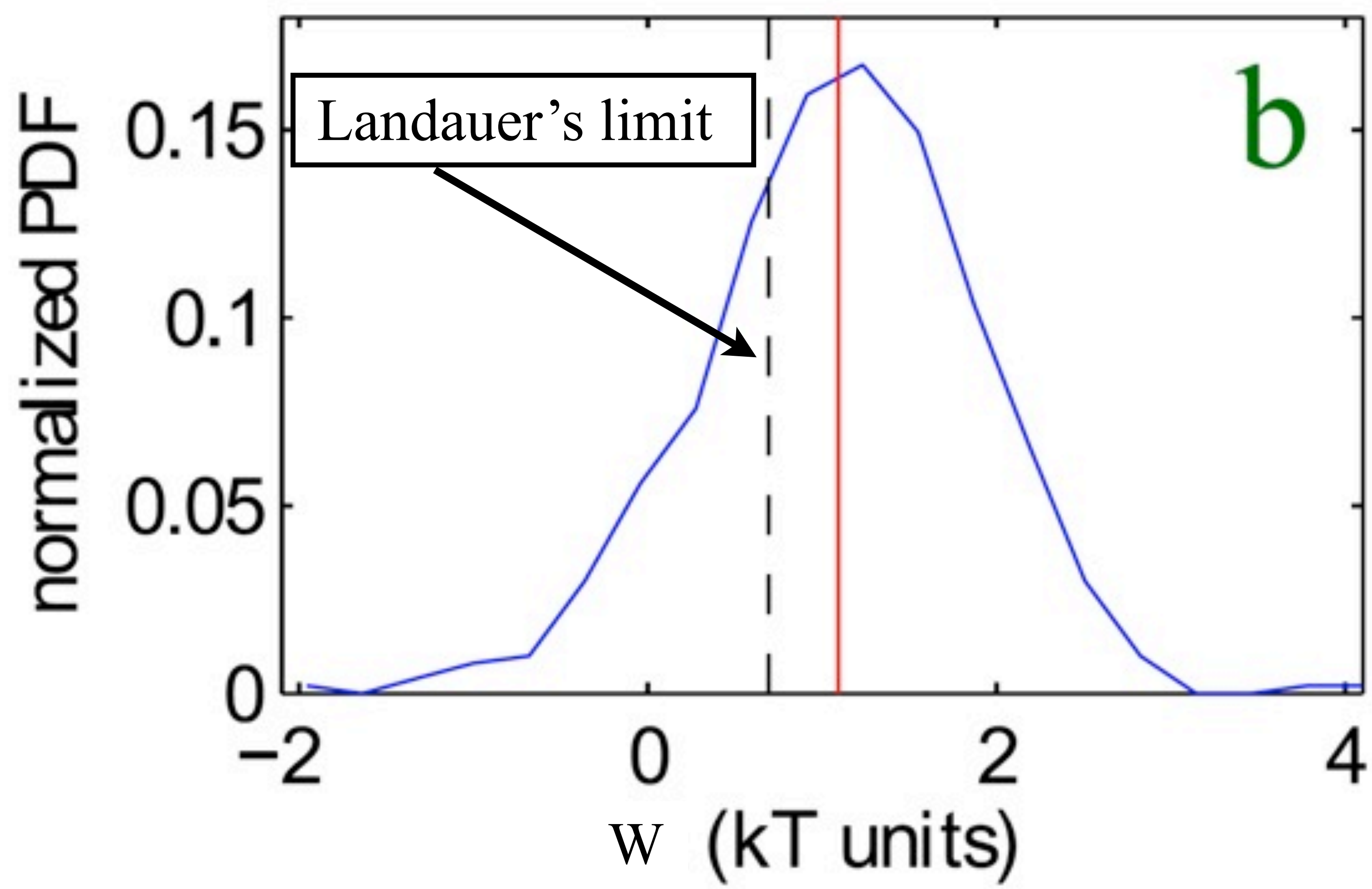
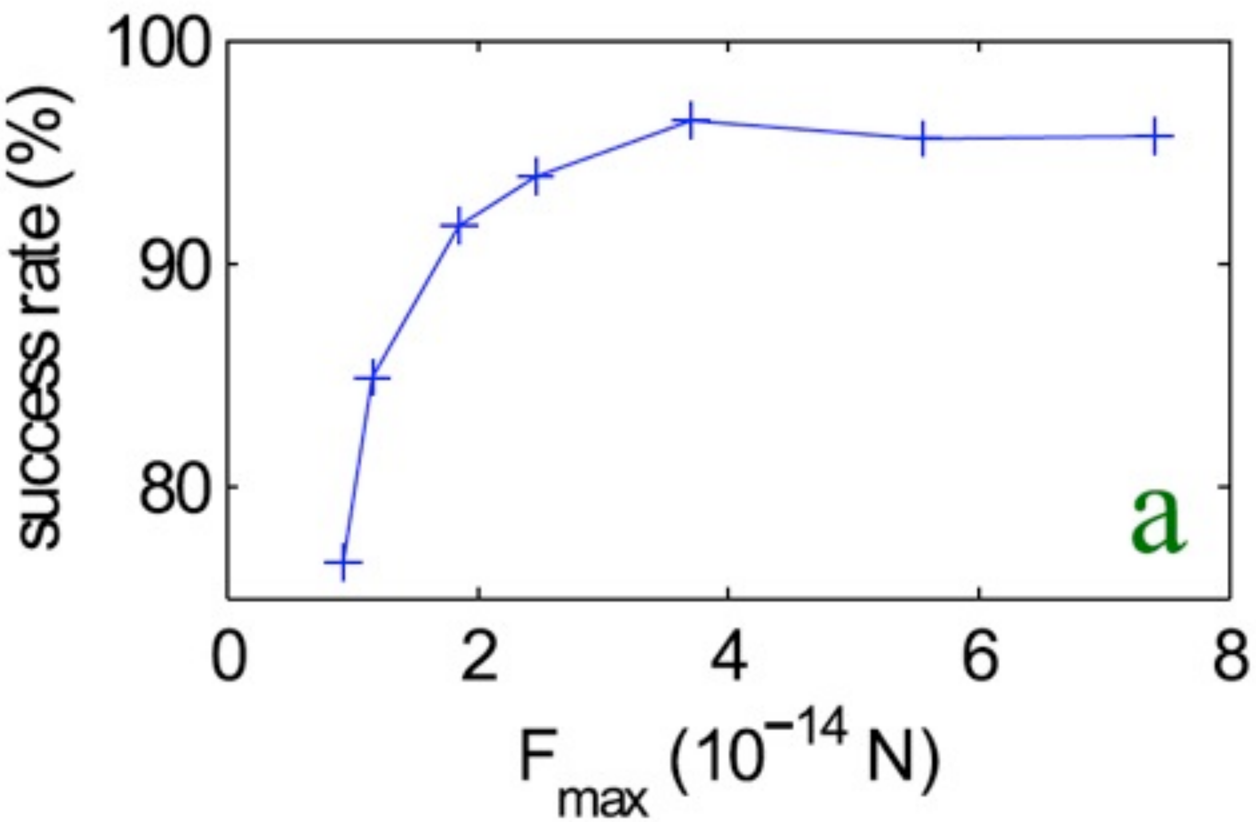


