## Modular, Compositional, and Executable **Formal Semantics for LLVM IR**

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- ICFP 2021
- Calvin Beck

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## LLVM Compiler Infrastructure [Lattner et al.]



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## Formal Semantics

Crellvm [Kang et al., 18]

K-LLVM [Li and Gunter, 20]

Vellvm [Zhao et al., 12]

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Alive [Lopes et al., 15]

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![](_page_7_Figure_16.jpeg)

![](_page_7_Picture_17.jpeg)

## Formal Semantics

- Crellvm [Kang et al., 18]
- K-LLVM [Li and Gunter, 20]
- Vellvm [Zhao et al., 12] This work's ancestor
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![](_page_8_Picture_13.jpeg)

![](_page_8_Figure_15.jpeg)

![](_page_8_Figure_17.jpeg)

![](_page_9_Picture_0.jpeg)

## The Vellvm Project

[Zhao and Zdancewic - CPP 2012]

#### Verified computation of dominators

## [Zhao et al. - POPL 2012]

Formal semantics of IR + verified SoftBound

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Verification of (v)mem2reg!

https://github.com/vellvm/vellvm-legacy

A success, but so monolithic it couldn't evolve!

![](_page_9_Figure_11.jpeg)

![](_page_10_Picture_0.jpeg)

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![](_page_11_Figure_13.jpeg)

![](_page_13_Figure_1.jpeg)

![](_page_14_Figure_1.jpeg)

![](_page_15_Figure_1.jpeg)

```
G \vdash pc, mem \rightarrow pc', mem'
```

![](_page_16_Figure_1.jpeg)

### $G \vdash pc, mem \rightarrow pc', mem'$

Program transformations modify G, invariants must relate the pc Lack of compositionality

![](_page_16_Picture_5.jpeg)

![](_page_17_Figure_1.jpeg)

# $G \vdash pc, mem \rightarrow pc', mem'$

Program transformations modify G, invariants must relate the pc Lack of compositionality

A single relation encompasses all aspects of the semantics

Lack of modularity

![](_page_17_Picture_7.jpeg)

![](_page_18_Figure_1.jpeg)

## $G \vdash pc, mem \rightarrow pc', mem'$

Program transformations modify G, invariants must relate the pc Lack of compositionality

A single relation encompasses all aspects of the semantics

Lack of modularity

The semantics does not compute, it is a relation Lack of executability

![](_page_18_Picture_8.jpeg)

![](_page_18_Picture_9.jpeg)

# This Paper: a Redesign for Vellvm

![](_page_19_Figure_1.jpeg)

![](_page_20_Figure_1.jpeg)

We consider here a computation whose interactions with the environment are read and writes to a state

![](_page_21_Figure_1.jpeg)

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![](_page_22_Figure_1.jpeg)

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![](_page_23_Figure_1.jpeg)

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![](_page_24_Figure_1.jpeg)

We consider here a computation whose interactions with the environment are read and writes to a state

![](_page_25_Figure_0.jpeg)

#### Events only specify the type of effects

![](_page_25_Figure_2.jpeg)

![](_page_25_Picture_4.jpeg)

![](_page_26_Figure_0.jpeg)

#### Events only specify the type of effects

![](_page_26_Figure_2.jpeg)

The semantics of effects is introduced

![](_page_27_Figure_0.jpeg)

![](_page_28_Figure_0.jpeg)

#### Write down the syntax $\mathscr{L}$ of your language 1.

- 1. Write down the syntax  $\mathscr{L}$  of your language
- 2. Inventory the effects of your language and write the corresponding event interface  $\mathscr{E}$

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## Compositionality

- Write down the syntax  $\mathscr{L}$  of your language
- Inventory the effects of your language and 2. write the corresponding event interface  $\mathscr{E}$
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- 4. Handle  $\mathscr{E}$  into an appropriate monad  $\mathscr{M}$ , get an *interpreter* for whole programs for free

