# Modular, Compositional, and Executable Formal Semantics for LLVM IR 

## ICFP 2021

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## LLVM Compiler Infrastructure



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[Lattner et al.]


LLVM IR


## LLVM IR

body:
$\% 6=$ load \%acc
$\% 7=$ load $\% 1$
$\% 8=\mathrm{mul} \% 6$, $\% 7$ store \%8, \%acc $\% 9$ = load $\% 1$
$\% 10=$ sub $\% 9$, 1
store \%10, \%1
br label \%start
$\% 3$ = load $\% 1$
$\% 4=$ icmp sgt $\% 3,0$
br \%4, label \%then, label \%else
entry:

## \%1 = alloca

\%acc = alloca store \%n, \%1 store 1 , \%acc br label \%start
loop:

## post:

$$
\% 12=\text { load \%acc }
$$

ret \%12
https://llvm.org/docs/LangRef.html


## Formal Semantics

Crellvm [Kang et al., 18]
K-LLVM [Li and Gunter, 20]
Vellvm [Zhao et al., 12]
Taming UB [Lee et al., 17]
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## Bug Finding

Alive [Lopes et al., 15]
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This work's ancestor
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## The Vellvm Project


[Zhao and Zdancewic - CPP 2012]
Verified computation of dominators
[Zhao et al. - POPL 2012]
Formal semantics of IR + verified SoftBound
[Zhao et al. - PLDI 2013]
Verification of (v)mem2reg!
https://github.com/vellvm/vellvm-legacy
A success, but so monolithic it couldn'† evolve!

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$$
G \vdash p c, \text { mem } \rightarrow p c^{\prime}, \text { mem }^{\prime}
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## Vellvm Legacy: Rough Approximation

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## Vellvm Legacy: Rough Approximation

## entry:

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## $\% 1=$ alloca

\%acc = alloca
store \%n, \%1 store 1, \%acc br label \%start

$\% 3=$ load $\% 1$
$\% 4=$ icmp sgt $\% 3,0$
br \%4, label \%then, label \%else

| $\circ 6=$ load \%acc | $\% 12=$ load $\%$ acc |
| :--- | :--- |
| $\% 7=$ load $\% 1$ | ret $\% 12$ |

\% $8=\mathrm{mul} \% 6$, $\% 7$ ret \% 12
$\% 9=$ load $\% 1$
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## Vellvm Legacy: Rough Approximation



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## This Paper: a Redesign for Vellvm

Vellvm
github.com/vellvm/vellvm

## A Tree Represents an Interactive Computation



We consider here a computation whose interactions with the environment are read and writes to a state

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Events only specify
the type of effects
The semantics of effects is introduced
interp handle The tree is interpreted via an event handler
St -> itree voidE (St * Nat)
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specific, user defined


A general recipe

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## Scaling to a Fully Fledged Language

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Event interface for an IR program

$$
\begin{aligned}
E & =L_{E}+{ }^{\prime} G_{E}+{ }^{\prime} M_{E} \\
& + \text { Call }_{E}+^{\prime} \text { Intrinsics }_{E} \\
& + \text { Pick }_{E}+{ }^{\prime} \text { UB }_{E} \\
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Compositional representation for (open) IR programs

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Compositional representation for (open) IR programs

## VIR

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structural representation
Level 0 itree VellvmE $\mathcal{V}_{u}$



Pieces of state get introduce


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|  | VIR |  |
| :---: | :---: | :---: |
|  | structural representation |  |
| Level 0 | itree VellvmE $\mathcal{V}_{u}$ |  |
|  |  |  |
| Level I | itree $E_{0} \mathcal{V}_{u}$ | Pieces of state ge introduce |
|  | $\downarrow$ global environment |  |
| Level 2 | stateT $E_{\text {Env }}\left(\right.$ itree $\left.E_{1}\right) \mathcal{V}_{u}$ |  |
|  | $\downarrow$ local environment |  |
| Level 3 | stateT $E n v_{L} * E n v_{G}\left(\right.$ itree $\left.E_{2}\right) \mathcal{V}_{u}$ |  |
|  | $\downarrow$ memory model |  |
| Level 4 | stateT $_{\text {Mem* }{ }^{\text {Env }} \text { * Env }}\left(\right.$ itree $\left.E_{3}\right) \mathcal{V}_{u}$ | Memory model based on CAV'15 |




## VIR

structural representation
Level $0 \quad$ itree VellvmE $\mathcal{V}_{u}$
$\downarrow$ intrinsics