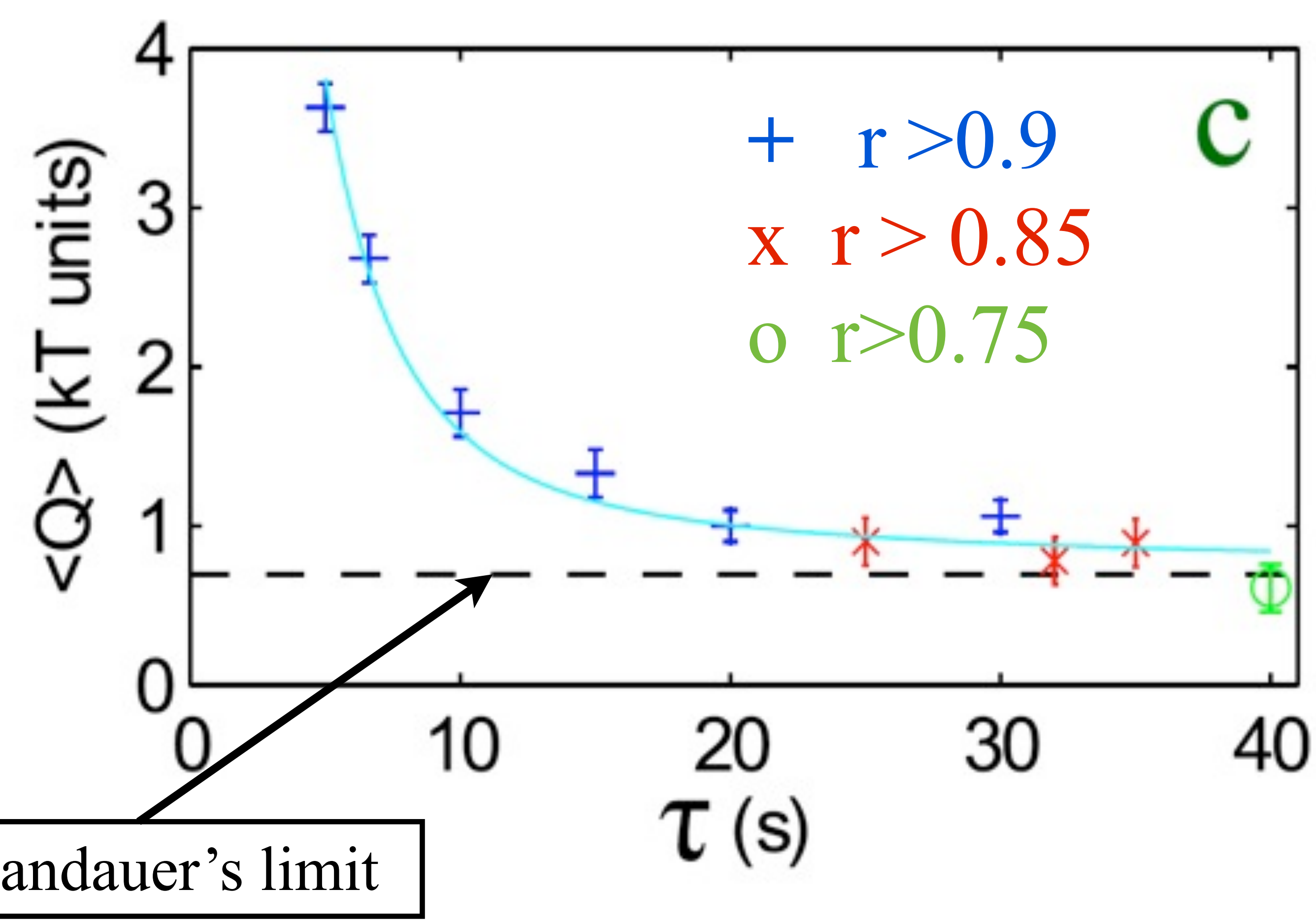
Results of the erasure procedure

$$\text{Success rate } r = \frac{\text{number of successful cycles}}{\text{Total number of cycle}}$$

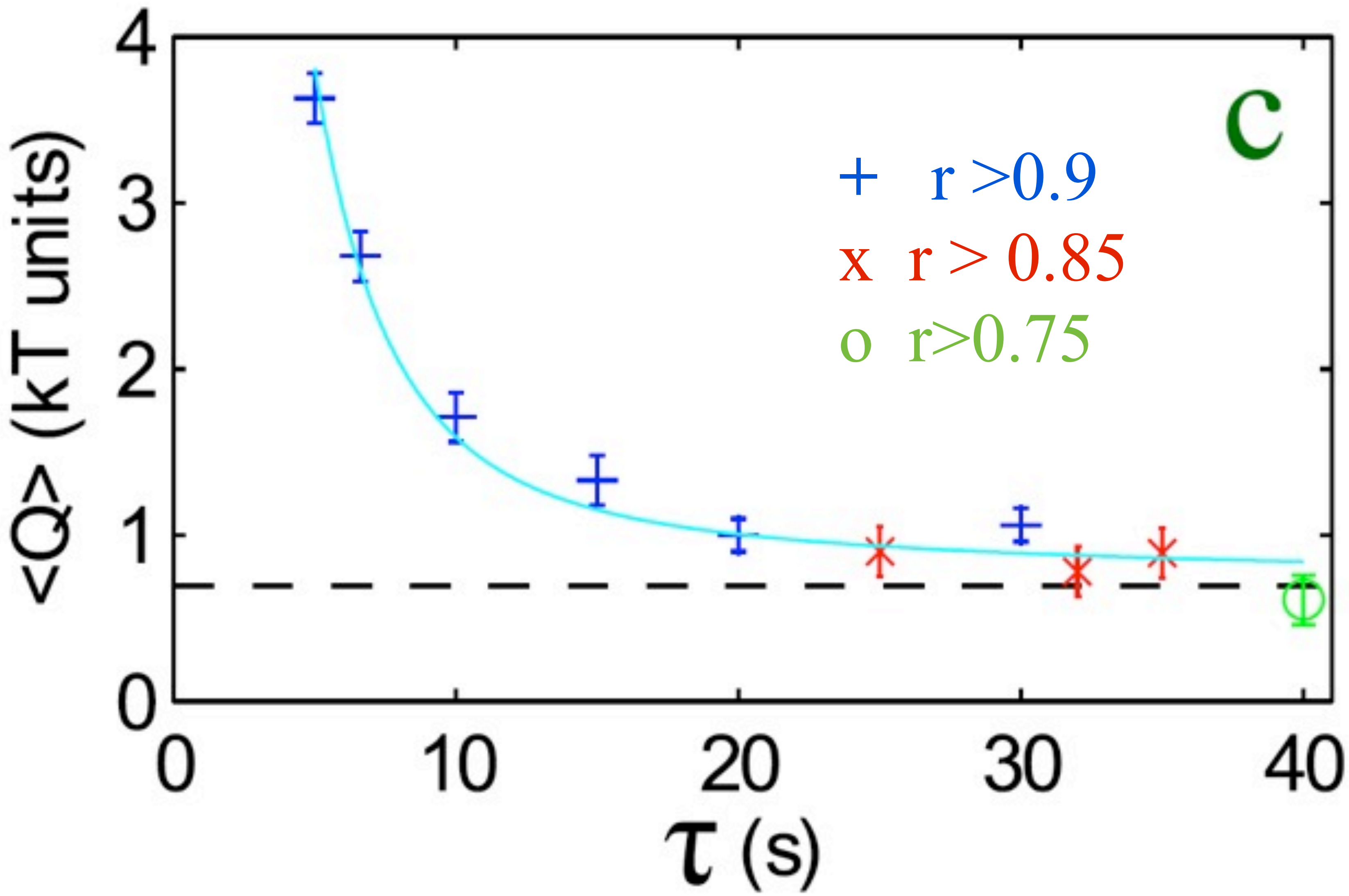
Qualitative observations :

- At constant τ : W and r increase with F_{max}
- At constant F_{max} : W decreases and r increases for increasing τ