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Compositionality

Modularity

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- As a bonus, extract the result to OCaml to 5. get a definitional interpreter

Compositionality

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Compositionality

Executability

# Scaling to a Fully Fledged Language

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![](_page_38_Picture_7.jpeg)

# Event interface for an IR program $E = L_E + G_E + M_E$ $+'Call_{E} +'Intrinsics_{E}$ $+'Pick_E +'UB_E$ $+'Debug_E +'Failure_E$

![](_page_38_Picture_12.jpeg)

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<u>Compositional representation</u> for (open) IR programs

![](_page_39_Picture_13.jpeg)

![](_page_39_Figure_14.jpeg)

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## <u>Compositional representation</u> <u>for (open) IR programs</u>

![](_page_40_Picture_13.jpeg)

![](_page_40_Figure_14.jpeg)

![](_page_40_Figure_15.jpeg)

## Level 0

### VIR

structural representation

itree VellvmE  $\mathcal{V}_u$ 

Level 0	itree V
Level I	itre

*structural representation* 

'ellvmE  $\mathcal{V}_u$ 

## ↓ intrinsics

e  $E_0 \mathcal{V}_u$ 

Level 0	itree V
Level I	itre

structural representation

'ellvmE  $V_u$ 

#### ↓ intrinsics

e  $E_0 \mathcal{V}_u$ 

## Pieces of state get introduce

![](_page_44_Picture_9.jpeg)

Level 0	itree V
Level I	itre
Level 2	$stateT_{Env_G}$

structural representation

'ellvmE  $V_u$ 

#### **↓** *intrinsics*

e  $E_0 \mathcal{V}_u$ 

J global environment

(itree  $E_1$ )  $\mathcal{V}_u$ 

Pieces of state get introduce

![](_page_45_Picture_10.jpeg)

	structural representation
Level 0	itree VellvmE $\mathcal{V}_u$
	intrinsics
Level I	itree $E_0 \mathcal{V}_u$
	🖌 global environment
Level 2	stateT $_{Env_G}$ (itree $E_1$ ) $\mathcal{V}_u$
	Iocal environment
Level 3	stateT $_{Env_L*Env_G}$ (itree $E_2$ ) $\mathcal{V}_u$

# Pieces of state get introduce

![](_page_46_Picture_4.jpeg)

Level 0	itree V
Level I	itre
Level 2	$stateT_{Env_G}$
Level 3	stateT $Env_L * E$
Level 4	<pre>stateT Mem * Env<sub>L</sub> *</pre>

![](_page_47_Figure_3.jpeg)

![](_page_48_Figure_1.jpeg)

![](_page_49_Figure_1.jpeg)

![](_page_49_Figure_3.jpeg)

![](_page_50_Figure_1.jpeg)

structural representation

# But How Does it Relate to LLVM IR?

# You can play with it yourself!

```
define i64 @factorial(i64 %n) {
 %1 = alloca i64
 %acc = alloca i64
 store i64 %n, i64* %1
 store i64 1, i64* %acc
 br label %start
start:
 %2 = load i64, i64* %1
 %3 = icmp sgt i64 %2, 0
 br i1 %3, label %then, label %end
then:
 %4 = load i64, i64* %acc
 %5 = load i64, i64* %1
 %6 = mul i64 %4, %5
 store i64 %6, i64* %acc
 %7 = load i64, i64* %1
 %8 = sub i64 %7, 1
 store i64 %8, i64* %1
 br label %start
end:
 %9 = load i64, i64* %acc
 ret i64 %9
define i64 @main(i64 %argc, i8** %arcv) {
 %1 = alloca i64
 store i64 0, i64* %1
 %2 = call i64 @factorial(i64 5)
 ret i64 %2
```

![](_page_52_Picture_2.jpeg)

![](_page_52_Picture_3.jpeg)