## But How Does it Relate to LLVM IR?

## You can play with it yourself!



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```
define i64 @factorial(i64 %n) {
    %1 = alloca i64
    %acc = alloca i64
    store i64 %n, i64* %1
    store i64 1, i64* %acc
    br label %start
start:
    %2 = load i64, i64* %1
    %3 = icmp sgt i64 %2, 0
    br i1 %3, label %then, label %end
then
    %4 = load i64, i64* %acc
    %5 = load i64, i64* %1
    %6 = mul i64 %4, %5
    store i64 %6, i64* %acc
    %7 = load i64, i64* %1
    %8 = sub i64 %7, 1
    store i64 %8, i64* %1
    br label %start
end:
    %9 = load i64, i64* %acc
    ret i64 %9
}
define i64 @main(i64 %argc, i8** %arcv) {
    %1 = alloca i64
    store i64 0, i64* %1
    %2 = call i64 @factorial(i64 5)
    ret i64 %2
}
```



Realistic* (sequential) subset, happy to take feature requests!

* See the paper for the details of the features we cover


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Realistic* (sequential) subset, happy to take feature requests!
Tested against clang over:

- A collection of unit tests
- A handful of significant programs from the HELIX project
-Early experiments over randomly generated programs using QuickChick
* See the paper for the details of the features we cover


## But Why Would it Be Any Useful?

## A (weak) bisimulation over itrees

Coinductive relation ignoring finite amounts of internal steps


Get us a first (fine) notion of equivalent programs

# Structural Equational Theory and Compositional Reasoning 

A battery of structural equational lemmas at the VIR level

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A battery of structural equational lemmas at the VIR level


Reasoning about control-flow graph composition

$$
\text { outputs }\left(c f g_{2}\right) \cap \operatorname{inputs}\left(c f g_{1}\right)=\varnothing \quad \text { to } \notin \operatorname{inputs}\left(c f g_{1}\right)
$$

$\llbracket c f g_{1} \cup c f g_{2} \rrbracket(f, t o) \approx \llbracket c f g_{2} \rrbracket(f, t o)$

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$$

Proof of a simple block-fusion optimization





Block-Fusion (proof)

Block-Fusion (result)

## Instruction level reasoning

To reason about instructions, we could get back down to comparing trees
$\llbracket \% x=$ load i64, i64* \%acc $\rrbracket_{i n s t r}$


## Instruction level reasoning

To reason about instructions, we could get back down to comparing trees
$\llbracket \% x=$ load $\mathrm{i} 64, \mathrm{i} 64^{*} \% \mathrm{acc} \rrbracket_{\text {instr }}$


Instead, we reason at the level of VIR through a battery of lemmas for each expression and instruction

$$
\begin{gathered}
\llbracket \mathrm{acc} \rrbracket_{\text {expr } \mathrm{g}} \mathrm{Im} \mathrm{~m} \approx \operatorname{Ret}(\mathrm{~m},(\mathrm{l},,(\mathrm{~g}, \mathrm{tt}))) \\
\text { read } \mathrm{m} \text { a i } 64=\mathrm{inr} \mathrm{uv}
\end{gathered}
$$

$\llbracket \% \mathrm{x}=\mathrm{load} \mathrm{i} 64, \mathrm{i} 64^{*} \% \mathrm{acc} \rrbracket_{\mathrm{instr}} \mathrm{g} \mathrm{I} \mathrm{m} \approx \operatorname{Ret}(\mathrm{m},($ Maps.add x uv l', $(\mathrm{g}, \mathrm{tt}))$ )
Representation functions can be made completely opaque

## Two main reasoning ingredients

## Strong equivalences at the VIR level over:

- the syntactic structure of the language
-the control flow
-the instructions, expressions and terminators.
Symbolic interpreter that can be run by rewriting during refinement proofs


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## Strong equivalences at the VIR level over:

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Symbolic interpreter that can be run by rewriting during refinement proofs
A primitive relational program logic:

- Weakening, conjunction, ... over the postcondition
- Sequential composition

Compositional construction of refinement proofs

## SPIRAL/HELIX

[Püschel, et al. 2005] [Franchetti et al., 2005, 2018] [Zaliva et al., 2015 2018, 2019]
DSL for high-performance numerical computing.


- Numerical computations compiled down to LLVM IR
- Formalized in Coq, targets Vellvm
- Bottom of the compilation chain proved* w.r.t. this technique
*Some operators are currently not proved



## Vellvm is Back!



A Coq formal semantics for a large fragment of LLVM IR coming with:

- a certified interpreter
- promising modularity
- a rich equational theory
- an equational style to refinement proofs


## A fertile ground is laid!

Verified analyses Verified optimizations
Concurrency
Back-ends