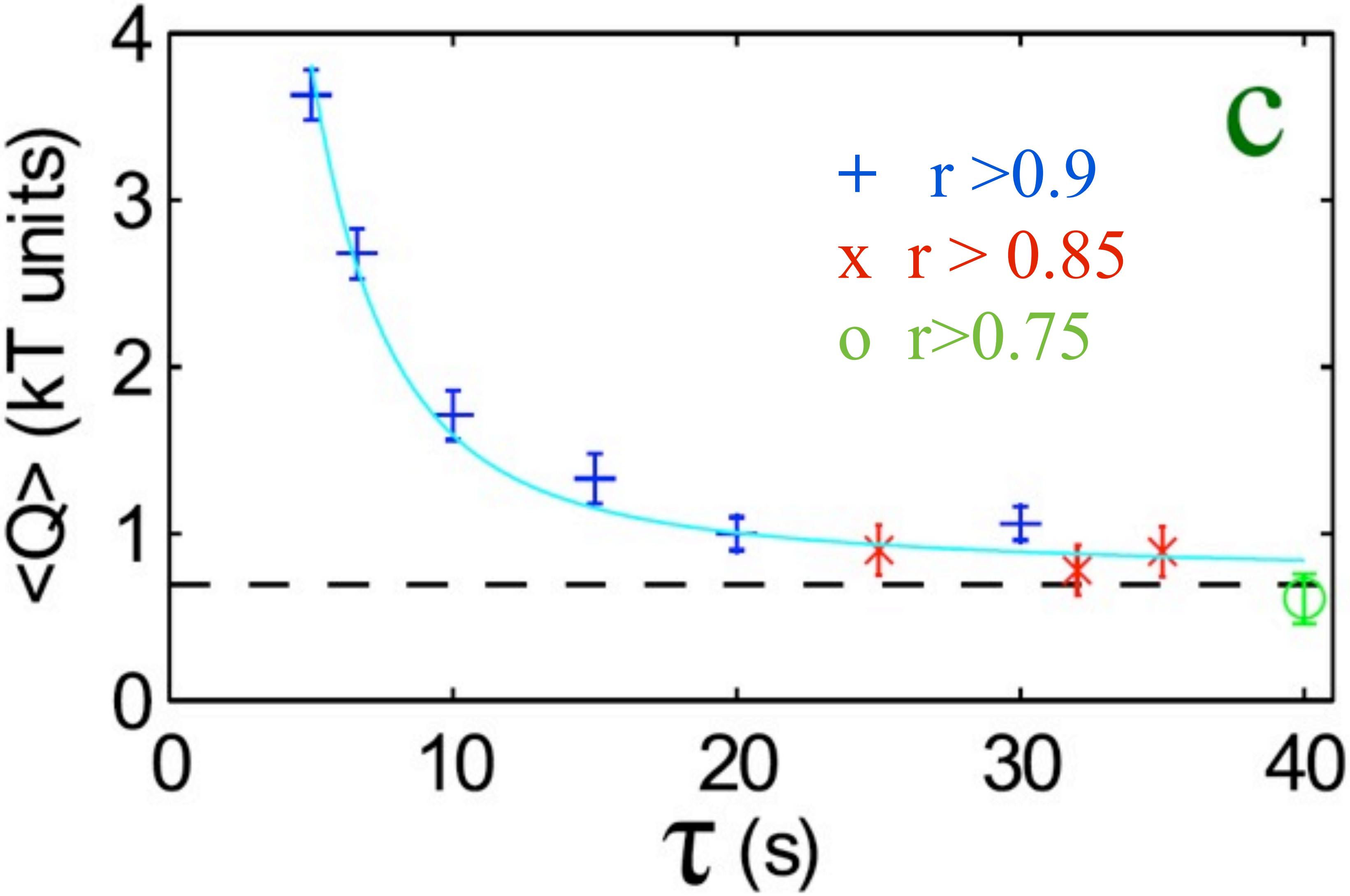
Landauer's limit



Landauer's limit as a function of r

$$< Q >^{r}_{\mathrm{Landauer}} = kT[\ln 2 + r \ln r + (1-r)\ln(1-r)]$$

Landauer's limit



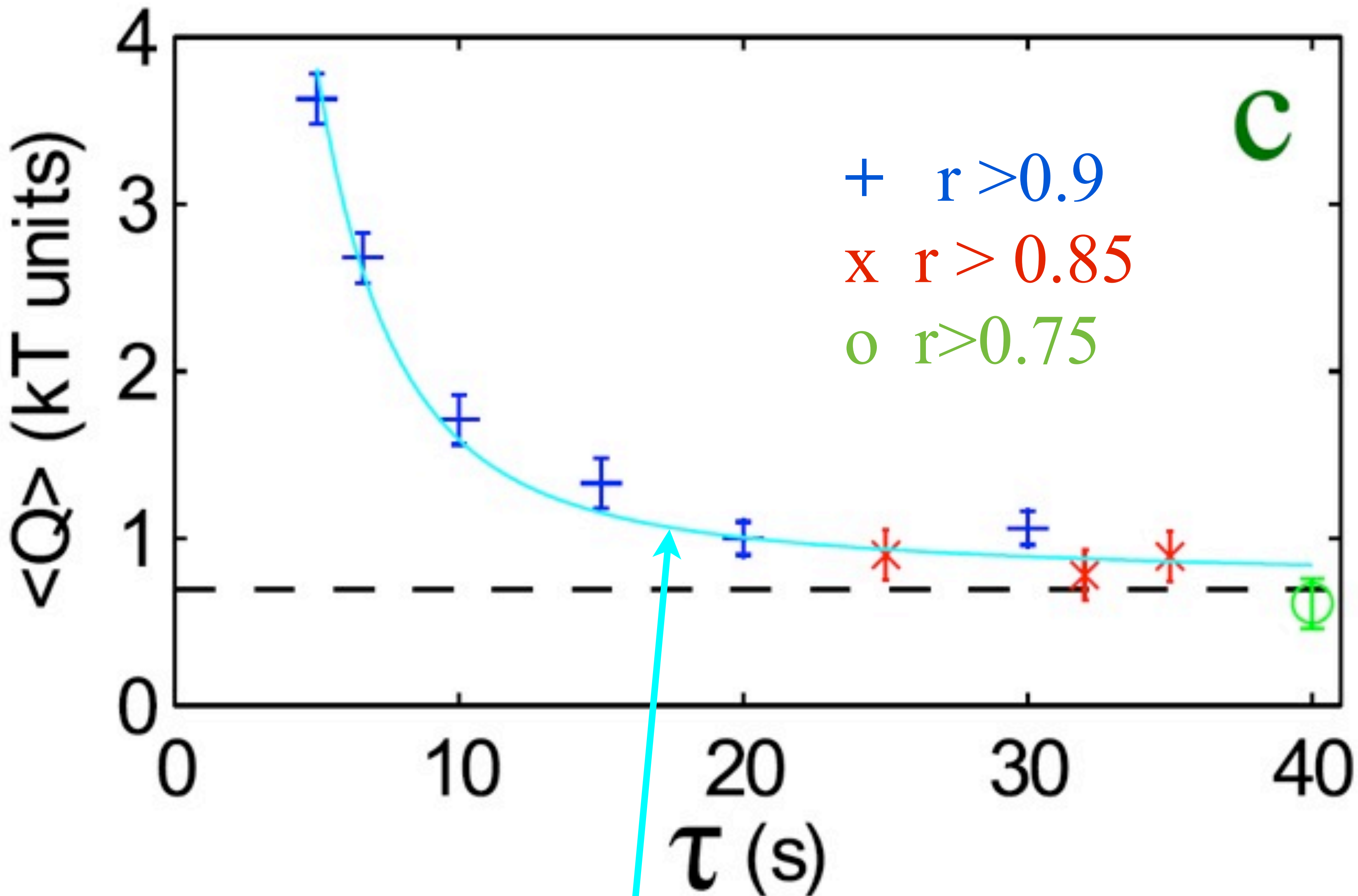
Landauer's limit as a function of r

$$< Q >^r_{\mathrm{Landauer}} = kT[\ln 2 + r \ln r + (1-r)\ln(1-r)]$$

At r=0.5 $< Q >^r_{\mathrm{Landauer}} = 0$

Indeed the Erasure Procedure left the initial state unchanged

Landauer's limit



$$\Delta F = \Delta U - T\Delta S$$

Asymptotic behaviour

$$\tau \to \infty$$

Sekimoto -Sasa J. Phys. Soc. Jpn. 66, 3326 (1997).