Tiny OCaml driver to crawl the tree

External calls Debugging messages Failure

![](_page_52_Picture_7.jpeg)

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![](_page_53_Picture_2.jpeg)

![](_page_53_Picture_3.jpeg)

\* See the paper for the details of the features we cover

![](_page_53_Picture_6.jpeg)

![](_page_53_Picture_7.jpeg)

![](_page_53_Picture_8.jpeg)

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  ret i64 %2
```

![](_page_54_Picture_2.jpeg)

## Tested against *clang* over:

- A collection of unit tests
- A handful of significant programs from the HELIX project
- Early experiments over randomly generated programs using QuickChick \* See the paper for the details of the features we cover

### Realistic\* (sequential) subset, happy to take feature requests!

![](_page_54_Picture_10.jpeg)

![](_page_54_Picture_11.jpeg)

![](_page_54_Picture_12.jpeg)

# But Why Would it Be Any Useful?

# A (weak) bisimulation over itrees

## Coinductive relation ignoring finite amounts of internal steps

![](_page_56_Picture_2.jpeg)

## Get us a first (fine) notion of equivalent programs

![](_page_56_Figure_4.jpeg)

![](_page_56_Picture_5.jpeg)

# Structural Equational Theory and Compositional Reasoning

A battery of structural equational lemmas at the VIR level

![](_page_57_Picture_2.jpeg)

# **Structural Equational Theory and Compositional Reasoning**

A battery of structural equational lemmas at the VIR level

Reasoning about control-flow graph composition

*to*  $\notin$  **inputs**(*cfg*<sub>1</sub>) outputs( $cfg_2$ )  $\cap$  inputs( $cfg_1$ ) = Ø

![](_page_58_Figure_5.jpeg)

![](_page_58_Picture_6.jpeg)

 $\llbracket cfg_1 \cup cfg_2 \rrbracket (f, to) \approx \llbracket cfg_2 \rrbracket (f, to)$ 

# **Structural Equational Theory and Compositional Reasoning**

A battery of structural equational lemmas at the VIR level

Reasoning about control-flow graph composition

outputs( $cfg_2$ )  $\cap$  inputs( $cfg_1$ ) = Ø *to*  $\notin$  **inputs**(*cfg*<sub>1</sub>)

Proof of a simple block-fusion optimization

![](_page_59_Figure_6.jpeg)

![](_page_59_Picture_7.jpeg)

 $\llbracket cfg_1 \cup cfg_2 \rrbracket (f, to) \approx \llbracket cfg_2 \rrbracket (f, to)$ 

				VI	R
				- - - -	struci
Level 0		-	itree	Vel	lvmE
				↓	intrin
Level I			itr	ree	$E_0 \mathcal{V}_u$
				. ↓	globa
Level 2		sta	ateT <sub>En</sub>	$v_G$ (i	tree
		Γ		↓	local
Level 3		stat	eT Env	*Env <sub>G</sub>	(itı
					memo
Level 4		stateT	Mem * Er	$v_L * En$	$v_G$ (i
····					prop
	state	T Mem * Env <sub>L</sub>	$*Env_G$ (	itre	e $E_4$
		mode	el unde	$f_{\tau}$	
	state	T Mem * Env <sub>L</sub>	$*Env_G$ (	itre	e $E_5$

![](_page_60_Figure_1.jpeg)

 $\sim^0_R$ 

#### ositional model

![](_page_60_Figure_3.jpeg)

				VI	R
					struci
Level 0		-	itree	Vel	lvmE
				↓	intrin
Level I			itr	ree	$E_0 \mathcal{V}_u$
				. ↓	globa
Level 2		sta	ateT <sub>En</sub>	$v_G$ (i	tree
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····					prop
	state	$T Mem * Env_L$	$*Env_G$ (	itre	e $E_4$
		mode	el unde	$f_{\tau}$	
	state	T Mem * Env <sub>L</sub>	$*Env_G$ (	itre	e $E_5$

![](_page_61_Figure_1.jpeg)