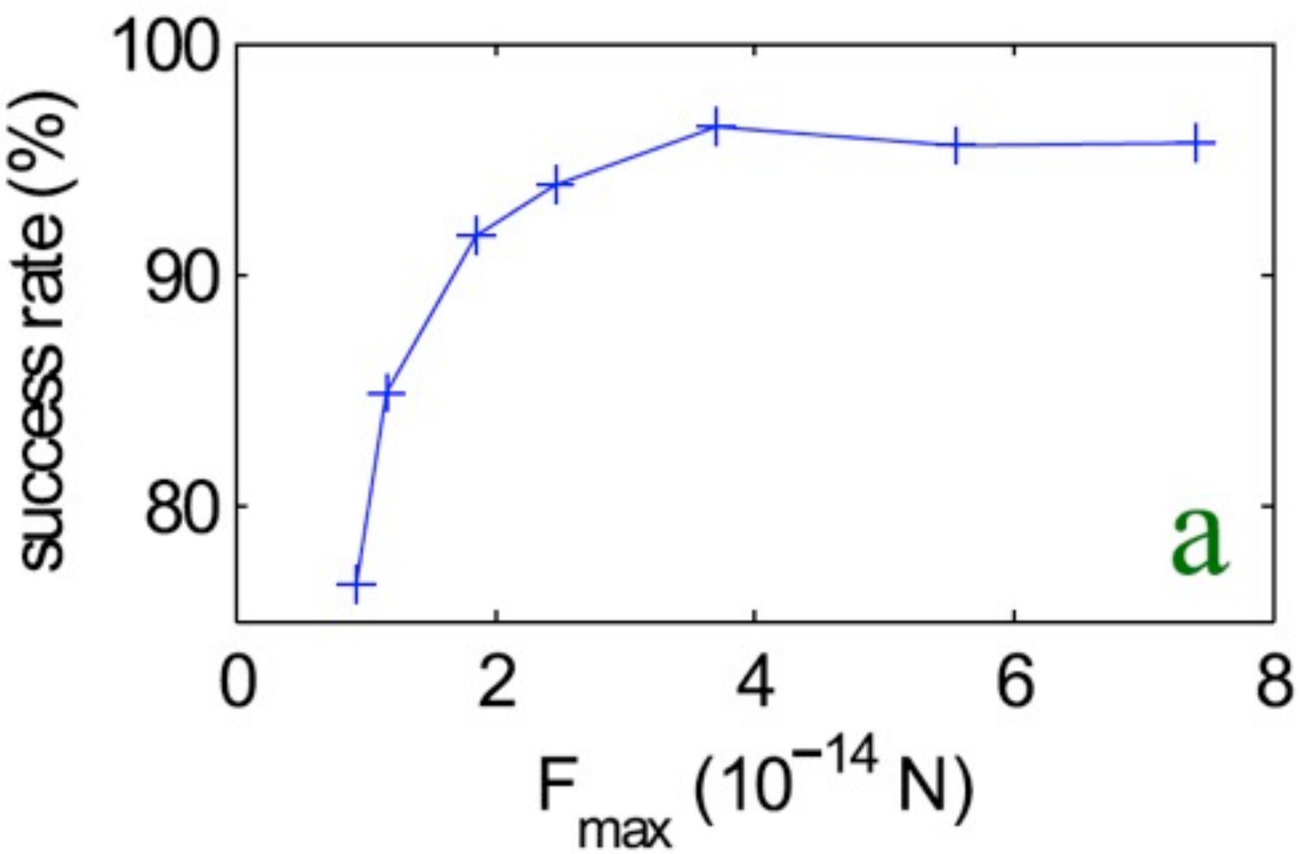
$$< W > \simeq \Delta F + B/\tau$$

As $< \Delta U >= 0$ then $\Delta F = -T\Delta S$ and $< Q >=< W > \simeq kT \ln 2 + B/\tau$

$$< Q >=< Q >_{\text{Landauer}} + [A \exp(-t/\tau_K) + 1] B/\tau$$

$\tau_K = \tau_o \exp[\frac{\Delta U}{k_B T}]$ is the Kramers time

The success rate r

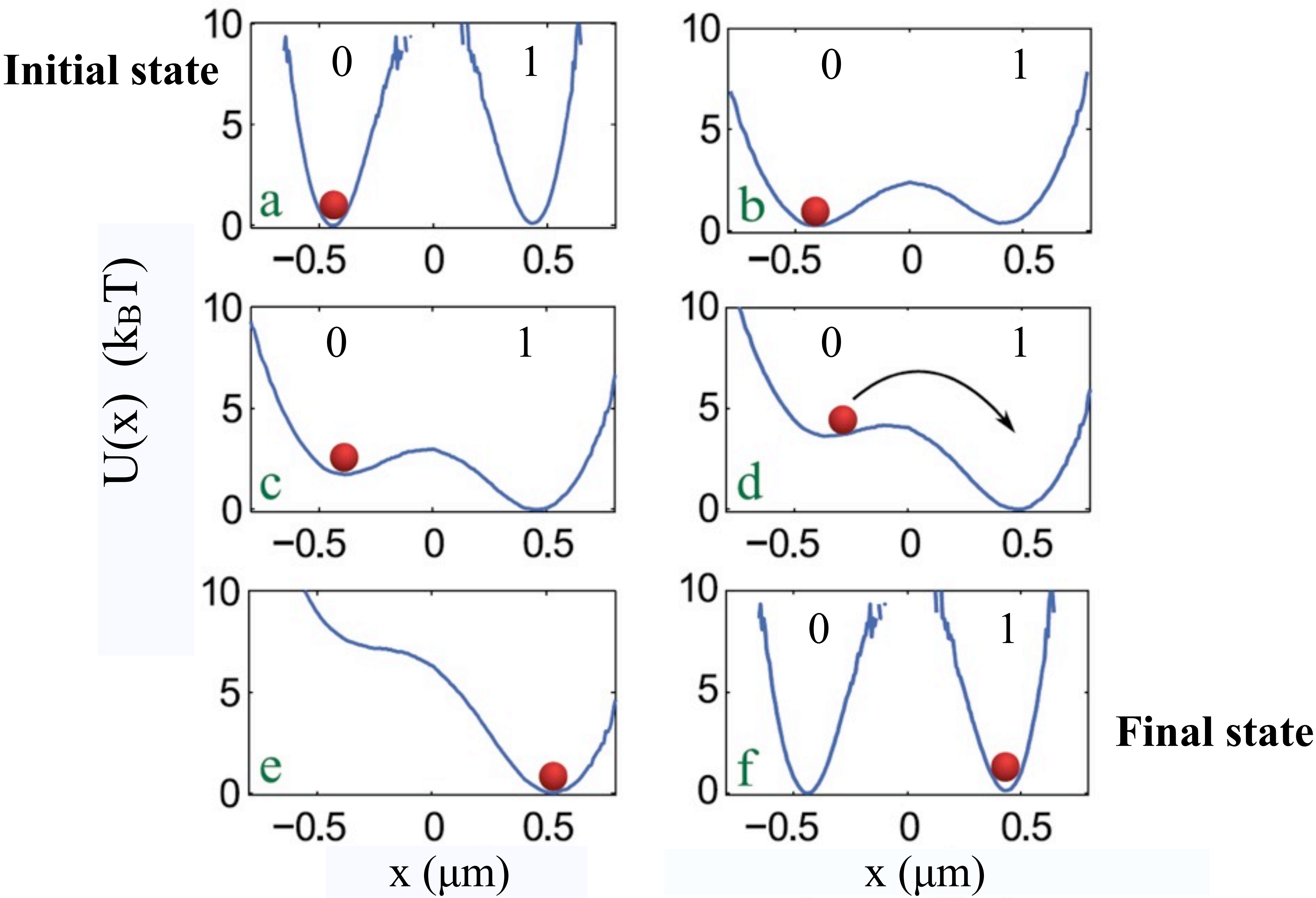


Why in the experiment r< 1 ?

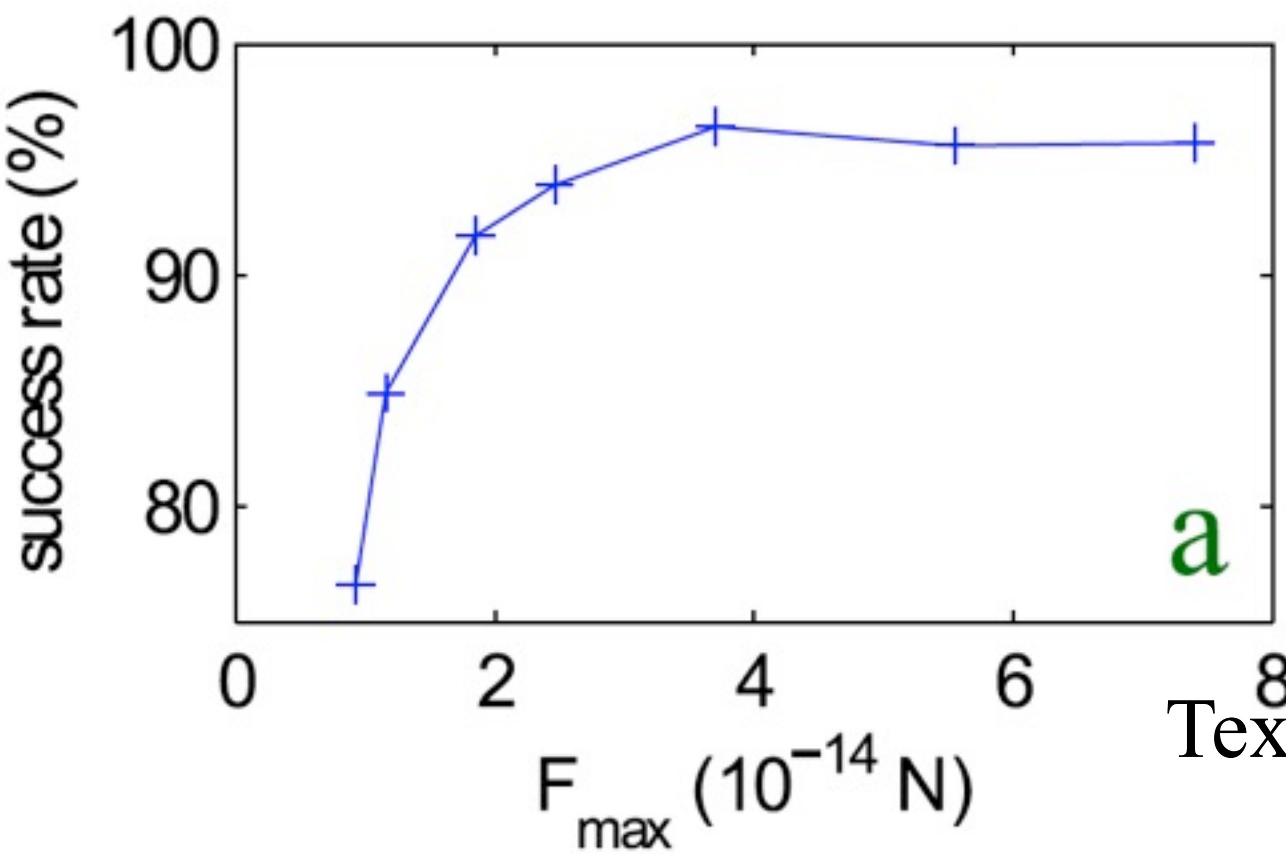
Is this result produced by 3D effects of the trap ?

Is the finite height of the initial barrier responsible of r<1 ?

The Erasure Procedure



The success rate r



Texte

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Numerical test

We use all the experimental parameters and procedure

with two different initial barriers $8k_BT$ and $15k_BT$

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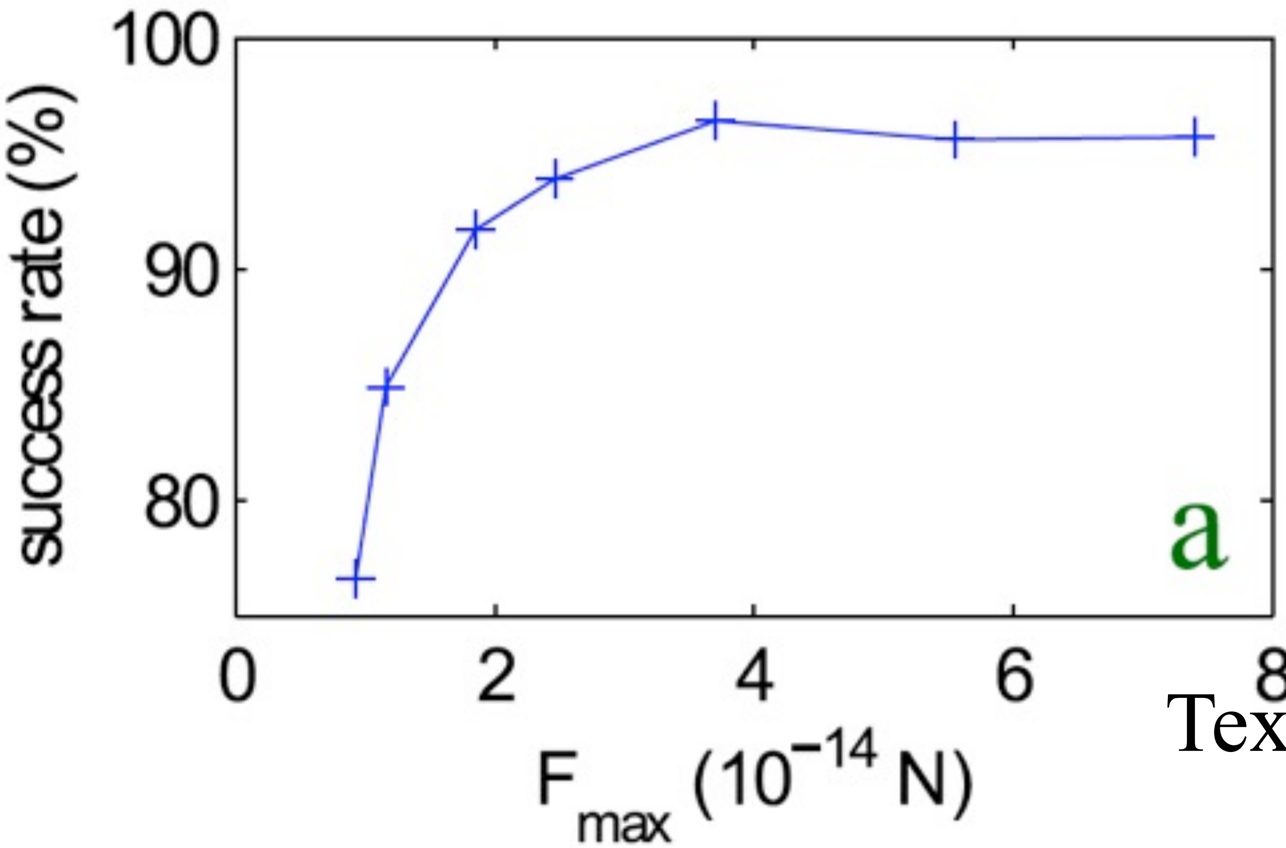

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$$\nu \dot{x} = -\frac{\partial U(x,t)}{\partial x} + F(t) + \eta$$

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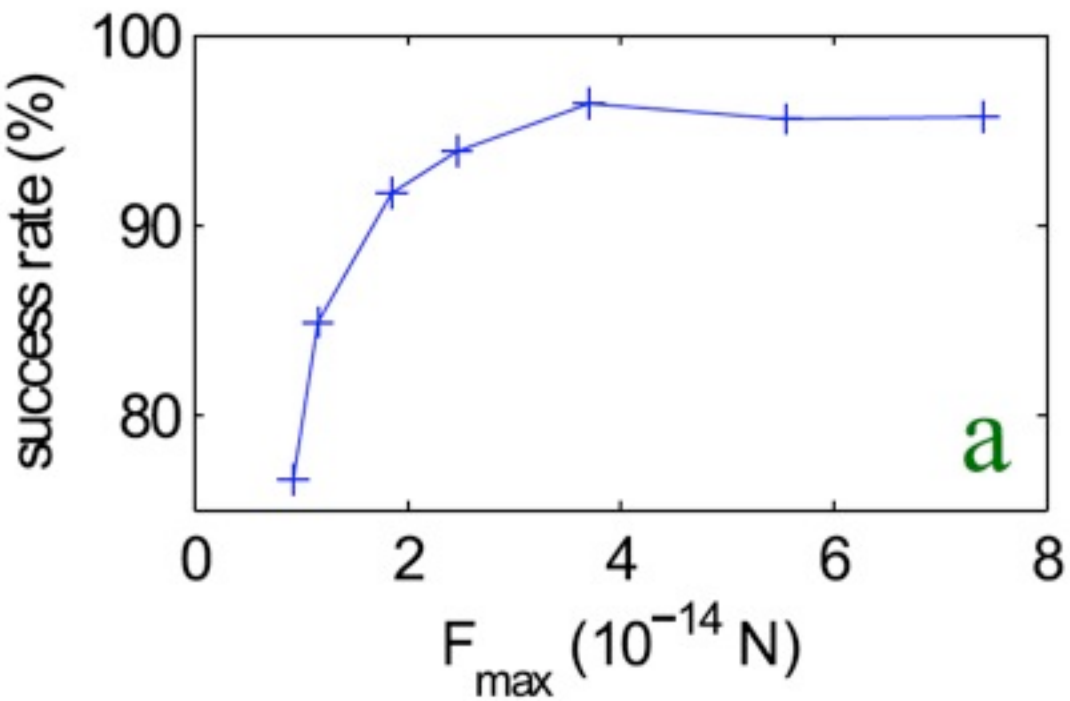