#### ositional model

![](_page_61_Figure_3.jpeg)

				VI	R
				· · ·	struci
Level 0		-	itree	Vel	lvmE
				↓	intrin
Level I			itr	ree	$E_0 \mathcal{V}_u$
				. ↓	globa
Level 2		sta	ateT <sub>En</sub>	$v_G$ (i	tree
		Γ		↓	local
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					memo
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····					prop
	state	$T Mem * Env_L$	$*Env_G$ (	itre	e $E_4$
		mode	el unde	$f_{\tau}$	
	state	T Mem * Env <sub>L</sub>	$*Env_G$ (	itre	e $E_5$

![](_page_62_Figure_1.jpeg)

				VI	R
				- - - -	struci
Level 0		-	itree	Vel	lvmE
				↓	intrin
Level I			itr	ree	$E_0 \mathcal{V}_u$
				. ↓	globa
Level 2		sta	ateT <sub>En</sub>	$v_G$ (i	tree
		Γ		↓	local
Level 3		stat	eT Env	*Env <sub>G</sub>	(itı
					memo
Level 4		stateT	Mem * Er	$v_L * En$	$v_G$ (i
····					prop
	state	T Mem * Env <sub>L</sub>	$*Env_G$ (	itre	e $E_4$
		mode	el unde	$f_{\tau}$	
	state	T Mem * Env <sub>L</sub>	$*Env_G$ (	itre	e $E_5$

![](_page_63_Figure_1.jpeg)

## Instruction level reasoning

To reason about instructions, we could get back down to comparing trees

[%x = load i64, i64\* %acc]<sub>instr</sub>

![](_page_64_Picture_3.jpeg)

![](_page_64_Figure_4.jpeg)

48

## Instruction level reasoning

To reason about instructions, we could get back down to comparing trees

 $[\%x = load i64, i64* \%acc]_{instr}$ 

Instead, we reason at the level of VIR through a battery of lemmas for each expression and instruction

 $[\%x = load i64, i64^* \%acc]_{instr} g l m \approx Ret (m, (Maps.add x uv l', (g, tt)))$ 

Representation functions can be made completely opaque

![](_page_65_Picture_8.jpeg)

![](_page_65_Figure_9.jpeg)

- $[acc]_{expr} g | m \approx Ret (m, (l', (g, tt)))$ 
  - read m a i64 = inr uv

## Two main reasoning ingredients

## Strong equivalences at the VIR level over:

- the syntactic structure of the language
- the control flow
- the instructions, expressions and terminators.

Symbolic interpreter that can be run by rewriting during refinement proofs

![](_page_66_Picture_6.jpeg)

## Two main reasoning ingredients

## Strong equivalences at the VIR level over:

- the syntactic structure of the language
- the control flow
- the instructions, expressions and terminators.

Symbolic interpreter that can be run by rewriting during refinement proofs

## A primitive relational program logic:

- Weakening, conjunction, ... over the postcondition
- Sequential composition

Compositional construction of refinement proofs

![](_page_67_Picture_10.jpeg)

# **SPIRAL/HELIX**

# [Püschel, et al. 2005] [Franchetti et al., 2005, 2018] [Zaliva et al., 2015 2018, 2019]

![](_page_68_Figure_2.jpeg)

DSL for high-performance numerical computing.

\* Some operators are currently not proved

![](_page_68_Figure_10.jpeg)

## **Vellvm is Back!**

![](_page_69_Picture_1.jpeg)

A Coq formal semantics for a large fragment of LLVM IR coming with:

- a certified interpreter
- promising modularity
- a rich equational theory
- an equational style to refinement proofs

## A fertile ground is laid!

Verified analyses Verified optimizations

Concurrency

Back-ends

![](_page_69_Picture_12.jpeg)

![](_page_69_Picture_14.jpeg)