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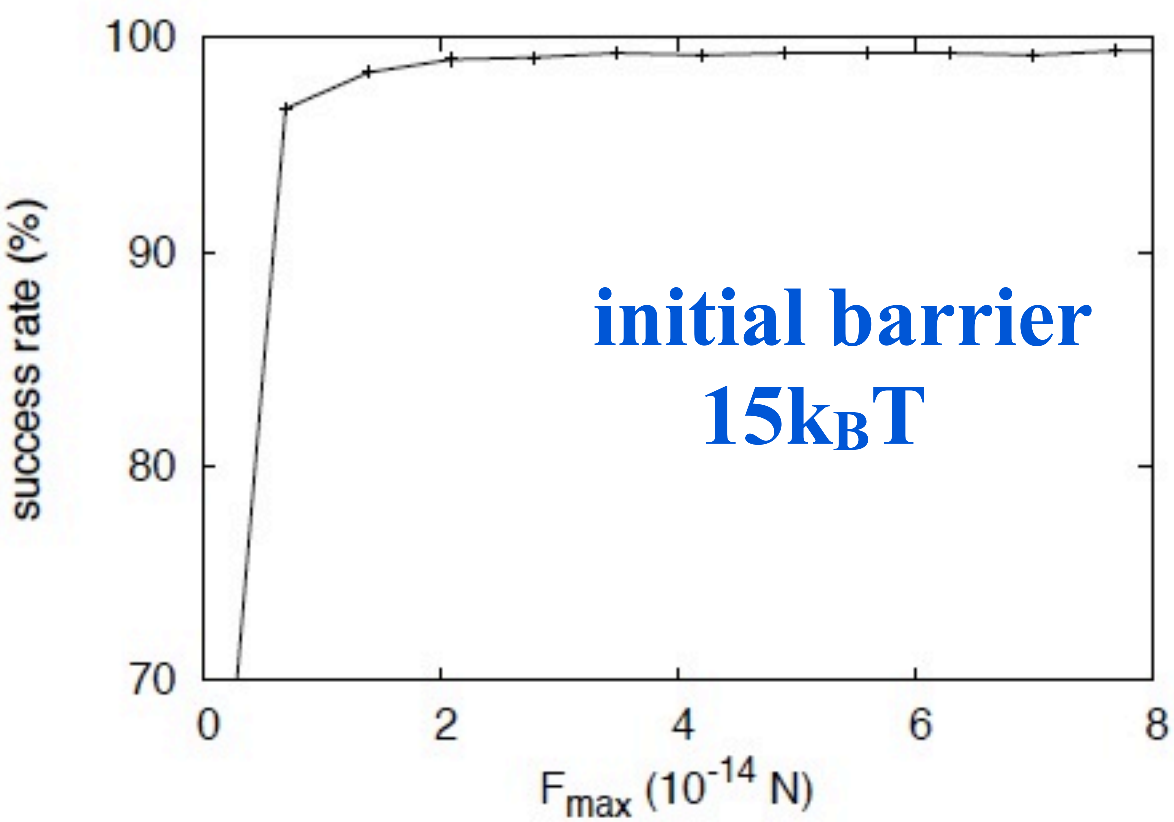

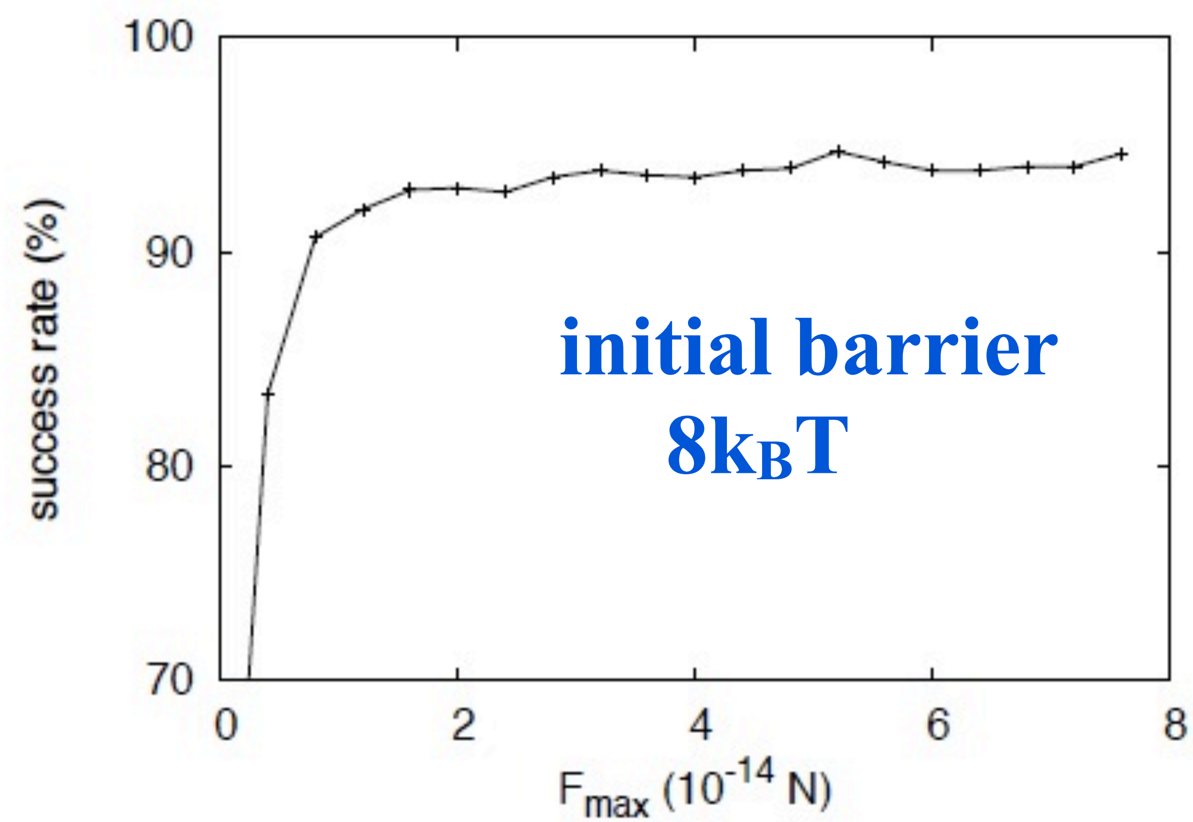

Conclusions

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- The asymptotic limit is reached in $1/\tau$ for $\tau > 3\,\tau_k$

- The fact that $r<1$ is due to the finite height of the initial barrier

- Thermal fluctuations play an important role to reach the limit

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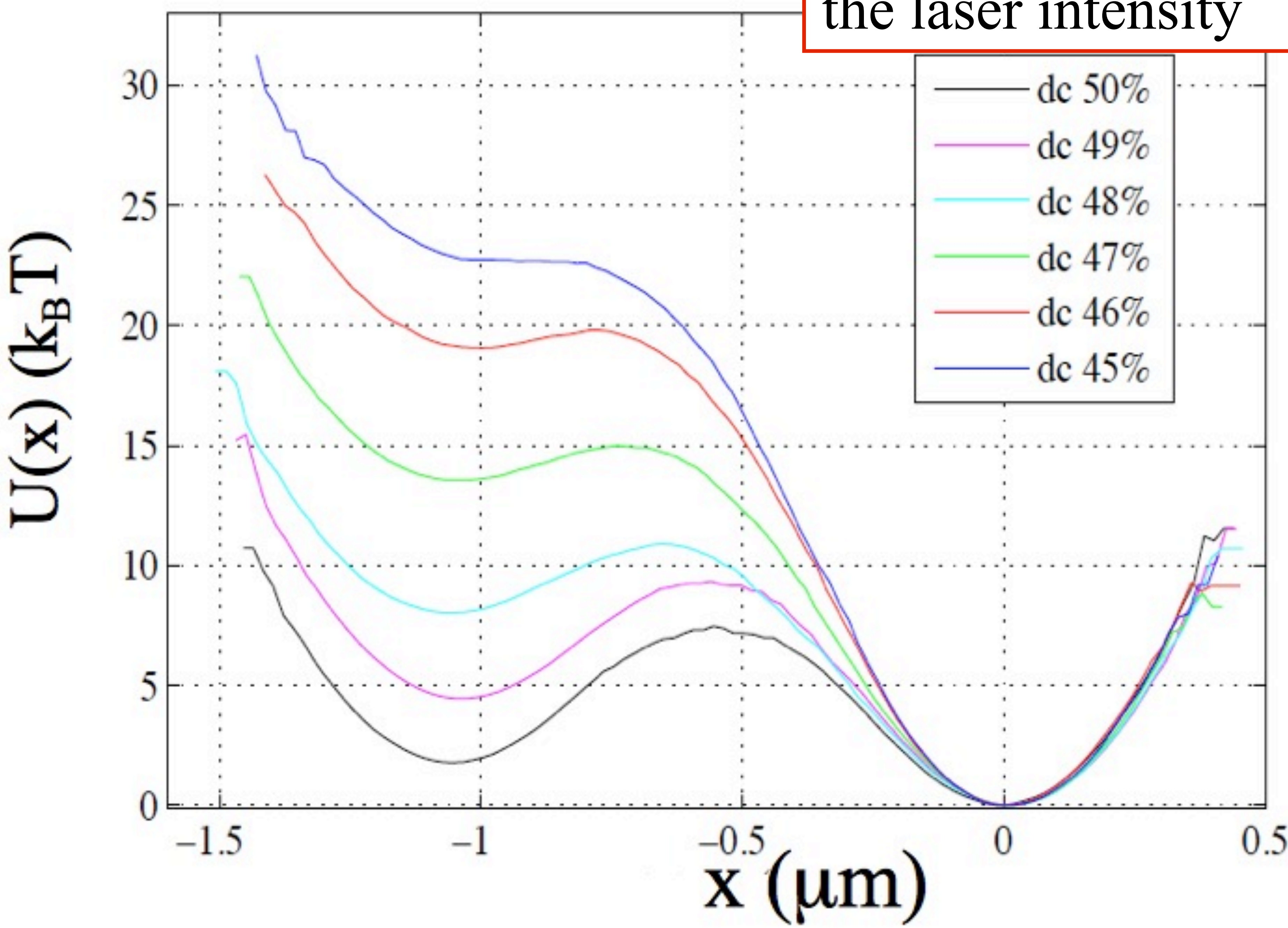
See recent paper on optimisation : E. Aurell, K. Gawedzki

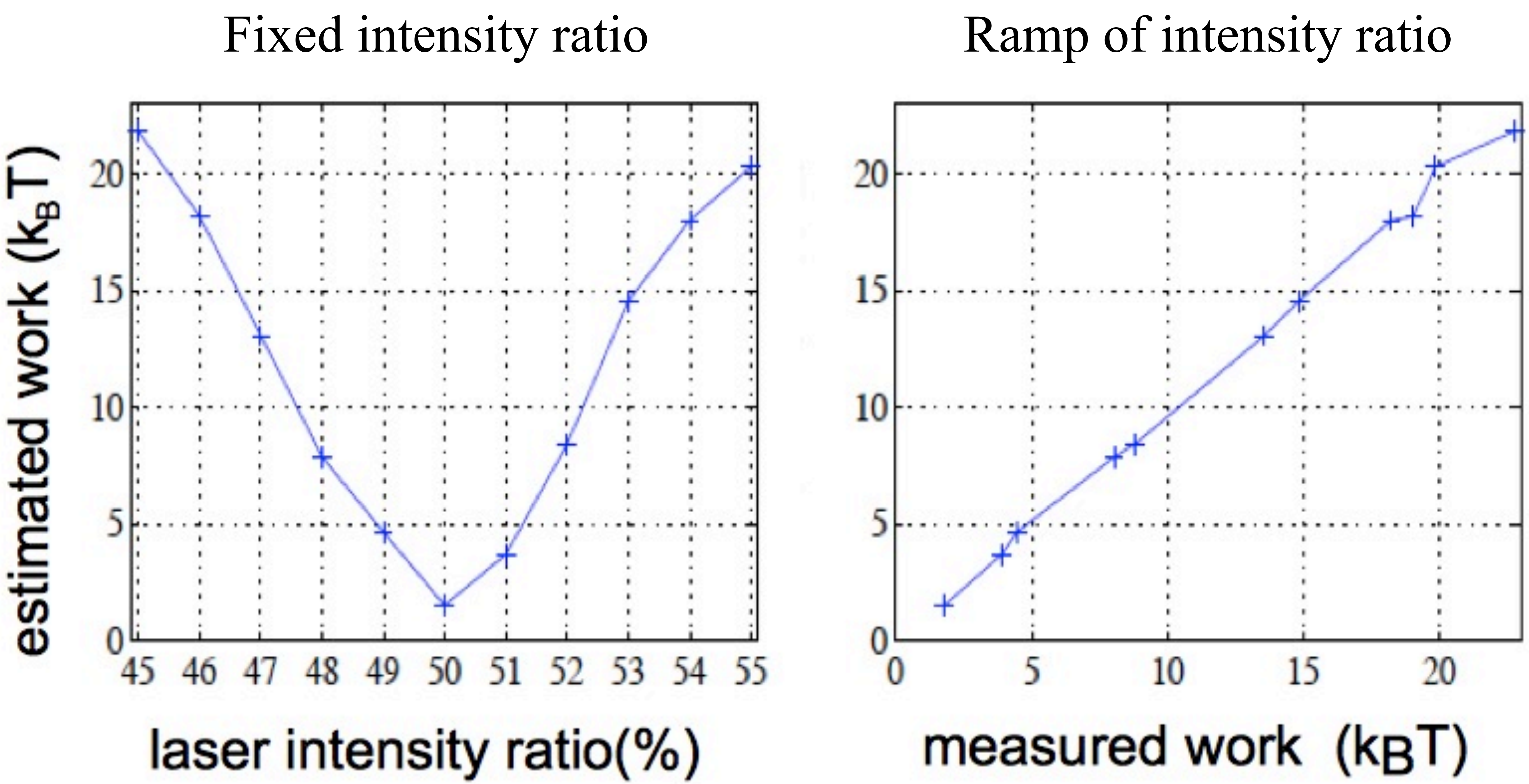
Other procedure (I)

Relative change of the laser intensity



The ramping time of the laser intensity has been changed from 1s to 50s

Other procedure (II)



The work is mainly due to the jump of the particle

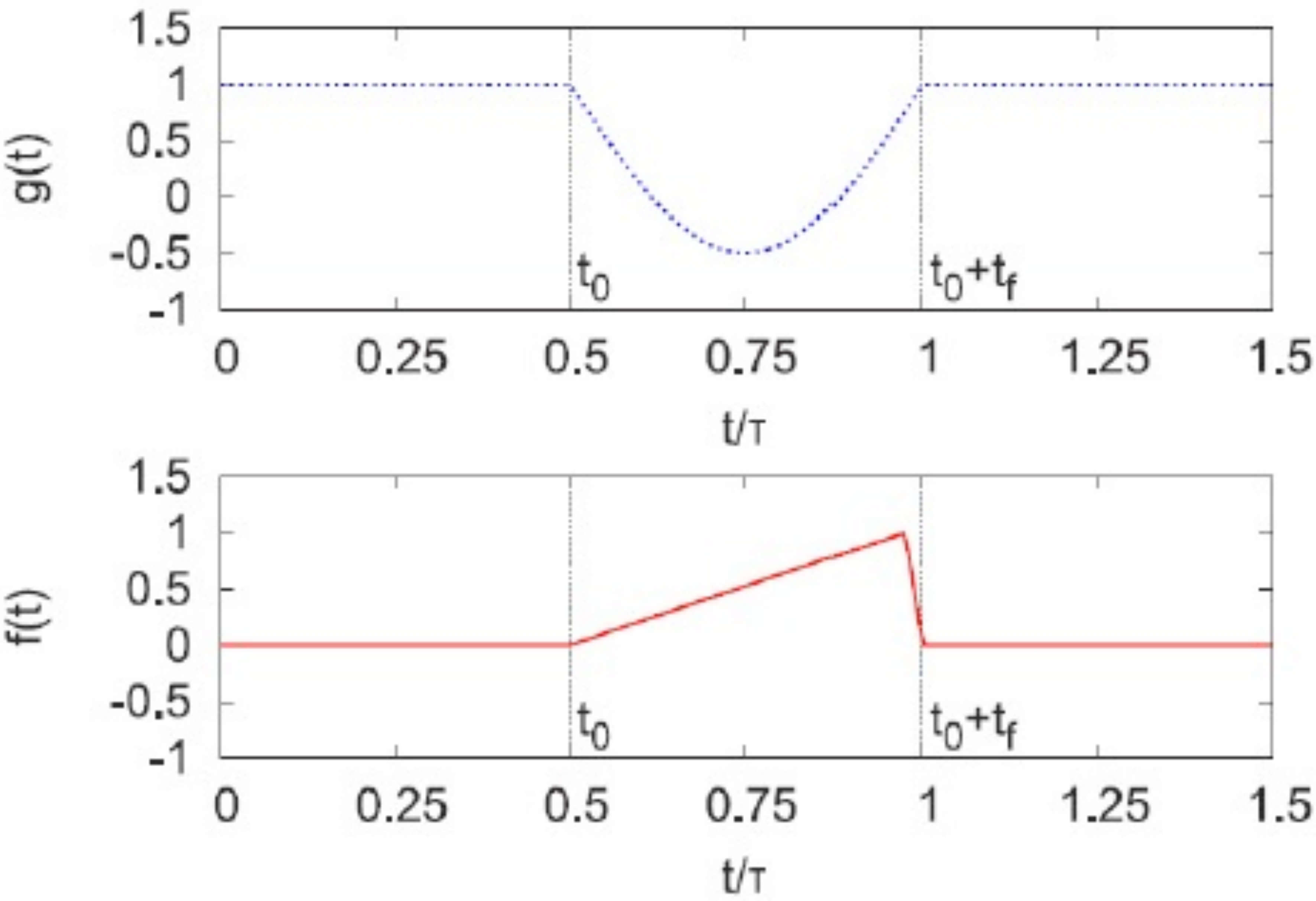
The Landauer limit can never be reached

Numerical results

Memory Erasure in Small Systems,
R. Dillenschneiderand E. Lutz, Phys. Rev. Lett. 102, 210601 (2009)

Potential : $V(x,t) = -\frac{1}{2}g(t)x^2 + \frac{1}{4}x^4$

External force : $Af(t)$



Non-dimensional numbers and the success rate

$$\bar{\tau} = \frac{\tau}{\tau_k}$$

Possibility of jumping the barrier without force

$$\bar{F} = \frac{\delta x \ F_{max}}{\Delta U}$$

The maximum external work overcomes the barrier

$\tau_K = \tau_o \exp[\frac{\Delta U}{k_B T}]$ is the Kramers time with $\tau_o \simeq 1s$

δx is the distance of the potential minima

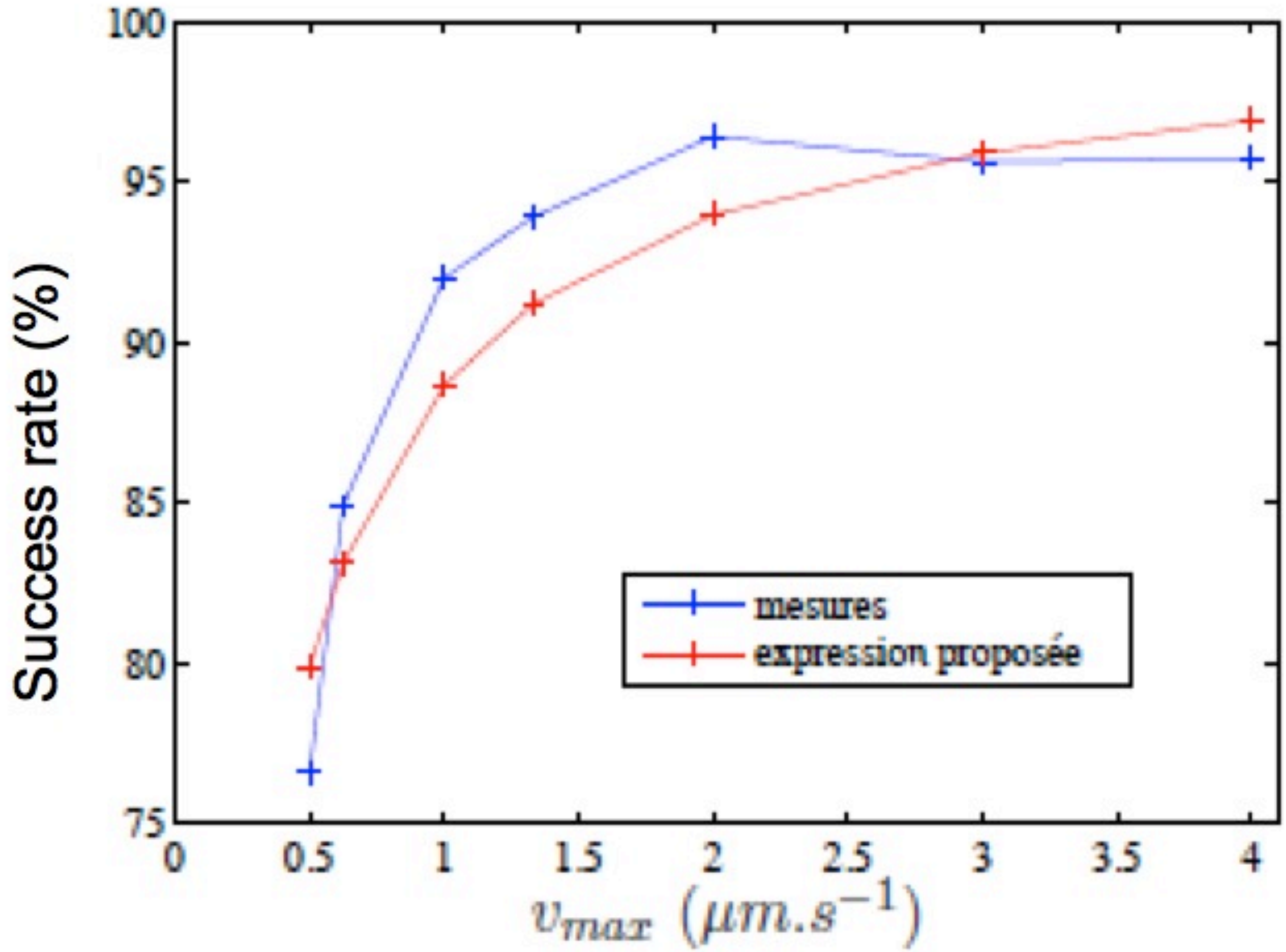
One can think that the success rate is :

$$r = \frac{1}{2}[1 + \exp(-\frac{1}{\bar{\tau}\bar{F}})]$$

Non-dimensional numbers and the success rate

Experimentally

$$r = \frac{1}{2}\left[1 + \exp\left(-\frac{a}{\bar{\tau}\bar{F}^2}\right)\right]$$



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